Chapter 15
How Do Students Share and Refine Models Through Dual Modelling Teaching: The Case of Students Who Do Not Solve Independently

Takashi Kawakami, Akihiko Saeki, and Akio Matsuzaki

Abstract The purpose of this chapter is firstly to show how students who could not solve an initial task by themselves shared and refined models through dual modelling teaching, and secondly to derive suggestions for dual modelling teaching. Through examining students' worksheets and protocols of video and audio records of the lessons, it was shown that unsuccessful modellers were able to change/modify their own models and classmates' ones and progress their dual modelling cycle by sharing different models. One crucial point for progression in the dual modelling cycle is the sharing of various models, which the modellers could not interpret or find independently. As an intervention strategy for the progression, teachers need to encourage students to share models that are related with both the initial task and their similar one, and to ensure a variety of ways to progress.

T. Kawakami (✉)
Faculty of Children's Studies, Nishikyushu University, Kamizono 3-18-15, Saga-shi, Saga, Japan
e-mail: kawakamita@nisikyu-u.ac.jp

A. Saeki
Graduate School of Education, Naruto University of Education, Nakashima 748, Takashima, Naruto-cho, Naruto-shi, Tokushima, Japan
e-mail: asaeki@naruto-u.ac.jp

A. Matsuzaki
Faculty of Education, Saitama University, Simo-Okubo 255, Sakura-ku, Saitama-shi, Saitama, Japan
e-mail: makio@mail.saitama-u.ac.jp

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15.1 Dual Modelling Cycle Framework (DMCF)

For responding to the diversity of modellers' modelling processes (Borromeo Ferri 2007) and variation between tasks in their individual modelling progress (e.g., Matsuzaki 2011; Stillman and Galbraith 1998), Saeki and Matsuzaki (2013) elaborated a *dual modelling cycle framework (DMCF)* theoretically (Fig. 15.1). DMCF is constructed using the modelling cycle of Blum and Leiß (2007) and is focused on switching between a first modelling cycle of an initial task and a second modelling cycle of a similar task. In DMCF, three types of cycles (single, double and dual) are distinguished with corresponding progression of the solution (Saeki and Matsuzaki 2013). In the case of using a dual modelling cycle, modellers progress the first modelling cycle by applying or referring to results from the second modelling cycle. DMCF provides opportunities for tackling, reflecting and applying tasks, where previously known or more accessible models are relevant in addressing the tasks.

Matsuzaki and Saeki (2013) illustrated a typical dual modelling cycle through a case study of undergraduate school students. Kawakami et al. (2012) implemented the experimental class based on DMCF for Year 5 students and reported typical successful cases of how students who could solve the initial task by themselves progressed their dual modelling cycle; however the study did not focus on the other students who were unsuccessful independently. The question remains: How does the teacher facilitate the students' progress of their dual modelling cycle? The purpose of this chapter is (a) to show how the students who could not solve the initial task by themselves share and refine models through the dual modelling teaching (Kawakami et al. 2012) and (b) to derive some suggestions for dual modelling teaching corresponding to a diversity of modellers.

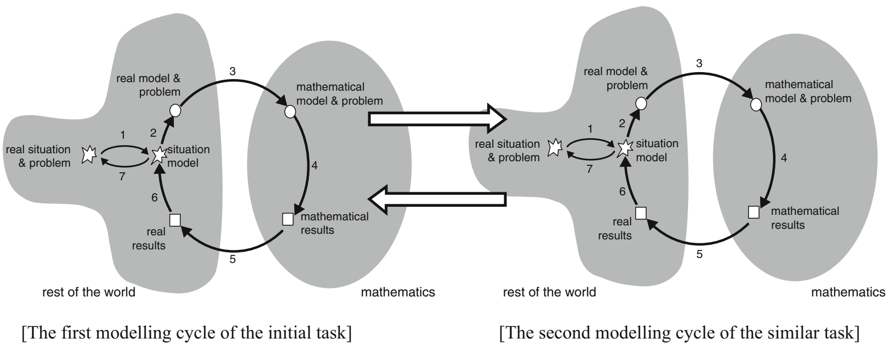


Fig. 15.1 Dual modelling cycle framework (Saeki and Matsuzaki 2013, p. 91)

15.2 Diversity of Student Models in Modelling Teaching Based on DMCF

15.2.1 The Teaching Material Based on DMCF

Oil Tank Task (Initial Task)
There are several types of oil tanks. Their heights are equal but their lengths of diameters are different. Are their lengths of spiral banisters equal or not? As conditions, angles to go up spiral banisters are all the same.

