Chapter 27
Mathematical Modellers' Opinions on Mathematical Modelling in Upper Secondary Education

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Abstract This chapter examines and discusses how professional mathematical modellers have learned about modelling as well as their opinions on the teaching of mathematical modelling in upper secondary education. An interview study showed that they developed most of their knowledge about mathematical modelling during their PhD studies and through their occupation by working with 'real modelling'. According to the interviewees mathematical modelling should be a part of mathematics education in upper secondary school, and in particular it should be more emphasized as a part of general education to develop students' critical awareness about how models are used in society. They also gave suggestions for approaches to teach modelling and examples of modelling problems to work with from their own workplace.

27.1 Introduction

Mathematical modelling is considered as a bridge between the mathematics learned and taught in schools and the mathematics used at the workplace (Sträßer et al. 2012). This view is also found in school mathematics curricula, in the section on the aim of the subject mathematics, as for example in Sweden where the subject syllabus for upper secondary school emphasizes, the use of mathematics in relation to workplace situations and to use investigating activities in an environment close to practice (Skolverket 2012). One such investigative activity is mathematical modelling, described as one of seven main teaching goals, so as to develop students' ability to "interpret a realistic situation and design a mathematical model, as well as use and assess a model's properties and limitations" (p. 2). These descriptions

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suggest the use of realistic modelling activities in mathematics classrooms with a relation to workplaces, at least if the modelling problem is chosen adequately.

However, even if the problems are chosen adequately there seems to be an accepted view among educational researchers in mathematics education that workplace mathematics is not identical to school mathematics. Workplace mathematics is more complex and strongly situation dependent. It also includes specific technologies, social, political and cultural dimensions that are not found in educational settings (e.g., Noss and Hoyles 1996; Wedege 2010). Mathematical modelling applied by different actors at the workplace also seems to be workplace specific and quite different from school situations (Mouwitz 2013). Frejd (2013) found that some professional modellers work in groups where the division of labour is specific and predefined (numerical analyst, meteorologist etc.) in contrast to school settings in Sweden where students spend much time on individual work with exercises from textbooks (Jablonka and Johansson 2010) with the goal to learn 'everything' about the modelling process. In addition, there are other aspects of mathematical modelling that appear in a limited way in Swedish mathematics classrooms, but are large parts of workplace practice, such as programming and that the consumer's (the company's) purpose of developing the model must be taken into consideration (Frejd 2013).

Assuming there is a gap between school mathematics and mathematics used in the workplace regarding mathematical modelling, it is of interest to chart and analyse how people who use mathematics in the workplace view mathematical modelling, especially professional mathematical modellers in different occupations. There is also a need to observe different types of modelling activities found in non-educational settings in search of potential links between the two practices. To this end, this chapter presents empirical research aiming to describe how professional mathematical modellers have learned mathematical modelling and their opinions on how it should be included in upper secondary school. The analysis has been guided by the following research questions:

- How do professional mathematical modellers describe their own learning of mathematical modelling?
- What opinions do professional mathematical modellers express in terms of goals of mathematics education in upper secondary school, goals of modelling in upper secondary school, suitable examples for use in secondary school, and mathematical modelling as a part of a general education?

Exploring and seeking answers to these research questions may contribute to development of new insights into pedagogy and curricula, links between school and the workplace, and how mathematical meanings are created in and out of school contexts as well as to informing curriculum developers and others about the role of modelling in the workplace.

27.2 Workplace Mathematics, Modelling and Modellers' Opinions

One goal of research on workplace mathematics is to explore how and what mathematics is used in specific professions (Noss and Hoyles 1996). Other goals concern the identification of similarities and discrepancies between what mathematics is needed in the workplace and what mathematics is taught in school (Triantafillou and Potari 2010), the analysis of communication between employers and visitors (Williams and Wake 2007), and the search for strategies that will improve a general curriculum that better prepares students for future work (Wake 2012).

