Risk Problems Identifying Optimal Pollution Level

George E. Halkos and Dimitra C. Kitsou

Abstract The determination of the optimal pollution level is essential in Environmental Economics. The associated risk in evaluating this optimal pollution level and the related Benefit Area (BA), is based on various factors. At the same time the uncertainty in the model fitting can be reduced by choosing the appropriate approximations for the abatement and damage marginal cost functions. The target of this paper is to identify analytically and empirically the Benefit Area (BA) in the case of quadratic marginal damage and linear marginal abatement cost functions, extending the work of (Halkos and Kitsos, Appl. Econ. 37:1475–1483, 2005, [9]).

Keywords Optimal pollution level · Risk · Benefit area

1 Introduction

Rationality in the formulation and applicability of environmental policies depends on careful consideration of their consequences for nature and society. For this reason it is important to quantify the costs and benefits in the most accurate way. But the validity of any cost–benefit analysis (hereafter CBA) is ambiguous as the results may have large uncertainties. Uncertainty in the evaluation of their effects is present in all environmental problems and this underlines the need for thoughtful policy design and evaluation. We may have uncertainty in the underlying physical or ecological processes, as well as in the economic consequences of the change in environmental quality.

As uncertainty may be due to the lack of appropriate abatement and damage cost data, we apply here a method of calibrating hypothetical damage cost estimates

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relying on individual country abatement cost functions. In this way a "calibrated" Benefit Area (BA^c) is estimated.

Specifically we try to identify the optimal pollution level under the assumptions of linear marginal abatement and quadratic marginal damage cost functions. That is, we consider another case of the possible approximations of the two cost curves improving the work in [9] by extending the number of different model approximations of abatement and damage cost functions and thus the assumed correct model eliminates uncertainty about curve fitting. The target of this paper is to develop the appropriate theory in this specific case.

2 Determining the Optimal Level of Pollution

Economic theory suggests that the optimal pollution level occurs when the marginal damage cost equals the marginal abatement cost. Graphically the optimal pollution level is presented in Fig. 1 where the marginal abatement (MAC $= g(z)$) and the marginal damage (MD $= \varphi(z)$) are represented as typical mathematical cost functions.

The intersection of the marginal abatement (MAC) and marginal damage (MD) cost functions defines the optimal pollution level (denoted as I in Fig. 1) with coordinates (z_0, k_0), $I(z_0, k_0)$. The value of z_0 describes the optimal damage reduction while k_0 corresponds to the optimal cost of attaining that. The area in $\mathbb{R}^2$ covered by the MAC and MD and the axis of cost is defined as the Benefit Area. That is, the point of intersection of the two curves, $I = I(z_0, k_0)$, reflects the optimal level of pollution with k_0 corresponding to the optimum cost (benefit) and z_0 to the optimum damage restriction. It is assumed (and we shall investigate the validity of this assumption subsequently) that the curves have an intersection and the area created by these curves (region AIB) is what we define as Benefit Area (see [15], among others), representing the maximum of the net benefit that is created by the activities of trying to reduce pollution.

Consider Fig. 1. Let A and B be the points of the intersection of the linear curves MD $= \varphi(z) = \alpha + \beta z$ and MAC $= \beta_0 + \beta_1 z$ with the "Y–axis". We are restricted to positive values. For these points $A = A(0, \alpha)$ and $B = B(0, \beta_0)$ the values of $a = \alpha$ and $b = \beta_0$ are the constant terms of the assumed curves that represent MD and MAC respectively.

Let us now assume that

$$\text{MAC}(z) = g(z) = \beta_0 + \beta_1 z, \quad \beta_1 \neq 0 \quad \text{and} \quad \text{MD}(z) = \varphi(z) = \alpha z^2 + \beta z + \gamma, \ \alpha > 0.$$

The intersections of MD and MAC with the Y–axis are $b = \text{MAC}(0) = \beta_0$ and $a = \text{MD}(0) = \gamma$, see Figs. 2, 3 and 4. To ensure that an intersection between MAC and MD occurs we need the restriction $0 < \beta_0 < \gamma$. yboxAssuming $\alpha > 0$ three cases can be distinguished, through the determinant of $\varphi(z)$, say D, $D = \beta^2 - 4\alpha\gamma$; (a) $D = 0$ (see Fig. 2), (b) $D > 0$ (see Fig. 3) while the case $D < 0$ is without

