A Cross-Efficiency Approach for Evaluating Decision Making Units in Presence of Undesirable Outputs

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Abstract. Data Envelopment Analysis (*DEA*) is a mathematical programming approach for measuring efficiency of Decision Making Units (*DMUs*). In traditional DEA, a ratio of weighted outputs to inputs is examined and, for each *DMU*, some optimal weights are obtained. The method of *cross-efficiency* is an extension to DEA by which a matrix of scores is computed. The elements of the matrix are computed by means of the weights obtained via usual models of DEA. The cross-efficiency may have some drawbacks, e.g., the cross-efficiency scores may be multiple due to the presence of several optima. To overcome this issue, secondary goals are used. However, this method has never been used for peer evaluation of *DMUs with undesirable outputs*. In this paper, our objective is to bridge this gap. For this end, we introduce a new secondary goal, test it on an empirical example with undesirable outputs, report the results, and finally, we give some concluding remarks.

Keywords: Data Envelopment Analysis, Cross-Efficiency, Secondary Goals, Undesirable Outputs.

1 Introduction

In mathematical programming, Data Envelopment Analysis (DEA) is known as a methodology that is used for evaluating the efficiency of decision making units (*DMUs*). In general, *DMUs* have multiple inputs and multiple outputs. The pioneer work of Charnes et al. [3] (well known as model CCR) uses the ratio of weighted multiple outputs over weighted multiple inputs for computing the efficiency scores of each *DMU*. The BCC model of Banker et al. [1] is

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another classical model in DEA. Following these models, different variants of CCR and BCC as well as some new models have been proposed for evaluating DMUs [5]. Despite the efficiency and wide usage of DEA in identifying the most performant $DMUs$, the classical models have some drawbacks. For example, self-evaluation character of DEA models is considered as an important issue [15]. In order to overcome these flaws, other approaches have been proposed [4, 7, 15, 19]. One of them is the technique of *cross-efficiency*. In this approach, a peer evaluation replaces the self-evaluation of $DMUs$ and the efficiency scores are computed through linking each DMU to all $DMUs$ [15]. The performance of cross-efficiency has been approved through numerous applications (see e.g., [2, 15, 19], and more references therein). However, cross-efficiency, at its turn, suffers from a major drawback: in general, the cross-efficiency scores are not unique [7, 15]. In order to overcome this problem, several secondary goals have been proposed and studied [7, 15–17, 19, 21].

Furthermore, among different applications of DEA, we find some applications related to sustainable development and environmental aspects. More precisely, among the usual outputs of a DMU, there may exist some outputs that are known as *undesirable outputs*. As an example, one can cite the excessive carbon emissions (as undesirable outputs) of a manufacturing firm. Due to importance of this class of problems, there have been several studies about undesirable inputs/outputs. The first paper is due to Fare et al. [9] that has been followed by some other researchers (see e.g., [10, 12, 18, 20, 23]).

There were some studies concerning the application of cross-efficiency in peer evaluation of $DMUs$ with undesirable inputs/outputs [24]. However, to the best of our knowledge, in all of such studies there is no emphasize on the drawbacks of the cross-efficiency approach. This paper aims at bridging the gap between cross-efficiency with secondary goal and the DEA models addressing undesirable outputs. In order to attain this objective, we bring together all necessary materials, such as suitable production possibility set (PPS) and a new secondary goal. This procedure produces a multi-objective model for which we use max-min model that focuses on worst-case evaluations [8, 11]. Some experiments have been carried out on a practical data set [22]. A comparative study on the results demonstrates interesting observations about the usefulness of the proposed approach.

The current paper is organized as follows: In Section 2, we present notations, the prerequisites, and the concept of cross-efficiency. Section 3 is an introduction to data envelopment analysis with undesirable outputs. In this section, a self-evaluation model as well as cross-efficiency models for undesirable outputs are presented. Section 4 is devoted to the numerical experiments on a practical data set and some comments on the observations are provided. Section 5 includes the concluding remarks as well as some directions for future research.