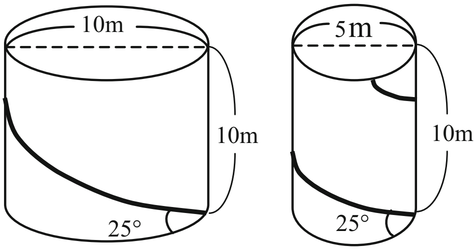
Toilet Paper Tube Task (Similar Task)
It is impossible to open an oil tank along the actual spiral banister. We can instead take a toilet paper tube as a similar shape to an oil tank, which is to be opened along its slit. Consider what the shape of an opened toilet paper tube would be.

The teaching material consists of an *Oil Tank Task* [OT task] (initial task) and a *Toilet Paper Tube Task* [TP task] (similar task). On the first modelling cycle of the OT task, the authors set up 3D models of oil tanks such as in Fig. 15.2. One of the 2D models of oil tanks is a rectangle model (Fig. 15.3). In this case, spirals of both tanks are the same by using mathematical ideas of equivalent transformation and translation of spiral. On the second modelling cycle of the TP task, we perceive that the 2D models of oil tanks are not only rectangle models but also parallelogram models (Fig. 15.4). In the case of a parallelogram model, spirals of both tanks are the same, which is supported by the mathematical idea of translation of the oblique side. Furthermore, we recognised that there was structural or relational correspondence between the rectangle model and the parallelogram model by using mathematical ideas of equivalent transformation and translation of spiral (Fig. 15.5).

15.2.2 Outline of the Modelling Teaching Based on DMCF

The experimental class (see also Kawakami et al. 2012) consisted of three lessons (45 min $\times$ 3). This class consisted of 33 Year 5 students (22 male, 11 female) aged 10–11 years old in a private elementary school in Japan. The data collection comprised video and audio recordings and artefacts of the lessons. In the first and second lesson, the students tackled the *Oil Tank Task* and the *Toilet Paper Tube Task*. In the OT task, the teacher showed 3D models (Fig. 15.2). Through discussion with the students, they drew 2D rectangle models of oil tanks to solve the OT task. However, they could not solve it because an oil tank is too big to open directly. So the teacher provided toilet paper tubes as similar shapes to the oil tank and students

Fig. 15.2 3D models of oil tanks



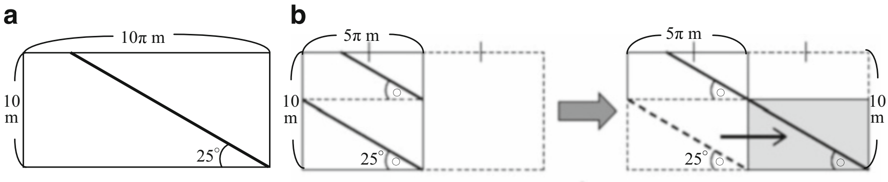
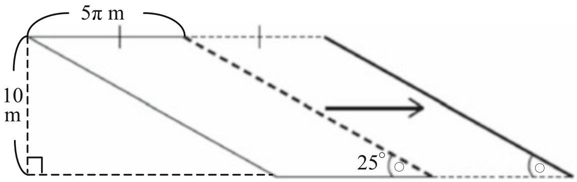


Fig. 15.3 The rectangle model: (**a**) original models, (**b**) half circumference models

Fig. 15.4 The parallelogram model



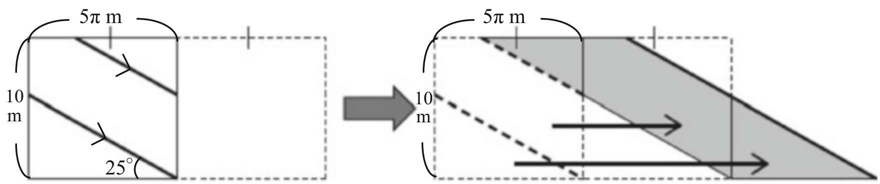


Fig. 15.5 Relationship between the rectangle model and the parallelogram model

made 2D parallelogram models in the TP task. At the end of the second lesson, they tackled the OT task again, with much referring to the TP task.