One central part of workplace mathematics concerns the use of mathematical models and modelling. At the Educational Interfaces between Mathematics and Industry-ICMI study conference several research papers related to engineering and modelling were presented (see Damlamian et al. 2013). Other examples of research literature including modelling and the use of mathematical models in the workplace, focus on employers that make their working decisions based on mathematical models in technological artefacts, with input and output of numerical values, like bankers (Noss and Hoyles 1996), telecom technicians (Triantafillou and Potari 2010) and operators in a chemical plant (William and Wake 2007). A common finding in these studies is that the underlying mathematical structure of the models used is not considered. Despite the facts the mathematical models sometimes are hidden in technology and the linguistic conventions of representing mathematical models used in the workplace (formula, graph, table) (Triantafillou and Potari 2010; Williams and Wake 2007) differ from those in mathematics education, it is argued that mathematical models together with metaphors and gestures facilitate communication of mathematics between workers and clients (Williams and Wake 2007). One of the "principles for strategic curriculum design" that support workplace mathematics (Wake 2012, p. 1686) emphasises communication about development and validation of mathematical models. Other principles suggested by Wake (2012) are: to take mathematics in practice into account; facilitate activities that pay attention to technology; and, to let students critique mathematics used by others.

There are researchers who discuss implications for education based on results from their own empirical research (observations and interviews) of the working practice of professional modellers (e.g., Drakes 2012; Gainsburg 2003). However, not much research is explicitly focused on modellers' opinions on mathematical modelling in secondary mathematics education. Previous research has shown that modellers in the Netherlands are sceptical of the use of 'messy' modelling problems in secondary education (Spandaw 2011). They argued that modelling is too complex and time consuming for students and that modelling projects are too complicated for teachers to supervise. Instead the aim of secondary education should be to teach basic skills in mathematics (i.e., algebra and analysis).

27.3 Methodology

Research within the field of workplace mathematics should consider two closely linked approaches, a *general approach* and a *subjective approach* (Wedege 2010). A *general approach* focuses on (general) demands from the labour market and the society for "formal" (school) mathematical competencies needed in a workplace, whereas a *subjective approach* focuses on the workers' abilities and their (subjective) needs in their specific workplace. Salling Olesen (2008) addressed both these approaches in a heuristic theoretical model, which is suggested as a helpful research tool for investigating the dynamics of workplace learning.

The model in Fig. 27.1 illustrates a relation between the three components *the societal work process* (division of labour, type of tasks and work organisation), *the knowledge available* (discipline, craft, methods and skills used in a workplace), and *the subjective working experiences* (individual/collective life history and their subjectivities like values, norms, emotions, etc. that appear to be profession specific). In the centre of Fig. 27.1 the words learning, experiences, practices, identifications and defences illustrate "that learning in the workplace occurs in a specific interplay of experiences and practices, identification and defensive responses" (Salling Olesen 2008, p. 118). "A teacher in mathematics may say *modelling activities are time consuming* (learned by experience), *but we don't have the time we need the classroom time to prepare for the final tests* (learned through practice) and *in mathematics we use the notation y*" (learned through identification), *but in*

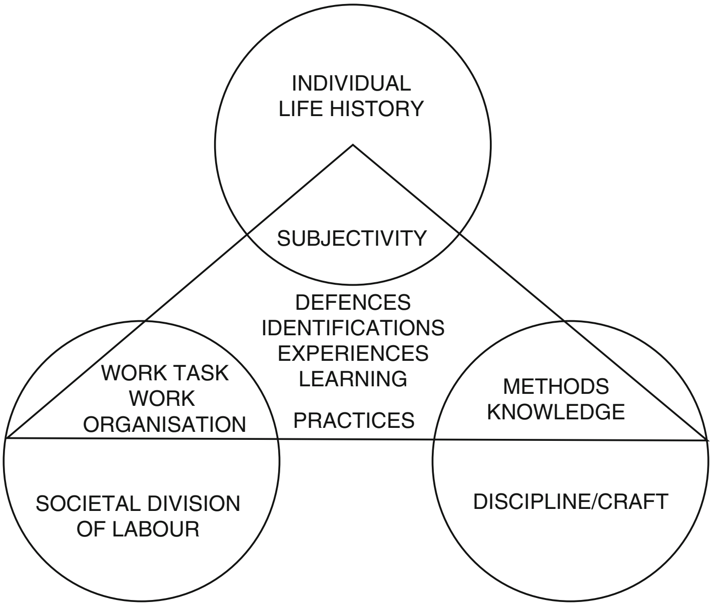

Fig. 27.1 A model to analyse workplace learning (Salling Olesen 2008, p. 119)

