Chapter 4
Extending the Reach of the Models and Modelling Perspective: A Course-Sized Research Site

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Abstract For over 30 years, researchers have engaged in inquiry within the Models and Modelling Perspective (MMP), taking as a fundamental principle that learners' ideas develop in coherent conceptual systems called *models*. Under appropriate conditions, such as in Model Eliciting Activities (MEAs), this research has shown how learners' models can grow through rapid cycles of development toward solutions involving creative mathematics. These externalized models, and other *thought-revealing artifacts,* can become rich objects for reflection by learners, for formative assessment by teachers, and for analysis of idea-development by researchers. This chapter describes a new research effort to expand the reach of this MMP tradition, engaging questions about the interconnected models and modelling processes of students and teachers at larger, course-length scales.

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G.A. Stillman et al. (eds.), *Mathematical Modelling in Education Research and Practice*, International Perspectives on the Teaching and Learning of Mathematical Modelling, DOI 10.1007/978-3-319-18272-8_4

4.1 Introduction

In this chapter we discuss ongoing work that is intended to carry forward the agenda of the Models and Modelling Perspective (MMP). We are in the process of assembling and testing a web-based research repository, dedicated to the creation and refinement of a suite of research tools to study the interacting and continually evolving modelling processes of students and teachers, in the context of a course-sized collection of curricular materials dealing with Quantification and Data Modelling. Our motivation and rationale for building this site resonates strongly with currents in the international research community, as seen at ICTMA 16. The site builds on the foundation of MMP research, which in turn was shaped by the perspectives of the American Pragmatists, as well as both cognitive and sociocultural constructivist perspectives. With it, we hope to support the kinds of extended inquiry and collaboration needed to broaden the reach of the MMP community.

We begin our account by reviewing the framing research questions of the MMP tradition. We then describe how these questions connect with claims about the nature of knowledge, and we show how MMP researchers have developed research tools to produce data and evidence to answer these questions. Next, we focus in on our ongoing research, describing the assumptions and conjectures that underlie our development of a course-sized research site to carry forward these lines of inquiry. As with prior MMP research, we show how our work will create and assemble new research tools to generate evidence for the new kinds of question we are asking. In particular, we suggest how it will support investigations into the various relations and interactions between pairs of constructs not always recognized as interdependent, such as: (a) student development and teacher development; (b) conceptual knowledge and procedural knowledge, facts, and skills; and (c) learning and assessment (both formative and summative).

4.2 Research Questions Addressed by the MMP

As a program of research, the MMP was developed explicitly to investigate the following kinds of questions about learning:

- How can we characterize realistic problem-solving situations where solutions demand elementary-but-powerful mathematical constructs and conceptual systems?
- What kinds of "mathematical thinking" are emphasized in such situations?
- What does it mean to "understand" the most important of these ideas and abilities?
- How do such competencies develop, and what can be done to facilitate their development?

- How can we document and assess the most important (deeper, higher-order, more powerful) conceptual achievements that are needed for full participation as citizens in increasingly complex societies and professions? and
- How can we identify students who have exceptional potential that is not adequately measured by standardized tests?

These questions are tightly connected with the portrayal of knowing and learning that has been developed through MMP research. Here, the MMP builds on perspectives originating with Piaget and Vygotsky as well as with the American Pragmatists (c.f., English et al. 2008; Lesh and Doerr 2003). In particular, Mousoulides et al. (2008) summarized key elements of the MMP debt to Dewey, Mead, James and Peirce:

- Conceptual systems are human constructs, and thus also are fundamentally social in nature (Dewey and Mead).
- The "worlds of experience" that humans strive to understand and explain are rarely static. They are most often products of human creativity, continually changing in response to the evolving needs of humans who create and re-create them (James).
- Meaning, in this setting, tends to be distributed across a variety of representational media (ranging from spoken language to written language, to diagrams and graphs, to concrete models, to experience-based metaphors). Each of these representational forms foregrounds some facets of experience and backgrounds others (Peirce).
- Knowledge is organized around concrete experiences at least as much as around abstractions. The ways of thinking needed to make sense of realistically complex situations nearly always must integrate ideas from more than a single discipline, textbook topic area, or theory (Dewey).
- In a world filled with technological tools for expressing and communicating ideas, it is naïve to suppose that all "thinking" goes on inside the minds of isolated individuals (Dewey).