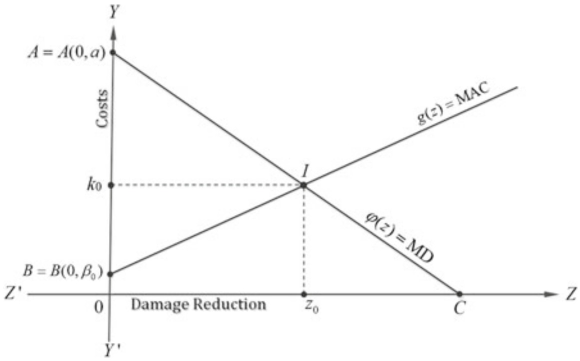

Fig. 1 Graphical presentation of the optimal pollution level (general case)

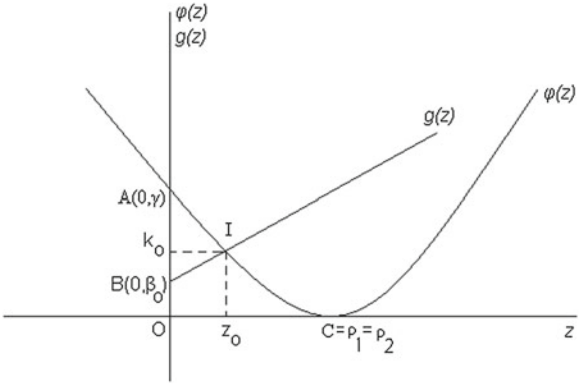

Fig. 2 $C = C\left(-\frac{\beta}{2\alpha}, 0\right), \alpha > 0$

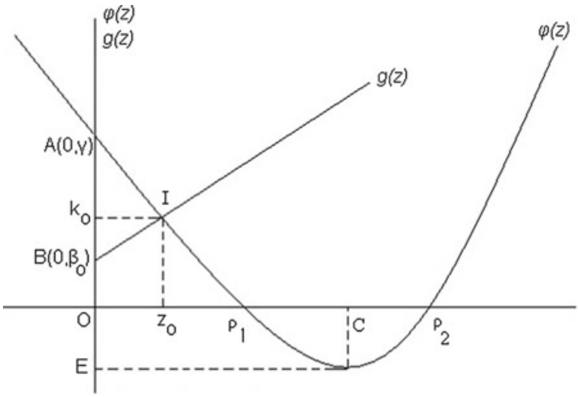

Fig. 3 $C = C\left(-\frac{\beta}{2\alpha}, 0\right)$, $E = E\left(0, \varphi\left(-\frac{\beta}{2\alpha}\right)\right)$, $\varphi\left(-\frac{\beta}{2\alpha}\right) = \min \varphi(z), \alpha > 0$

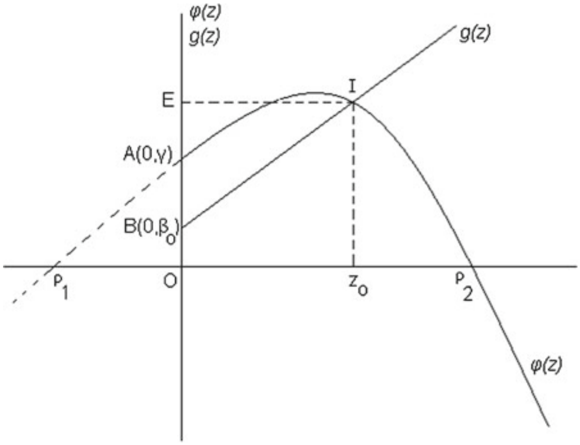

Fig. 4 $C = C\left(-\frac{\beta}{2\alpha}, 0\right)$, $E = E\left(0, \varphi\left(-\frac{\beta}{2\alpha}\right)\right)$, $\varphi\left(-\frac{\beta}{2\alpha}\right) = \min \varphi(z)$, $\alpha < 0$

economic interest (due to the complex–valued roots). Cases (a) and (b) are discussed below, while for the dual $\alpha < 0$ see Case (c). For more details see also [14].