In the third lesson, the students shared three types of classmates' solutions. At first, the teacher illustrated the solution by focusing on the area of the parallelogram model and calculating the length of the spiral. However, they could not calculate the height of the parallelogram model using their own mathematical skills. Next, the teacher illustrated the solution by measurement of the spiral in rectangle models and parallelogram ones. The teacher said, "We found the approach by calculating is difficult, but we can give an answer by measuring the reduced drawing". Finally, the teacher highlighted Kob's solution by translation of the spiral where he applied equivalent transformations in the rectangle model (Fig. 15.6a) and Matsu's

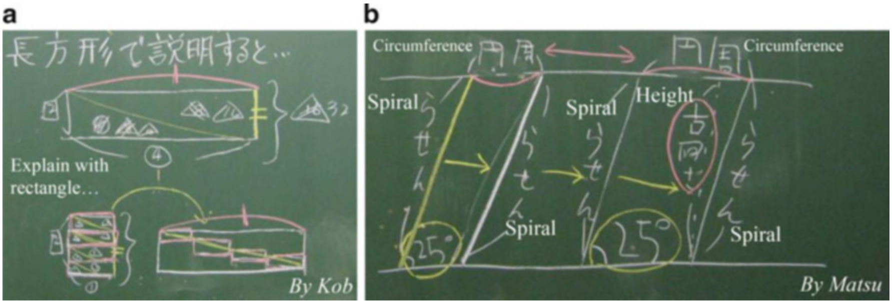


Fig. 15.6 (**a**) The rectangle models (**b**) The parallelogram models

approach of translation of the spiral in the parallelogram model (Fig. 15.6b). By illustrating two kinds of models, the teacher intended students to find the relationship between the first modelling cycle and the second modelling cycle.

Kob drew his idea on the blackboard (see Fig. 15.6a) and tried to explain that the length of the spiral was equal by applying the rate of area and equivalent transformation in the case of a quarter-length of the diameter. However, it was difficult for most of his classmates to understand, as the compound idea in Kob's explanation confused them. Then the teacher complemented his idea by using Tomi's idea in the case of half-length diameter as the following protocols show:

Teacher: Tomi cut the rectangle in half along this dotted line and translated the upper half [of the] rectangle to the bottom. Then the spiral is connected.
Students: I see!
Teacher: Tomi thought about the case as the half-length of the diameter. You have to think about the case of one third and the quarter-length of diameter too.

On the other hand, it was easy for the class to understand the parallelogram approach as the following protocols show:

Teacher: OK, we will look at the approach by parallelogram. Matsu, explain your idea. [Matsu drew his idea on the blackboard as in Fig. 15.6b.]
Matsu: It is probable that these spirals are all the same . . . and the length and angle are all the same . . . so I think oblique sides of the parallelogram are all the same.
Teacher: So how is this spiral [oblique side of parallelogram] moved?
Students: Translation.
Teacher: That's right. Matsu said, "The length of the spiral is equal because the oblique side of the parallelogram is translated".

15.2.3 Types of Student Solutions

Table 15.1 shows how 33 students tackled the *Oil Tank Task* again at the end of the second lesson. Sixteen students solved it by themselves. In tackling the OT task again, ten students answered by translation of the spiral in the 2D parallelogram model of the oil tank (Fig. 15.6a) (*Type A*), and 5 students answered by translation of spiral in the 2D rectangle model of the oil tank (Fig. 15.6b) (*Type B*). Only one student answered by measurement of spiral in both rectangle models and parallelogram models (*Type C*). On the other hand, 17 students could not solve the OT task by themselves. In *Type A*, four students could not solve it, although they used the parallelogram model. In *Type B*, nine students could not solve it, although they used the rectangle model. In *Type D*, four students could not solve it, although they used other models (e.g., rotation rate of spiral in cylinder model).