4.3 Claims About the Nature of Knowing and Learning

MMP research, rooted in these perspectives and pursuing questions such as the ones listed above, has illuminated the nature of knowing and learning in authentic problem-solving settings. A key feature of such settings is that they challenge learners to engage in original mathematical work (i.e., to produce mathematics that is *new to them*), rather than merely applying mathematics learned from an authoritative source. There are many dimensions to the image of knowledge that has emerged from this research; in this section we indicate three such dimensions that have been influential in guiding inquiry into both teacher and student knowledge. Then, in the following section we describe how the MMP develops and refines research tools to operationalize these dimensions of knowledge, permitting them to be externally expressed in *thought-revealing artifacts* created by the learners themselves.

Dimension 1 Practical knowledge is understood as an *interpretation system* for making sense of phenomena. In particular, this means that in realistic problem settings, experts distinguish themselves from non-experts not only by what they *do* but also by what they *see* in such situations. Moreover, learners' concrete past experiences and accumulated models also serve as *lenses* through which they can view and interpret new situations. Adopting this view also suggests that many aspects of learner's knowledge will be tacit and instinctive. Learners may be able to use these knowledge resources to guide actions before they can subject them to analysis as hierarchical, logical structures of rules and procedures. That said, interpretation systems have the property that they are *both* structures for action *and* structures that can be reflected upon. That is, knowledge as models and interpretation systems can serve as (a) windows or lenses which one can *look through* to view the world, or (b) objects in themselves which one can *look at* and analyze.

Dimension 2 Knowledge is constituted as much by *connections* forged by the learner among big ideas of the domain and between these ideas and prototype situations, as by an 'intrinsic' understanding of the big ideas themselves. Adopting this view also suggests that many aspects of knowledge may be highly situated, multiply-determined, and bound up with particular concrete experiences. The process of learning may therefore be expected to be multi-dimensional and non-linear, in spite of attempts to rationalize the *teaching* of material in the form of logical, linear sequences. Thus, in articulating learning goals, a distinction must be made between (a) lists of names of topics that should be emphasized in teaching, (b) operational definitions of what it means for students to have learned these big ideas, and (c) over-time accounts of students' growing appreciation of the significance and interrelatedness of these big ideas (see Learning Progress Maps, below).

Dimension 3 As discussed above, important kinds of knowledge are characterized by the ability to see situations in a certain way, or by having the skill or competency to act in a certain manner. Mastery of this kind of knowledge involves knowing *when* and *where* to apply it, as one conceptual tool among a repertoire. Adopting this perspective leads to the practical construct of *problem-solving personae*: stances or roles adopted by problem-solving individuals and groups, in response to the situation at hand. Expertise in this dimension involves recognizing that no single behavior, technique, or heuristic is valuable independent of context. Moreover, research into these personae increasingly suggests that while they may have a logical or technical core, they also involve "soft" aspects of knowledge, including attitudes, feelings, and beliefs (both about oneself and the domain).

4.4 Research Tools and the Data Generated by Inquiry Within the MMP

For each facet of what it means to know and understand, MMP researchers seek means to operationalize that dimension of knowing. Their history of success in doing this has supported the conviction that to conceptualize something is to be able to externalize it in thought-revealing artifacts, given an appropriate research setting to do so. Thus, for each dimension of knowledge we seek to study, MMP researchers produce and refine tools to facilitate the generation of a relevant type of research data and evidence. In this section we illustrate this with reference to the three facets of knowing discussed above.

Dimension 1: Knowledge as Interpretation Systems Increasing awareness of the complexity and diversity of learners' emerging knowledge in this dimension has been a principal driver of the genre of research activities known as Model-Eliciting Activities (MEAs). Research has shown that learners can externalize their local, situated systems of sense-making (which the MMP calls their *models*) given compelling problem settings and the means to represent these systems. Each element of the design of MEAs—both the activities themselves and their implementation in classroom settings—is driven by the goal of optimizing the processes of idea development and improving our view into the models and modelling processes of learners as they emerge in time.

Dimension 2: Making Connections Here we investigate students' work in constructing connections among the big ideas of a domain, as well as teachers' sense of inherent connections, and teachers' sense of the connections that their students are actually making. Researching this dimension of knowledge implies additional design criteria for MEAs (in particular, that these occasions for authentic mathematical production are also sufficiently *open* to permit different learners to incorporate different combinations of big ideas in productive ways). In addition, a focus on connections has also provoked the development of a series of specific research tools. Learning Progress Maps support learners and teachers in recognizing and reporting connections that are made between ideas at different stages in model development. Concept Analysis Wheels support and document emergent thinking about the organization of the discipline as a whole and the relations among its structures.