2 Technical Materials: Prerequisites, Cross-Efficiency and Secondary Goal

In this section, we present the technical materials and notations that we are going to use through out this paper. Furthermore, we present a new secondary goal that will be used in used in evaluating $DMUs$.

2.1 Prerequisites

Suppose that we have a set of $DMUs$ and each DMU_j (for $j \in \{1, \ldots, n\}$) produces s different outputs from m different inputs. The following optimization model describes the multiple form of the well known CCR model for evaluating DMU_d (where $d \in \{1, \ldots, n\}$):

$$(CCR): \quad E^*_{dd} = \max \quad \frac{\sum_{r=1}^{s} u_{rd} y_{rd}}{\sum_{i=1}^{m} v_{id} x_{id}} \tag{1}$$

$$\text{s.t: } E_{dj} = \frac{\sum_{r=1}^{s} u_{rd} y_{rj}}{\sum_{i=1}^{m} v_{id} x_{ij}} \leq 1 \; : \quad j = 1, \ldots, n, \tag{2}$$

$$u_{rd} \geq 0 \; : \quad r = 1, \ldots, s, \tag{3}$$

$$v_{id} \geq 0 \; : \quad i = 1, \ldots, m. \tag{4}$$

The optimal value of this model, i.e., E^*_{dd}, is the optimal relative CCR efficiency score of DMU_d (where $d \in \{1, \ldots, n\}$).

Let us suppose that (v^*_d, u^*_d) is the vector of optimal weights for DMU_d. Using these optimal weights, the cross-efficiency of DMU_j is computed as follows:

$$E^*_{dj} := \frac{\sum_{r=1}^{s} u^*_{rd} y_{rj}}{\sum_{i=1}^{m} v^*_{id} x_{ij}} : j = 1, \ldots, n.$$

We observe that E^*_{dj} reflects a peer evaluation of DMU_j with respect to DMU_d. If we consider DMU_d as a target DMU and solve the CCR model for each $d = 1, \ldots, n$, we will get n CCR scores and their corresponding optimal weights. Using these optimal weights, we can compute $(n-1)$ cross-efficiency values. If we put all of these CCR and cross-efficiency values in a single $n \times n$ matrix, we get $[E_{ce}]$, that is the *cross-efficiency matrix* of the set of $DMUs$. For each column j of $[E_{ce}]$, we can compute the average of its elements that is considered as the overall performance of DMU_j [15, 21] and is known as the cross-efficiency score of DMU_j. These average scores can then be used for ranking $DMUs$.

It is well-known that the optimal weights obtained through the CCR model are not necessarily unique [7, 15, 17, 21]. Due to this fact, we may have different cross-efficiency scores and, consequently, this drawback may reduce the usefulness of cross-efficiency as a ranking method. In order to overcome this issue, the general approach consists in introducing *secondary goals* [7, 15–17]. More precisely, a complementary model is introduced and solved for the aim of screening out the alternative solutions and getting solutions that satisfy the conditions imposed by the secondary goals. Introducing secondary goals will produce new models to solve and make possible to screen out and select suitable weights. However, this approach is not as perfect as we expect and consequently, different tries have been done to find the best secondary goals [7, 15–17]. In the sequel, we propose a new secondary goal by taking into account some technical points.

2.2 A Linear Model as Secondary Goal

In the sequel, we present a model as *secondary goal*. The objective of introducing a secondary goal consists in screening multiple optima in order to get a suitable efficiency score. First of all, we solve the CCR model for evaluating DMU_d and assume that E^*_{dd} is the CCR-efficiency score of DMU_d. Then, we consider the following linear programming model:

$$(LP-CCR): \quad \max \left(\sum_{r=1}^{s} u_{rd} y_{rd} - E^*_{dd} \sum_{i=1}^{m} v_{id} x_{id} \right) \tag{5}$$

$$\text{s.t:} \sum_{r=1}^{s} u_{rd} y_{rj} - E^*_{jj} \sum_{i=1}^{m} v_{id} x_{ij} \leq 0 \ : \quad j = 1, \ldots, n, \tag{6}$$

$$\sum_{r=1}^{s} u_{rd} + \sum_{i=1}^{m} v_{id} = 1, \tag{7}$$

$$u_{rd} \geq 0 \ : \quad r = 1, \ldots, s, \tag{8}$$

$$v_{id} \geq 0 \ : \quad i = 1, \ldots, m. \tag{9}$$

Where E^*_{jj} is the efficiency score for DMU_j (for $j \in \{1, \ldots, n\}$) and the constraint (7) is a normalization constraint. In fact, the contraint (7) excludes the null vector from the set of feasible solutions. Suppose that (v^*_d, u^*_d) is the vector of optimal weights for $(LP - CCR)$. The following theorem states that the optimal value of this model is equal to *zero*.