Kawakami et al. (2012) analysed the solutions of *Type A* and *Type B* with typical successful cases of how the students who could solve the OT task by themselves progressed the dual modelling cycle. These successful cases give us suggestions to consider what is ideal modelling teaching based on DMCF. However, there are other students who could not solve the OT task by themselves as indicated in Table 15.1. How do they progress the dual modelling cycle through shared classmates' models? How does the teacher facilitate the students' progress of the dual modelling cycle? In the next section, we focus on the solutions of two female students (Kawa's case in *Type A* and Kato's case in *Type B*) and analyse how their original models in the first and second lessons were changed or modified by examining their worksheets and protocols.

15.3 Analysis of Responses of Students who Could not Solve Independently

15.3.1 Kawa's Case

15.3.1.1 Lesson 1: First Trial of *Oil Tank Task* and *Toilet Paper Tube Task*

In the OT task, Kawa drew two rectangle models and put a spiral only in the left rectangle model (Fig. 15.7a), however she did not identify which is the case of the long diameter or of the short diameter. The left rectangle model seems to be mathematically correct. In the TP task, she drew a parallelogram model, however she made a mistake in identifying the length of the spiral as the length of the circumference (Fig. 15.7b).

Table 15.1 Types of student solutions to the *Oil Tank Task* ($N = 33$)

Solution type	Frequency solved	
	Independently	Not independently
A Parallelogram model	10	4
B Rectangular model	5	9
C Parallelogram and rectangular model	1	0
D Other models	0	4

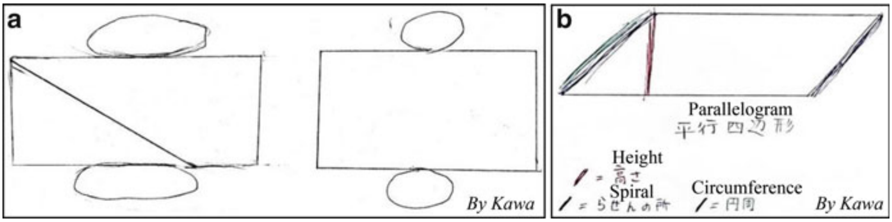


Fig. 15.7 (**a**) The rectangle models (**b**) The parallelogram model

15.3.1.2 Lesson 2: Second Trial of *Oil Tank Task* Based on the *Toilet Paper Tube Task*

Returning to the OT task, Kawa tried to solve it by the parallelogram model in which a pair of adjacent sides are 10 cm and 5 cm; however she could not solve the task. Her parallelogram model of the oil tanks did not correspond to Fig. 15.7b and to Fig. 15.2. She did not understand the structure of the parallelogram model enough.

15.3.1.3 Lesson 3: Final Trial of *Oil Tank Task* Through a Class Presentation

After the class presentation, Kawa wrote Kob's explanation by the rectangle model in the case of a quarter-length of the diameter (Fig. 15.8a). She described the equivalent transformation from a quarter diameter oil tank to the original diameter oil tank by using arrows and the phrase, "Cut and paste here . . . spiral is just connected!" So she showed that the spiral was connected as the result of equivalent transformation. She accepted Kob's explanation to be applicable to other diameter length oil tanks by her phrase, "We can vary the length of diameter!" She accepted Kob's rectangle model by referring to Tomi's idea in the case of half-length diameter. Additionally, she wrote a parallelogram model after Matsu's explanation (Fig. 15.8b). She did not copy Matsu's statement from the blackboard, but interpreted Matsu's statement and her classmates' statement of translation by using her own expressions "Circumference is only extended" and "Stretch". In the end, she concluded that changing the length of the diameter of the oil tank has

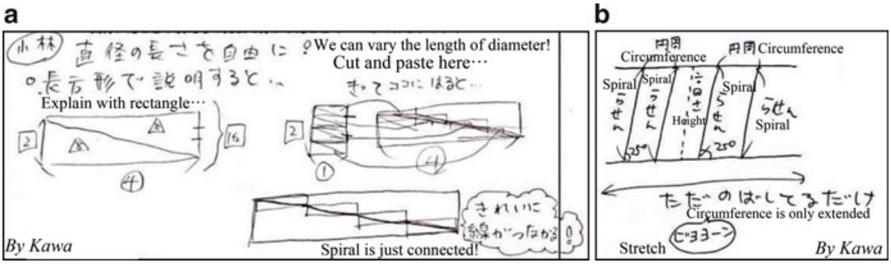


Fig. 15.8 (**a**) The accepted rectangle model (**b**) The accepted parallelogram model

nothing to do with determining the length of the spiral banister of it even if its height or its angle stays the same.