physics we use ÿ (learned through a defensive response). The suggested theoretical model is helpful for this chapter, since "the model pays particular attention to the cultural nature of the knowledge and skills with which a worker approaches a work task, whether they come from a scientific discipline, a craft, or just as the established knowledge in the field" (p. 118). The three components (*the societal work process, the knowledge available* and *the subjective working experiences*) may indicate the origin of the given opinions on how modelling should be taught and learned.

In this research project semi-structured interview questions were developed to pay attention to Salling Olesen's (2008) model (for further details see Frejd 2013). The interview questions addressing the research questions are:

1. Is mathematical modelling something that was a part of your education in school or something you learned in your vocation? How?
2. What are the goals of mathematics education in upper secondary school?
3. Is mathematical modelling something that should be brought up in mathematics education at upper secondary school? (if yes) How?
4. Do you have examples from your own practice that may be suitable to use in secondary education?
5. Modelling as a part of a general education (personal finance, global warming, etc.), should that be more emphasized in school and should that be a part of mathematics education?[1]

Nine mathematical modellers in different areas of expertise were invited and accepted to participate in this study. The following abbreviations are used in this chapter when referring to the modellers: climate modeller [CLI], military defence modeller [MD], modeller in physics [PHY], finance modeller [FIN], insurance modeller [INS], construction engineering modeller [CE], traffic simulations modeller [TS], biology modeller [BIO], and scheduling modeller [SCH]. Participants were a convenience sample as two persons were previously known to the authors, four others were either recommended by colleagues or interview participants to be invited and three were found on a web search. The interviews that were conducted and audio taped lasted from 40 min up to 90 min and were later transcribed, summarized and analysed based on the five questions together with the three components of Salling Olesen (2008).

All of the participants have at least a PhD degree in either mathematics, financial economics, physics, biology, or technology and all participants claimed during the interviews that mathematical modelling plays a major role in their profession. The participants' workplace problems were initially derived from reality with the aim to describe, predict, explain or create physical or social reality. However, they did not present coherent descriptions of what modelling means and how they work with modelling.

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[1] The last part of question 5 was added after the three first interviews.

27.4 Results and Analysis

The following sections (Sects. 27.4.1, 27.4.2, 27.4.3, 27.4.4, and 27.4.5) address the research questions in relation to the results and analysis of interview questions 1–5.

27.4.1 Source of Learning of Mathematical Modelling

Four of the participants indicated that they studied university courses only in pure mathematics and no explicit courses in mathematical modelling. Three of the participants had some formal teaching in modelling in either a course in applied mathematics, a set of seminars dealing with problems from industry, or in courses that included some aspects of modelling like simulations with Matlab. The other two participants indicated that they had experienced a more extensive teaching in modelling, referring to courses in optimization and courses on how to solve differential equations related to realistic tasks. All participants were insistent they had learnt mathematical modelling mainly during their PhD studies and in their occupation, by working with ‘real modelling’ problems either through schooling by supervisors or by working individually or in a group. Some participants also claimed that they had learned about modelling by reading scientific papers and books. Opinions expressed were that modelling problems in education and in the workplace and in research are quite different, where problems in education often are restricted and limited, whilst the problems in workplaces are more complex. The following excerpts from interviews illustrate that workplace learning of modelling (*the societal work process*) is the fundamental part in the experts’ opinions on learning modelling due to the nature of their working problems:

> When you get to the real problems [in research and in the workplace], not the idealised and clarified problems that can be solved, then you need modelling. (PHY)
>
> When you enter the workplace you “encounter problems that you never have encountered before, you will not simulate three processes rather 5,000”. (INS)
>
> In teaching modelling the problems are clarified, ‘they must be solvable within a reasonable time’ and you have the data, but in reality they are more complex. (TS)

The participants’ opinions on the extent to which modelling was a part of their own education may originate from Olesen’s (2008) three components *the societal work process, the knowledge available* and *the subjective working experiences*. For the climate modeller, for instance, *subjective working experiences* are mainly based on working in teams where his particular task is to solve well-defined problems (e.g., solve differential equations), *societal work process,* and *the knowledge available* or the skills needed are methods for solving PDEs, also emphasised as a skill he required during his university studies.