Dimension 3: Building and Using Problem-Solving Personae To study the broad spectrum of problem-solving behaviors and their deployment as situations shift over time, MMP researchers have developed a family of Reflection Tools (RTs). For instance, RTs have been created to study shifting roles in group interactions; changes in the affective dimension of "flow" (Csikszentmihalyi 1997) in the course of problem-solving; shifts in groups' self-assessment of their progress; and so forth. For each behavior or experiential aspect, a separate research tool is developed to support learners in the generation of thought-revealing artifacts focused on that aspect. The wide range of these tools reflects the complexity of problem-solving personae that have emerged in the context of MEAs.

4.5 Extending the Questions; Expanding the Toolkit

Building on the successes of prior MMP research, we aim to study new levels and dimensions of the models and modelling processes of students and teachers. For example,

- How can learners' models best be extended and expanded in classroom settings where multiple problem solving teams have worked in parallel?
- How can learners connect their models and a domain's big ideas with procedural skills and with a familiarity and facility with tools of the domain?
- How do knowledge and models develop and mature at larger time scales?

While it has been possible to pursue many of the research questions discussed earlier in this chapter through studies involving a single MEA or a sequence of several of them, questions like the ones above require new design structures that can be deployed over a course-sized experience. In particular, these questions require attention to students' and teachers' ways of unpacking the models created in MEAs, incorporating them into normal classroom discourse; integrating them into shared canonical ways of thinking and generating mathematical interpretations; and linking these models and big ideas with a host of techniques, tools, and procedural skills. These activities have not yet been a focus of the majority of MMP research. Studying models and modelling at these scales also demands the development of a series of new research tools. These will be based on our existing assumptions and conjectures but also designed to produce evidence that will test and revise or refine those assumptions and conjectures and support iterative cycles of development—that is, modelling at the researcher level.

4.6 Some Assumptions and Conjectures

In this section, we outline some of the assumptions and conjectures that form a basis for our initial development of a course-sized research site. We focus on a subset that are most relevant to currents in the international research discourse. In particular, we describe our relations to research on (a) learning progressions, and (b) explicitly teaching problem solving and heuristics.

4.6.1 Learning Progressions

Inquiring into the organization of knowledge and learning experiences as they unfold in time, many researchers in the international community have engaged the notion of a *learning progression*. In many manifestations of this approach, the objective is to identify a *trajectory* among the big ideas of a discipline. This may be grounded in mathematical structure, historical development, and/or studies of connections among ideas that are resonant with conceptions that have been identified in learners before instruction has occurred. These documented connections are then used to rationalize a fixed, linear order in which the big ideas are presented.

Our conjecture is that while this notion may be relevant for the study of *teaching* or instructional planning, it does not adequately address the question of possible productive connections among ideas from the viewpoint of the *learner*. For example, in prior work with MEAs in the context of courses focused on data modelling, we have found that the big ideas of a course can be productively connected in a variety of possible ways, and that connections develop in multiple dimensions simultaneously. Moreover, the earliest emergence of big ideas often occurs in modelling solutions, where learners formulate embryonic concepts and procedures by assembling ideas from a variety of textbook topic areas. Thus, facilitating model development may demand a responsive instructional approach, which unpacks these partially-formed insights and ideas, sorting them out and supporting more explicit connections among knowledge elements. In particular, this leads us to the conjecture that there may not be a single, a priori way to establish these connections for all learners independent of their idiosyncratic and situated modelling work.

Our approach on this matter is critical to designing our course-sized research site, affecting the very metaphors we have for learning at larger time scales. In particular, we associate the learning-progression perspective with a *ballistic* model of learning, in which a teach-first, apply-later philosophy is applied to propel classroom learners through course materials, at least in its initial exposure to the big ideas of the curriculum. In contrast, we explore the viability of alternatives to a ballistic model, which we unpack below.

4.6.1.1 Alternative Model #1: Learning as Finding One's Way Around in a Terrain

Instead of envisioning concept development using a ballistic metaphor, where a single point moves along a path in space, we find it useful to substitute a metaphor where the discipline is conceptualized as a multi-dimensional terrain, and where big ideas are conceived as something akin to mountains in a topographic view (see also, Zawojewski et al. 2013). Within this metaphor, we have found that development often proceeds in ways that resemble the formation and passage of interacting weather systems over the region. This metaphor honors the value and specificity of each manner of moving through the domain while also recognizing that different routes and systems are possible, each with its own characteristic strengths and weaknesses. Facilitating classroom inquiry in which different students or groups are exploring different regions in the disciplinary topography, however, requires the support of new instructional principles as well as additional tools and structures. If successful, the classroom group's exploration of a course's terrain would yield a range of possible "views" of the topographically high points (the big ideas), constructing powerful personal models that illustrate connections among them