Kawa accepted Kob's rectangle model and Matsu's parallelogram model because she could accept the mathematical structure of both models. Furthermore, she concluded the OT task by the rectangle model and parallelogram model, and generalized it by the latter.

15.3.2 Kato's Case

15.3.2.1 Lesson 1: First Trial of *Oil Tank Task* and *Toilet Paper Tube Task*

In the OT task, Kato drew two rectangle models and put spirals on both models. The left rectangle model (Fig. 15.9a) is for the long diameter and right one is for another. However, the latter model is not mathematically correct. In the TP task, she indicated the relationships between the spiral and circumference correctly in a parallelogram model (Fig. 15.9b).

15.3.2.2 Lesson 2: Second Trial of *Oil Tank Task* Based on the *Toilet Paper Tube Task*

Returning to the OT task, Kato used a rectangle model and calculated the length of the circumference and the spiral by using a reduced drawing in which the diameter is 4 cm. This gave a circumference of $4 \times 3.14 = 12.56$ cm. However, she could not solve the task. Her model is not correct mathematically because the spiral is disconnected.

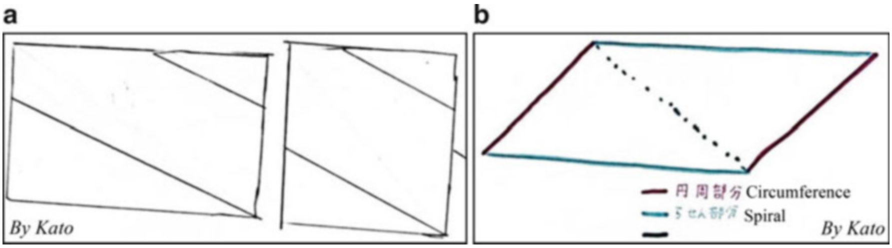


Fig. 15.9 (**a**) The rectangle models (**b**) The parallelogram model

15.3.2.3 Lesson 3: Final Trial of *Oil Tank Task* Through a Class Presentation

After the class presentation, Kato wrote Kob's explanation using the rectangle model in the case of the quarter-length of the diameter (Fig. 15.10a). It seems that she did not accept the relationship of the spiral in the rectangle model and/or apply the mathematical idea of equivalent transformation. Additionally, she wrote a parallelogram model after Matsu's explanation (Fig. 15.10b). She did not copy Matsu's statement from the blackboard, but interpreted Matsu's statement and her classmates' statement of transformation by her expression "You can give a value to the circumference". In the end, she concluded, as did Kawa, that changing the length of diameter of the oil tank has nothing to do with determining the length of the spiral banister of it.

Kato did not adopt Kob's rectangle model because she could not accept the mathematical structure of the rectangle model and/or apply the mathematical idea of equivalent transformation, however, she accepted Matsu's parallelogram model and the relation between circumference and spiral in it. Furthermore, she concluded the OT task and generalised it by using the parallelogram model.

15.4 Discussion

In the first two lessons, Kawa made rectangle models of the oil tank in the first modelling cycle (Fig. 15.7a) and the parallelogram model of the toilet paper tube in the second modelling cycle of DMCF (Fig. 15.7b). However, her parallelogram model was not correct mathematically. Returning to the first cycle from the second cycle, she used the parallelogram model and tried to solve the *Oil Tank Task*, however she could not solve it. From the viewpoint of DMCF, her modelling process is a "double" modelling cycle because she could not progress the first modelling cycle by findings from the second modelling cycle. In other words, she could not transition from the second modelling cycle to the first modelling cycle well. One of the factors of unsuccessful modelling is that she did not accept the relationship between the circumference and the spiral in the parallelogram model.