27.4.2 Goals of Mathematics Education in Upper Secondary School

The modellers' opinions of the goals of mathematics education in secondary education seem to be mainly *subjective*. Examples of opinions are: to gain knowledge to handle everyday situations (INS); to train mathematical skills (CE); to get a tool or a language to apply in different situations (BIO); to understand the role of mathematics in society and as a part of our cultural heritage (FIN); to learn logical thinking (SCH); and knowledge for further studies (TS). In addition, the goal of mathematics education to develop democratic citizens is addressed:

> we [citizens] will be deceived by politicians with different agendas if we don't know mathematics,..., everyone needs this [mathematics] or else our democracy will erode (PHY)

The opinions expressed may be found in school documents like syllabuses (e.g., Skolverket 2012). There was also a variety in what participants stressed as important to learn in terms of concepts versus processes. For example, one participant emphasized understanding:

> The possibility of understanding is more important than memorizing... understand why and how it [mathematics] is used is more important than to understand how to do it. (INS)

Another participant focused more on the skills, stating that

> The goal is to train a skill to be able to [make] unhindered use [of] things like standard functions,...you should know the laws of logarithms and the trigonometric identities, it is not something that you need to look up every time, if you forget you should be able to derive them. (CE)

27.4.3 Goals of Modelling in Upper Secondary School

According to six of the participants, modelling should be a part of secondary education, the other three being uncertain. A repeated argument for including modelling in upper secondary school is that modelling motivates students to learn and apply mathematics in different situations. A doubt expressed may relate to their working tasks, *societal work process*, which is that the students are not "mature [enough] and do not have practical experience" (MD) since "if it is going to be realistic it gets very complex" (INS). The participants gave a range of suggestions for how to include modelling in mathematics education that seem to be grounded in *subjective working experiences*. The most extreme suggestion was a paradigm shift in mathematics education, "instead of teaching mathematics I will teach mathematics as a tool to solve problems" (BIO). Other less extreme approaches, such as to explicitly express to the students that they actually use models, which now are implicit (SCH) or that the teacher may adapt the models based on students knowledge (CE). A suggestion of using interdisciplinary weekly assignments was put forward (FIN), which may include both general knowledge of how mathematics is used in society and more deepened practice on how to model. One idea that may be implemented in a classroom setting is for the students to work in small groups as described in the following excerpt (INS):

> [in these groups] the students can meet a practical problem, like a new pharmaceutical product has been developed and the question is can we use it? How can we design an experiment to evaluate that? [Let students] ask people, so they will meet the reality... I don't think that the problem always needs to be solved mathematically, they don't need to know all the mathematical models, but they need to reason about them. Then the teacher, with help of the students, presents a way to solve the problem with mathematics... and hopefully the students will realise that mathematics actually supports them.

The role of the teacher was described as an important factor for adequate teaching. The teacher "needs to be passionate about it [modelling], because it is nothing you do like this [snapping his fingers] you need to think carefully" (FIN). One way to make teachers more interested and gain inspiration is to let them listen to presentations given by modellers and researchers on how they work with modelling (TS). Another way is to change the problems in the textbooks (TS).

27.4.4 Examples from Practice Suitable for Use in Secondary School

Most of the participants' opinions (seven out of nine) were that it is relatively easy to adapt some modelling problems from their own *societal work process* to be used in secondary school. One suggestion for upper secondary students is to investigate the models used for score cards for loans, for which the following situation might be adaptable:

> If you go into a bank and apply for a loan, then they will collect information about you. For example your age, your income, where you live, you savings, your loans etc. There is loads of information that they put into a model, which the bank has developed, and the output is a p/d number. The p/d number is probability of default, which is the likelihood that you will be unable to meet its debt obligations and if that probability is not too high, compared to a set number, then you will not get the loan. But if you get the loan the p/d number [...] will effect the interest you have to pay. (FIN)

Other examples are: to explore the predator–prey relation with Excel (BIO); to analyse how temperature affects the magnetism of a piece of iron (PHY); work with 'realistic' linear optimisation problems and networks problems (SCH); estimate safety distances when cars are following each other on the freeway (TS); trying to identify factors that should be part of the development of pension funds and how a pension fund might be organised (INS).