Theorem 1. *The optimal value of* $(LP - CCR)$ *model is equal to zero.*

Proof: Since E^*_{dd} is the optimal value for CCR with the optimal vector (u^*, v^*) ≥ 0, we have

$$\frac{\sum_{r=1}^{s} u^*_{rd} y_{rd}}{\sum_{i=1}^{m} v^*_{id} x_{id}} = E^*_{dd} \text{ and } \frac{\sum_{r=1}^{s} u^*_{rd} y_{rj}}{\sum_{i=1}^{m} v^*_{id} x_{ij}} \leq 1 : j = 1, \ldots, n; j \neq d.$$

If we define E^*_{jj} as the optimal value of CCR for DMU_j, then

$$\frac{\sum_{r=1}^{s} u^*_{rd} y_{rj}}{\sum_{i=1}^{m} v^*_{id} x_{ij}} \leq E^*_{jj} : j = 1, \ldots, n; j \neq d.$$

In other words

$$\sum_{r=1}^{s} u^*_{rd} y_{rd} - E^*_{dd} \sum_{i=1}^{m} v^*_{id} x_{id} = 0 \text{ and } \sum_{r=1}^{s} u^*_{rj} y_{rd} - E^*_{jj} \sum_{i=1}^{m} v^*_{id} x_{ij} \leq 0 : \forall j \neq d.$$

Let us define $k = \frac{1}{\sum_{r=1}^{s} u_{rd}^* + \sum_{i=1}^{m} v_{id}^*}$ (we know that $\sum_{r=1}^{s} u_{rd}^* + \sum_{i=1}^{m} v_{id}^* \neq 0$). We observe that (ku^*, kv^*) is a feasible solution for $LP - CCR$, with 0 as the objective value. Furthermore, $(LP - CCR)$ in a maximization problem of a non-positive objective function. Hence, we come up with the conclusion that the optimal value of $(LP - CCR)$ is equal to 0. ■

Corollary: According to Theorem 1, we conclude that the efficiency score obtained by $(LP - CCR)$ is equal to the efficiency score provided by the model CCR.

In order to screen the multiple optima of CCR, we consider the following Multi-Objective model as the *Secondary Goal* (MO-SG).

$$
\begin{aligned}
&(MO - SG): \\
&\max_{j=1,\ldots,n; j\neq d} \left(\sum_{r=1}^{s} u_{rd} y_{rj} - E_{jj}^* \sum_{i=1}^{m} v_{id} x_{ij} \right) \\
&s.t. \\
&\sum_{r=1}^{s} u_{rd} y_{rj} - E_{jj}^* \sum_{i=1}^{m} v_{id} x_{ij} \leq 0 \quad : j = 1, \ldots, n; j \neq d, \\
&\sum_{r=1}^{s} u_{rd} y_{rd} - E_{dd}^* \sum_{i=1}^{m} v_{id} x_{id} = 0, \\
&\sum_{r=1}^{s} u_{rd} + \sum_{i=1}^{m} v_{id} = 1, \\
&u_{rd} \geq 0 \quad : r = 1, \ldots, s, \\
&v_{id} \geq 0 \quad : i = 1, \ldots, m.
\end{aligned}
$$

Where E_{dd}^* represents the amount of efficiency score for DMU_d and is obtained by solving CCR. The purpose of *(MO-SG)* model consists in providing optimal weights for DMU_d with fixing the efficiency of this unit and maximizing the efficiency score of other $DMUs$.

The scientific literature in multi-objective optimization includes different approaches for solving (*MO-SG*) (see e.g., [8]).