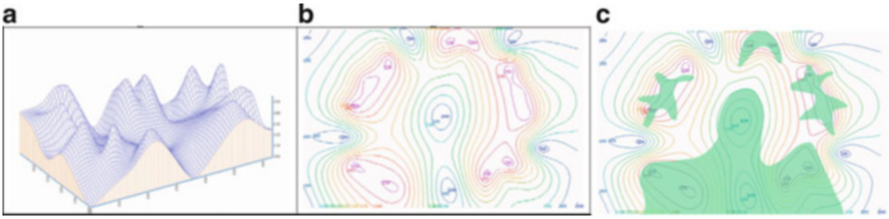

Fig. 4.1 The ideas of a course conceived as a terrain (**a**) (**b**) (c.f., Zawojewski et al. 2013, p. 492). Mapping progress (**c**) (c.f., Zawojewski et al. 2013, p. 498)

and make use of different forms of procedural and conceptual knowledge. Learning Progress Maps (prototypes shown in Fig. 4.1) are intended to support this instructional work and to study the utility of the terrain metaphor.

4.6.1.2 Alternative Model #2: An Evolutionary Model for the Development of Ideas

Prior MMP research also suggests another alternative to the ballistic model: one rooted in evolutionary theory. This metaphor highlights the importance of the *diversity* of ideas within problem-solving groups; of conditions for these diverse ideas to be placed in *communication* with each other; of an immanent mechanism for the *selection* of some ideas over others to be pursued; and of a means for the *survival* and *accumulation* of changes in ideas over iterative cycles. These four factors—diversity, communication, selection, and accumulation—are critical elements of nearly all evolutionary processes that involve living organisms in complex ecosystems. Hence in this view, the classroom is seen as an ecology in which a diversity of ideas evolves, by analogy with the diversity of species in a natural ecosystem. The classroom group is analogous to a team of naturalists, understanding each of these idea-species both in terms of their history and in terms of the relations with the conceptual environment. Like the terrain metaphor, this evolutionary metaphor honors the organic and nonlinear development of knowledge that the MMP has illuminated.

4.6.2 Teaching Problem Solving and Heuristics

Another current in international research involves revived interest in the explicit teaching of problem-solving strategies. Some attempts to teach heuristics or metacognitive strategies are rooted in work by Pólya (1945) and Schoenfeld (1985, 1987), among others. This approach holds that context-free instruction in heuristics may make the actual practices of problem solving more generally learnable. However, doing so by "cleaning up" *descriptive* accounts of real process and presenting them as *prescriptive* guides to follow, can be based on a conception of problem solving that sees the situated "messiness" of actual problems as incidental. Moreover, these problem-solving approaches based on Pólya's and Schoenfeld's work hold that instruction in abstract concepts should precede "application" of these concepts to solve particular problems. In contrast, MMP researchers view messiness and intuition as intrinsic elements of the modelling process. Problem-solving individuals and groups appropriate the ideas they are working with and make them effective tools for problem solving in idiosyncratic ways, relying on dimensions of knowledge and understanding that emerge *along with* their solutions (*because of* their problem-solving activity). This view is far from a situation in which students are taught to recognize mathematical structures in the "givens" of a problem and apply corresponding techniques.

How, then, does the MMP tradition answer the question of how to guide problem-solving practice as it unfolds? At their simplest, models can be seen as representations that capture what the modeller sees as the essence of the situation they are attempting to conceptualize. They can employ a variety of media (like stories, drawings, etc). As with other human attempts at representation, their development tends to involve an iterative series of drafts—some building on prior drafts, others exploring new directions, techniques, or emphases. Even though no single draft may be the "best" in all dimensions, it is still possible to compare drafts against one another, identify their strengths and weaknesses, and isolate the best parts of different drafts, *with reference to a given representational purpose*.

Thus, an answer to the question of guidance comes with the "Self-Evaluation Principle" of MEAs. Critical to the specification of MEAs is the *purpose* for which the model will be used by a client. Given such a description, learners themselves can identify the degree to which their current models are useful or powerful tools for those specific purposes. Thus usefulness and power, with reference to the specifications of the problem, enable learners themselves to assess and regulate their work, rather than appealing to the authority of the teacher or to guidelines in a decontextualized model of problem-solving behavior.