27.4.5 Mathematical Modelling as a Part of a General Education

All participants indicated that modelling should be emphasized more in school as a part of general education. It is important to understand results, and be able to form an opinion and critically examine statistics to become a democratic citizen (CLI). This is expressed in the following interview excerpt where it is claimed that

> [Modelling] is the key response to the question, why do we need mathematics? If someone wants to understand and be a part of the society, affect society and take part in decisions made in everyday life, in family, at work, in society, in the world then you must know these things (PHY)

In a society decision-making often depends on economic considerations, which creates a need for awareness of mathematics and modelling (TS). The subjective opinions above of the climate modeller, physics modeller and traffic simulation modeller are examples of arguments for modelling being more emphasized in school.

Opinions about whether modelling as a general education should be a part of mathematics education or not seemed to be *subjective* and differed between the participants. Three participants expressed that modelling should be taught mainly in mathematics education.

> I definitely think that it [modelling] should be central part of mathematics education (INS).

The other participants suggested that modelling may be used also in relation to other subjects (FIN) like in physics, chemistry, biology and social science (CE) as well as in home economics (TS).

27.5 Discussion and Implications

To summarize, the modellers interviewed have mainly learned mathematical modelling during their doctoral studies and through their occupation, by working with 'real modelling' problems. The modellers have used different learning strategies either through guidance by supervisors or working in a group but a few were self-taught through personal reading. One (*societal*) reason for this not being through their education is the nature of their working problems – they are not the idealised and clarified problems like the ones they found in their education. The modellers' opinions on how modelling should be taught and learned differed. However, most of the modellers' (*subjective*) opinions are that mathematical modelling should be a part of mathematics education in upper secondary school. They gave examples of how to implement modelling in mathematics education and gave suggestions as to how their own workplace problems may be adaptable to be used in secondary education. All modellers agreed that mathematical modelling should be more emphasized as a part of general education to develop students' critical views on how models are used in society.

The results found in this study contradict to some extent those by Spandaw (2011), where a majority of 12 scientists interviewed in the Netherlands were sceptical about using modelling problems in secondary mathematics education. A few of the Swedish modellers also expressed a concern that the modelling might be too complex for upper secondary education, but the overall opinion was that modelling may be very useful and motivational for students. The modellers in the Netherlands argued that the main goal for mathematics education in secondary school is to learn basic skills in algebra and analysis (Spandaw 2011), an opinion raised also in this study. However, other more general goals mentioned were related to modelling, such as becoming a democratic citizen and opinions that the emphasis on mathematics education should not only be on procedures but also on why and how mathematics is used in society. The need to consider these and other social aspects of modelling in future mathematics curriculum design is discussed by Jablonka (2010).

As the modellers in this study had limited experience of the present secondary mathematics education, their opinions must be taken as *opinions* and *subjective*. Nevertheless, they gave suggestions from their workplace for approaches to teach modelling as well as proposals of suitable modelling problems to work with in school that may be investigated in further research studies, indicating how modelling can function as a link between school mathematics and workplace mathematics (cf. Sträßer et al. 2012). What was not discussed during the interviews, however, was the difference in objectives for using modelling. In education, modelling is a mathematical classroom activity either as an aim in itself (to develop modelling competencies) or as an aim to develop a broader mathematical ability (modelling as a didactical tool to learn mathematics) (see e.g., Blum and Niss 1991), whereas in the workplace mathematical modelling is "the gateway into the use of mathematics" (Sträßer et al. 2012, p. 7872). The role of the teacher as mediator between workplace mathematics and school mathematics objectives is therefore crucial for all efficient collaborations between school and the workplace (Wake 2013), which was also highlighted by the modellers' opinions in this study on mathematical modelling in upper secondary education.