3 Undesirable Outputs

In this section, we present an approach for peer evaluation of $DMUs$ in presence of undesirable outputs. Undesirable outputs can be in different forms. Perhaps, some of the most notable examples of undesirable outputs are *excessive carbon emissions* in a manufacturing firm and *tax payments* in business operations. Due to importance of these factors, it is necessary to study the ways of reducing undesirable outputs and ranking the most efficient firms in terms of the least undesirable productions. In this paper, we aim at introducing a procedure for peer evaluation of $DMUs$ (having some undesirable outputs). This becomes possible by means of the technique of cross-efficiency and using the new secondary goal that we presented in Section (2.2).

3.1 Data Envelopment Analysis for Undesirable Outputs

For $j \in \{1, \dots, n\}$, suppose that DMU_j is a unit with the input vector x_i such that the vectors v_j and w_j are, respectively, the desirable and undesirable output vectors of DMU_j. Each DMU_j uses m inputs x_{ij} $(i = 1, \dots, m)$ to produce s desirable outputs v_{rj} $(r = 1, \dots, s)$ and h undesirable outputs w_{tj} $(t = 1, \dots, h)$. Kousmanen et al. [13, 14] introduced the following linear programming model for evaluating DMU_d (where $d \in \{1, \dots, n\}$)

$$\min \theta \tag{10}$$

$$s.t. \quad \sum_{j=1}^{n} (\eta_j + \mu_j) x_{ij} \le \theta x_{id} \;:\quad i = 1, \dots, m, \tag{11}$$

$$\sum_{j=1}^{n} \eta_j v_{rj} \ge v_{rd} \;:\quad r = 1, \dots, s, \tag{12}$$

$$\sum_{j=1}^{n} \eta_j w_{tj} = w_{td} \;:\quad t = 1, \dots, h, \tag{13}$$

$$\sum_{j=1}^{n} (\eta_j + \mu_j) = 1, \tag{14}$$

$$\eta_j \ge 0, \mu_j \ge 0 \;:\quad j = 1, \dots, n. \tag{15}$$

Where $\eta_j, \mu_j \ge 0$ (for $j = 1, \dots, n$) are structural variables for the production possibility set (PPS) (see [13, 14]). This model seeks for decreasing the inputs of DMU_d by rate of θ. The model (10)-(15) is an input oriented model with an optimal value of $\theta^* \in (0, 1]$. If $\theta^* = 1$, we say that DMU_d is technical efficient. If we write the dual of (10)-(15), we get the following model that we name it *Undesirable Multiple Form* (UMF):

$$E^*_{dd} = \max \left(\sum_{r=1}^{s} q_{rd} v_{rd} + \sum_{t=1}^{h} \pi_{td} w_{td} + \alpha_d \right) \tag{16}$$

$s.t.$

$$-\sum_{i=1}^{m} p_{id} x_{ij} + \sum_{r=1}^{s} q_{rd} v_{rj} + \sum_{t=1}^{h} \pi_{td} w_{tj} + \alpha_d \le 0 \;:\quad j = 1, \dots, n, \tag{17}$$

$$-\sum_{i=1}^{m} p_{id} x_{ij} + \alpha_d \le 0 \;:\quad j = 1, \dots, n, \tag{18}$$

$$\sum_{i=1}^{m} p_{id} x_{id} = 1, \tag{19}$$

$$p_{id} \geq 0 \ : \quad i = 1, \ldots, m, \tag{20}$$

$$q_{rd} \geq 0 \ : \quad r = 1, \ldots, s. \tag{21}$$

Where the dual variables p_{id} (for $i = 1, \ldots, m$) are defined as the weights for the inputs of DMU_d and q_{rd} (for $r = 1, \ldots, s$) (respectively, π_{td} (for $t = 1, \ldots, h$)) are the weights for desirable outputs (respectively, undesirable outputs) of DMU_d.