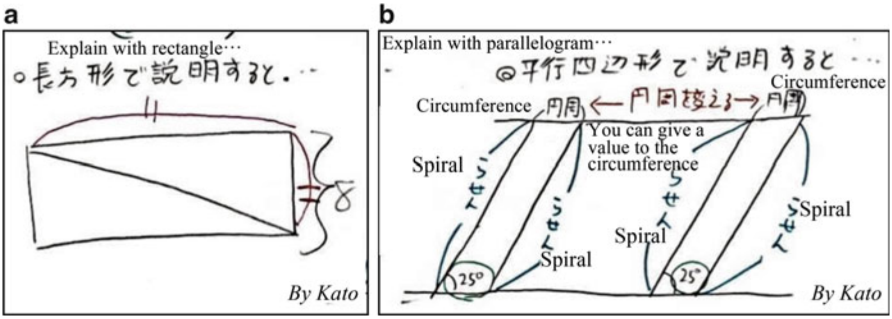


Fig. 15.10 (**a**) The unaccepted rectangle model (**b**) The accepted parallelogram model

In this dual modelling teaching, students confirmed the correspondence between each part in the toilet paper tube and the parallelogram as its development (Kawakami et al. 2012). If the parallelogram model is put on a different orientation, the relationship between the circumference and slit (spiral in oil tank) is changed (Saeki and Matsuzaki 2013). This might influence Kawa's "reconstruction of the figurative context" (Busse and Kaiser 2003, p. 4) offered in the OT task. Through the third lesson, she modified the parallelogram model which she had mistaken by objectifying and interpreting her shared classmates' parallelogram model in her own ways (Fig. 15.8b). She accepted her classmates' rectangle model too (Fig. 15.8a). Additionally, she concluded the OT task and generalized it by using both her accepted parallelogram model and the rectangle one. Her modelling process is a "dual" modelling cycle.

On the other hand, in the first and second lessons, Kato made rectangle models of the oil tank in the first modelling cycle (Fig. 15.9a) and a parallelogram model of the toilet paper tube in the second modelling cycle (Fig. 15.9b), however her rectangle models were not correct mathematically. Returning to the first cycle from the second cycle, she used the rectangle model and tried to solve the OT task, however she could not solve it. Her modelling process is a "double" modelling cycle as was Kawa's in the first and second lessons. Through the third lesson, she concluded and generalised the OT task in objectifying and interpreting only accepted classmates' parallelogram model in her own ways (Fig. 15.10b). When she meant to solve it, she changed the rectangle model to a parallelogram model. Her modelling process is also a "dual" modelling cycle. The reason she did not accept her classmates' rectangle model might be her misunderstanding of the relationship of the spiral to a rectangle and/or the analogy of rectangle models and parallelogram models. In this teaching experiment, the teacher could not spend time confirming the mathematical structure of the rectangle model (Kawakami et al. 2012).

Some unsuccessful modellers were able to change/modify their own models and classmates' ones and progress their dual modelling cycle by sharing different models such as Kawa's case and Kato's case. Kawa and Kato interpreted classmates' models in their own ways and solved them again by applying these models.

Similarly to Brown and Edwards (2013), students' communication of their solutions to the modelling task enhanced their mathematical understandings and modelling. On the other hand, Kawa and Kato sometimes could not transition from the second modelling cycle to the first cycle well because they did not accept the mathematical structure of the models and/or they could not apply mathematical ideas. This also illustrates that individual modeller's prior knowledge related to task context including the mathematical content of a task (e.g., Matsuzaki 2011; Stillman 2000) influences which models are able to be shared.

15.5 Conclusion

One crucial point for progression in the dual modelling cycle is to share various models, which modellers could not interpret or find. As an intervention strategy to stimulate the transition between modelling cycles, teachers need to encourage students to share and use the models that are related with both the initial task and the similar task. For example, teachers need to confirm with students structural or relational correspondences between initial task models and similar task ones and common mathematical ideas in both models. If they do so, unsuccessful modellers can reinterpret and, if necessary, modify the solution of the similar task as a model to reach the solution of the initial task or might integrate both solutions to generalise the conclusion. Thus, teachers have to ensure there are various types of modelling progress for each modeller through dual modelling teaching.

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