We should note in passing that we believe that the kind of situated knowing described above is also a feature of effective decision making in *teaching* practice. This has strong implications for our models of professional development. A view like ours, which emphasizes contextual understanding, will favor models of teacher professional development organized as *in situ* reflections on practice. A core element in the design of our course-sized research site must therefore be to offer the means for teachers to engage in "inline" modelling of their own students' learning processes and to become "connoisseurs of student work," sharing these emerging skills and sensitivities with colleagues in a low-overhead manner and with an eye to their usefulness and power in actual teaching settings.

4.7 Implications for Design

In recent years, MMP research has helped to lay the foundation for the inquiry we are currently undertaking. For instance, various efforts to assemble MEAs into larger, coherent instructional unities have led to the emergence of Model Development Sequences (MDSs), which support instruction that makes the most of MEAs in practical classroom contexts. In particular, a given MDS may include the following, in addition to one or more MEAs:

- Reflection Tool Activities, in which student groups turn their attention to describing individual and group level processes, functions, roles, conceptions, and beliefs. These include Ways of Thinking Sheets, various surveys and questionnaires, Concept Maps, Observation Sheets, Self-Reflection Guides, and Quality Assurance Guides for the products created in MEA activities.
- Product Classification Activities, in which students categorize the kinds of thinking involved in their solutions to MEAs.
- Model Extension Activities, often involving dynamic mathematics software, in which the class extends and formalizes promising elements of mathematical thinking that have appeared in student solutions.
- Model Adaptation Activities, where students generalize models, transferring them to related but different problems from those they originally were created to solve.

These MDS elements are designed to be highly modular, in order to accommodate (as well as to reveal) the needs and intentions of teachers that engage with them. One example of how these components might be laid out across multiple days in an instructional unit is shown in Fig. 4.2 (in this example, a 4-day sequence).

Importantly, each of the elements of the MDS acts as a research tool as well as an instructional component. At the research level, the data produced by these elements will serve to illuminate processes of learning and idea development that are critical to answering our research questions. Similarly, many of the other research tools described above (including Learning Progress Maps and Concept Analysis Wheels) will generate new research data when developed *iteratively* by students and teachers as the course unfolds.

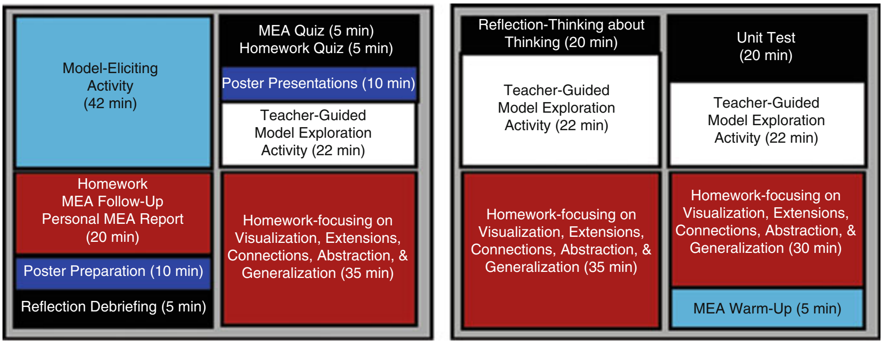

Fig. 4.2 A 4-day segment from a model development sequence

4.8 Conclusion: Contributions of a Course-Sized Research Site

In building our research site, we aim to illuminate new dimensions of knowing and learning, and we have already established initial versions of many of the research tools that will support this work. As suggested in the discussion of the design of MDS units, we are undertaking this work with a commitment to flexibility-in-use that we describe as *designing for scale*. Our course materials are presented in modularized and easily modifiable, reconfigurable, and extendable components.

Thus, teachers and researchers will be able to engage with these materials in a variety of ways, each embodying a unique (teacher or researcher level) model of knowledge development. The data and evidence produced by this diversity of models will help support choices amongst them, to refine our own assumptions and conjectures, and to iteratively shape our own understandings so that more useful and powerful conceptions survive and accumulate. From our description, it should be clear that we are conceiving this work as a modelling process for ourselves. We aim for our course-sized research site to provide value also to the broader teacher and researcher community,

- by facilitating the development, sharing, and testing of new Research Tools for different facets of research into dimensions of learning as they emerge;
- by offering a shared setting for the refinement of the *design principles* for research tools to produce evidence about learning at scales higher than the MEA;
- by fostering the accumulation of knowledge in teacher and researcher communities, exposing our process and inviting broad participation in constructing the site; and
- by encouraging the formation of collaborative communities of teachers and researchers, allowing participants to identify possible collaborators through shared interests in research tools and facets of problems of research or instruction.

This expansion of the MMP perspective builds on a long history of research success, whilst opening the way to have a new level of practical impact on classroom instruction.

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