Let us suppose that $(p_d^*, q_d^*, \pi_d^*, \alpha_d^*)$ is the optimal solution vector of *UMF* model in evaluating DMU_d (where $d \in \{1, \ldots, n\}$). The cross-efficiency of DMU_j, by using the profile of weights that has been provided by DMU_d, is computed as follows [6]:

$$E_{dj}^* = \frac{\sum_{r=1}^{s} q_{rd}^* v_{rj} + \sum_{r=1}^{h} \pi_{td}^* w_{tj} + \alpha_d^*}{\sum_{i=1}^{m} p_{id}^* x_{ij}} \quad : \quad j = 1, \ldots, n \tag{22}$$

By solving *UMF* model for each $d \in \{1, \ldots, n\}$ and using (22), we can construct an $n \times n$ matrix $[E_{ce}^*]$ that is the *matrix of cross-efficiency.*

We know that the *UMF* model may have alternative solutions and consequently, we may have several cross-efficiency matrices. In order to overcome this problem, we can proceed as we explained in Section 2. To this end, we do a *cross-efficiency evaluation with secondary goal* for each DMU_d (for $d = 1, \ldots, n$). First of all, we solve the *UMF* model for each DMU_j ($j = 1, \ldots, n$) and we get E_{jj}^*, then we consider the (MO-SG) model as the following version in which the undesirable outputs are taken into account:

$$(MO - SG)_{undesirable} :$$

$$\max_{j=1,\ldots,n; j \neq d} \left(\sum_{r=1}^{s} q_{rd} v_{rj} + \sum_{t=1}^{h} \pi_{td} w_{tj} + \alpha_d - E_{jj}^* \sum_{i=1}^{m} p_{id} x_{ij} \right) \tag{23}$$

$s.t.$

$$\sum_{r=1}^{s} q_{rd} v_{rd} + \sum_{t=1}^{h} \pi_{td} w_{td} + \alpha_d - E_{dd}^* \sum_{i=1}^{m} p_{id} x_{id} = 0, \tag{24}$$

$$\sum_{r=1}^{s} q_{rd} v_{rj} + \sum_{t=1}^{h} \pi_{td} w_{tj} + \alpha_d - E_{jj}^* \sum_{i=1}^{m} p_{id} x_{ij} \leq 0 \ : j = 1, \ldots, n, \tag{25}$$

$$- \sum_{i=1}^{m} p_{id} x_{ij} + \alpha_d \leq 0 \ : j = 1, \ldots, n, \tag{26}$$

$$\sum_{r=1}^{s} q_{rd} + \sum_{t=1}^{h} \pi_{td} + \sum_{i=1}^{m} p_{id} = 1, \tag{27}$$

$$p_{id} \geq 0 \ : i = 1, \ldots, m, \qquad (28)$$
$$q_{rd} \geq 0 \ : r = 1, \ldots, s. \qquad (29)$$

We note that this model is similar to (MO-SG); however, this model takes into account the undesirable factors of $DMUs$. By keeping the score of DMU_d at its own level, the model (23)-(29) seeks for the optimal weights such that the efficiency scores of other $DMUs$ are *maximized*. In order to solve this multi-objective model, many approaches can be chosen from the literature of multi-objective programming. One of the classical approaches consists in using the *max-min method* [8, 11]. In this approach, we introduce a supplementary variable, e.g., ϕ for transforming the multi-objective model (23)-(29) to the following linear programming (LP) model (where $J = \{1, \ldots, n\}$).

$$\max \phi \qquad (30)$$
$$s.t.$$
$$\phi \leq \sum_{r=1}^{s} q_{rd} v_{rj} + \sum_{t=1}^{h} \pi_{td} w_{tj} + \alpha_d - E^*_{jj} \sum_{i=1}^{m} p_{id} x_{ij} \ : j \in J \setminus \{d\}, \qquad (31)$$
$$\sum_{r=1}^{s} q_{rd} v_{rd} + \sum_{t=1}^{h} \pi_{td} w_{td} + \alpha_d - E^*_{dd} \sum_{i=1}^{m} p_{id} x_{id} = 0, \qquad (32)$$
$$\sum_{r=1}^{s} q_{rd} v_{rj} + \sum_{t=1}^{h} \pi_{td} w_{tj} + \alpha_d - E^*_{jj} \sum_{i=1}^{m} p_{id} x_{ij} \leq 0 \ : j \in J, \qquad (33)$$
$$- \sum_{i=1}^{m} p_{id} x_{ij} + \alpha_d \leq 0 \ : j \in J, \qquad (34)$$
$$\sum_{r=1}^{s} q_{rd} + \sum_{t=1}^{h} \pi_{td} + \sum_{i=1}^{m} p_{id} = 1, \qquad (35)$$
$$p_{id} \geq 0 \ : i = 1, \ldots, m, \qquad (36)$$
$$q_{rd} \geq 0 \ : r = 1, \ldots, s. \qquad (37)$$