References

Blum, W., & Niss, M. (1991). Applied mathematical problem solving, modelling, applications, and links to other subjects: State, trends and issues in mathematics instruction. *Educational Studies in Mathematics, 22*(1), 37–68.

Damlamian, A., Rodrigues, J. F., & Sträßer, R. (Eds.). (2013). *Educational interfaces between mathematics and industry. Report on an ICMI-ICIAM-study*. Cham: Springer.

Drakes, C. I. (2012). *Mathematical modelling: From novice to expert*. Doctoral dissertation, Simon Fraser University.

Frejd, P. (2013). *Mathematical modelling discussed by mathematical modellers*. Paper presented at CERME8, Manavgat-Side, Turkey, 6–10 Feb 2013. (To appear in the proceedings.)

Gainsburg, J. (2003). *The mathematical behavior of structural engineers*. Doctoral dissertation, Stanford University.

Jablonka, E. (2010). Reflections on mathematical modelling. In H. Alrø, O. Ravn, & P. Valero (Eds.), *Critical mathematics education: Past, present and future. Festschrift for Ole Skovsmose* (pp. 89–100). Rotterdam: Sense Publishers.

Jablonka, E., & Johanssson, M. (2010). Using texts and tasks: Swedish studies on mathematics textbooks. In B. Sriraman et al. (Eds.), *The first sourcebook of Nordic research in mathematics education* (pp. 269–282). Charlotte: Information Age Publishing.

Mouwitz, L. (2013). Rationality of practice and mathematical modelling: Connections, conflicts, and codifications. In G. A. Stillman, G. Kaiser, W. Blum, & J. P. Brown (Eds.), *Teaching mathematical modelling: Connecting to research and practice* (pp. 563–572). Dordrecht: Springer.

Noss, R., & Hoyles, C. (1996). The visibility of meanings: Modelling the mathematics of banking. *International Journal of Computers for Mathematical Learning, 1*(17), 3–31.

Salling Olesen, H. (2008). Workplace learning. In P. Jarvis (Ed.), *The Routledge international handbook of lifelong learning* (pp. 114–128). London: Routledge Mental Health.

Skolverket. (2012). *The subject syllabus for mathematics*. Retrieved April 1, 2013, from http://www.skolverket.se/polopoly_fs/1.174554!/Menu/article/attachment/Mathematics.pdf

Spandaw, J. (2011). Practical knowledge of research mathematicians, scientists and engineers about the teaching of modelling. In G. Kaiser, W. Blum, R. Borromeo Ferri, & G. Stillman (Eds.), *Trends in teaching and learning of mathematical modelling* (pp. 679–688). New York: Springer.

Sträßer, R., Damlamian, A., & Rodrigues, J. F. (2012). Educational interfaces between mathematics and industry (ICMI-ICIAM-Study 20). *Pre proceedings of ICME12*, *The 12th international congress on mathematics education*, Seoul, July 8–15, 2012, pp. 7863–7874.

Triantafillou, C., & Potari, D. (2010). Mathematical practices in a technological workplace: The role of tools. *Educational Studies in Mathematics, 74*, 275–294.

Wake, G. (2012). Seeking principles of design for general mathematics curricula informed by research of use of mathematics in workplace contexts. *Pre proceeding of ICME12, The 12th International congress on mathematics education*, Seoul, 8–15 July 2012, pp. 1679–1688.

Wake, G. (2013). Drawing on understanding of workplace practice to inform design of general mathematics curricula. In A. Damlamian, J. F. Rodrigues, & R. Sträßer (Eds.), *Educational interfaces between mathematics and industry. Report on an ICMI-ICIAM-study* (pp. 309–316). Cham: Springer.

Wedege, T. (2010). Researching workers' mathematics at work. In A. Araújo, A. Fernandes, A. Azevedo, & J. F. Rodrigues (Eds.), *Educational interfaces between mathematics and industry. Proceedings of EIMI 2010 Lisbon conference* (pp. 565–574). Portugal: Centro International de Matemática & Bedford: COMAP.

Williams, J., & Wake, G. (2007). Metaphors and models in translation between college and workplace mathematics. *Educational Studies in Mathematics, 64*(3), 345–371.