To sum up, the following procedure outlines the process of forming the matrix of cross-efficiency by using the proposed secondary goal and for evaluating $DMUs$ having undesirable outputs.

Procedure of Cross-Efficiency with Secondary Goal (PCE-SG) (in presence of undesirable outputs)

Step 1. For each $j = 1, \ldots, n$, solve the model *UMF* to obtain E^*_{jj}.
Step 2. For each $d = 1, \ldots, n$, solve the *max-min* model (30)-(37) to obtain the solution vector $(p^*_d, q^*_d, \pi^*_d, \alpha^*_d)$ as the profile of weights for DMU_d.

Step 3. For each $d = 1, \ldots, n$, use the vector $(p_d^*, q_d^*, \pi_d^*, \alpha_d^*)$ in (22) to construct the d^{th} row of the matrix of cross-efficiency.

Once the matrix of cross-efficiency is formed, we can compute the *cross-efficiency score (CES)* of each DMU_j (for $j = 1, \ldots, n$). The cross-efficiency score of DMU_j is computed by taking the average of the entries on the j^{th} column of the matrix (see also [6, 15]), i.e.,

$$\overline{E_j^*} := \frac{\sum_{d=1}^{n} E_{dj}^*}{n} \tag{38}$$

4 Numerical Example

In this section, the presented cross-efficiency approach is used to evaluate the efficiency of Chinese provinces in matter of the environmental criteria [22]. In this evaluation, the data of 16 Chinese provinces are considered. Table 1 includes the whole data. In this table, each province is considered as a DMU and it has two inputs (the total energy assumption (x_1) and the total population (x_2)) and four outputs, including one desirable output (GDP, denoted by v) and three undesirable outputs: the total amounts of industrial emissions of waste water (w_1), waste gas (w_2), and waste solids (w_3) [22].

We apply the procedure (PCE-SG) to form the matrix of cross-efficiency. Then, we use (38) to compute the cross-efficiency score (CES) of each DMU.

In order to solve the linear programming (LP) models, we use the standard solver *Lingo 11* on a *Pentium Dual-Core CPU*, with 4GB RAM and 2.10 GHz. The computational time for solving LP models is negligible. The results are presented in Table 2. In this table, we present the meaningful information of the cross-efficiency matrix. More precisely, for each DMU, we present: minimum score (min), maximum score (max), average of scores (CES) (that is the *Cross-Efficiency Score (CES)* as we defined in previous sections), standard deviation of scores (SD), range of scores ($range$), and the *rank* of the DMU.

The ranking of $DMUs$ is based on the CES values, that is known as the *cross-efficiency score* [6, 15]. We note that the *max* values correspond to the efficiency scores in conventional CCR model. The CCR model ranks the $DMUs$ by a self-evaluation procedure. However, by using the cross-efficiency, we take its advantage as a weighting procedure that helps us to rank $DMUs$ by a peer evaluation of all $DMUs$. In this context, we use the new secondary goal that lets us to consider the best performance of all DMU.

As we observe in Table 2, through the procedure (PCE-SG), the information provided by the method of cross-efficiency helps us in ranking $DMUs$ in an explicite way and (in this example) without any tie. This fact shows the pure advantage of using a peer evaluation of $DMUs$ through the method of cross-efficiency and, particularly, the proposed secondary goal. More precisely, by means of (PCE-SG) we can take into account more information about all $DMUs$ and the ranking is based on these information.

Table 1. The data corresponding to 16 $DMUs$: inputs (total energy assumption, total population) and desirable outputs (GDP) and undesirable outputs (total industrial emission of waste water, waste gas, waste solids)

	Inputs		Desirable Output	Undesirable Outputs		
DMU	x_1	x_2	v	w_1	w_2	w_3
1	6570	15096	12153.03	8713	4408	1242.4
2	5874	10473	7521.85	19441	5983	1515.7
3	15576	30378	7358.31	39720	23693	14742.9
4	19112	38429	15212.49	75159	25211	17221.4
5	7698	24349	7278.75	37563	7124	3940.5
6	23709	68371	34457.30	256160	27432	8027.8
7	15567	45472	22990.35	203442	18860	3909.7
8	8916	32097	12236.53	142747	10497	6348.9
9	19751	83974	19480.46	140325	22186	10785.8
10	13708	50862	12961.10	91324	12523	5561.5
11	13331	56820	13059.69	96396	10973	5092.8
12	24654	84981	39482.56	188844	22682	4740.9
13	7030	25284	6530.01	65684	12587	2551.8
14	16322	72471	14151.28	105910	13410	8596.9
15	8032	40460	6169.75	32375	9484	8672.8
16	8044	33504	8169.80	49137	11032	5546.7

Table 2. The results of cross-efficiency (CE) method for ranking 16 $DMUs$

DMU	min	max	$range$	SD	CES	$rank$
1	0.10	0.94	0.84	0.264	0.48	13
2	0.23	1	0.77	0.267	0.64	7
3	0.12	1	0.88	0.338	0.73	3
4	0.19	1	0.81	0.281	0.71	4
5	0.15	0.88	0.73	0.199	0.54	11
6	0.25	0.92	0.67	0.255	0.61	9
7	0.23	1	0.77	0.298	0.65	6
8	0.19	1	0.81	0.275	0.79	1
9	0.11	0.77	0.66	0.211	0.49	12
10	0.12	0.64	0.52	0.165	0.46	14
11	0.19	1	0.81	0.312	0.77	2
12	0.15	0.68	0.53	0.182	0.44	15
13	0.12	1	0.88	0.328	0.71	5
14	0.09	0.64	0.55	0.168	0.43	16
15	0.07	1	0.93	0.330	0.60	10
16	0.12	0.95	0.83	0.270	0.62	8

As we observe in Table 2, there are 8 efficient $DMUs$ in terms of traditional efficiency evaluation (see the column "max") and this is a drawback of self-evaluation. According to Table 2, there is just one cross-efficient DMU, that is DMU_8. Particularly, DMU_8 is efficient not only in the traditional sense, but also in terms of the *cross-efficiency with secondary goal*. In terms of efficiency, DMU_8 dominates the other $DMUs$. It is important to note that the dominance of DMU_8 with respect to the others has a strong sense, i.e., DMU_8 is efficient by taking into account the *best performance* of all $DMUs$.

Finally, in this example, the procedure (PCE-SG) is used for evaluating $DMUs$ in presence of *undesirable outputs*. This procedure helps to identify the best $DMU(s)$ and produces a ranking approach. By means of these results, we can recognize the necessary improvements that must be done in the inputs and the outputs (desirable and/or undesirable) of any inefficient DMU in order to obtain efficient $DMUs$ with the least undesirable factors.

5 Conclusion

In this paper, we discussed about a cross-efficiency approach for evaluating the performance of $DMUs$ in presence of undesirable outputs. More precisely, we studied a cross-efficiency approach by introducing a new *secondary goal*. This approach consists in constructing a multi-objective model that can be transformed to an LP model in many ways, for example, the classical *max-min method* [8, 11]. The introduced approach has been extended (in a natural way) for evaluation of $DMUs$ with *undesirable outputs*. We applied our approach on a real-world data set [22] involving *undesirable outputs*. According to the results, our approach is useful in peer-evaluation of $DMUs$ and helps decision makers in ranking units by taking into account the best performance of all $DMUs$; particularly, in the case of ties in conventional DEA scoring systems. The current study is in progress by considering alternative secondary goals. The complementary results will be reported in future.

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