Case (a): $\alpha > 0, \quad D = \beta^2 - 4\alpha\gamma = 0$. In this case there is a double real root for MD(z), say $\rho = \rho_1 = \rho_2 = -\frac{\beta}{2\alpha}$. We need $\rho > 0$ and hence $\beta < 0$. To identify the optimal pollution level point $I(z_0, k_0)$ the evaluation of point z_0 is the one for which

$$\mathrm{MD}(z_0) = \varphi(z_0) \Leftrightarrow g(z_0) = \mathrm{MAC}(z_0) \Leftrightarrow \alpha z_0^2 + \beta z_0 + \gamma = \beta_0 + \beta_1 z_0 \Leftrightarrow$$

$$\alpha z_0^2 + (\beta - \beta_1) z_0 + (\gamma - \beta_0) = 0. \tag{1}$$

Relation (1) provides the unique (double) solution when $D_1 = (\beta - \beta_1)^2 - 4\alpha(\gamma - \beta_0) = 0$ which is equivalent to

$$z_0 = -\frac{\beta - \beta_1}{2\alpha} = \frac{\beta_1 - \beta}{2\alpha}. \tag{2}$$

As z_0 is positive and $\alpha > 0$ we conclude that $\beta_1 > \beta$. So for the conditions are: $\alpha > 0, \beta_1 > \beta, 0 < \beta_0 < \gamma$ we can easily calculate

$$k_0 = \mathrm{MAC}(z_0) = \beta_0 + \beta_1 \frac{\beta_1 - \beta}{2\alpha} > 0, \tag{3}$$

and therefore $I(z_0, k_0)$ is well defined. The corresponding Benefit Area ($\mathrm{BA_{QL}}$) in this case is

$$
\begin{aligned}
\mathrm{BA}_{\mathrm{QL}} = (\mathrm{ABI}) = \int_0^{z_0} \varphi(z) - g(z)dz = \int_0^{z_0} \alpha z^2 + (\beta - \beta_1)z + (\gamma - \beta_0)dz = \\
\left[\tfrac{\alpha}{3}z^3 + \tfrac{1}{2}(\beta - \beta_1)z^2 + (\gamma - \beta_0)z\right]_{z=0}^{z_0} = \\
\tfrac{\alpha}{3}z_0^3 + \tfrac{1}{2}(\beta - \beta_1)z_0^2 + (\gamma - \beta_0)z_0.
\end{aligned} \tag{4}
$$

Case (b): $\alpha > 0$, $D = \beta^2 - 4\alpha\gamma > 0$. For the two roots ρ_1, ρ_2, we have $|\rho_1| \neq |\rho_2|$, $\varphi(\rho_1) = \varphi(\rho_2) = 0$ and we suppose $0 < \rho_1 < \rho_2$, see Fig. 3. The fact that $D > 0$ is equivalent to $0 < \alpha\gamma < (\beta/2)^2$, while the minimum value of the MD function is $\varphi(-\beta/(2\alpha)) = (4\alpha\gamma - \beta^2)/(4\alpha)$.

Proposition 1 *The order* $0 < \rho_1 < \rho_2$ *for the roots and the value which provides the minimum is true under the relation*

$$\beta < 0 < \alpha\gamma < \left(\frac{\beta}{2}\right)^2. \tag{5}$$

Proof The order of the roots $0 < \rho_1 < \rho_2$ is equivalent to the set of relations:

$$D > 0, \quad \alpha\varphi\left(-\frac{\beta}{2\alpha}\right) < 0, \quad \alpha\varphi(0) > 0, \quad 0 < \frac{\rho_1 + \rho_2}{2}. \tag{6}$$

The first is valid, as we have assumed $D > 0$. For the imposed second relation from (6) we have $\alpha\varphi(-\frac{\beta}{2\alpha}) < 0 \Leftrightarrow \alpha\frac{4\alpha\gamma - \beta^2}{4\alpha} < 0 \Leftrightarrow D > 0$, which holds. As both the roots are positive $\rho_1, \rho_2 > 0$, then the product $\rho_1\rho_2 > 0$ and therefore $\frac{\gamma}{\alpha} > 0 \Leftrightarrow \alpha\gamma > 0$. The third relation $\alpha\varphi(0) = \alpha\gamma > 0$, in (6) is true already and $0 < \frac{\rho_1+\rho_2}{2} \Rightarrow 0 < -\frac{\beta}{2\alpha}$ equivalent to $\beta < 0$. Therefore we get $\beta < 0 < \alpha\gamma < (\frac{\beta}{2})^2$.

We can then identify the point of intersection $I(z_0, k_0)$, $z_0 : \mathrm{MAC}(z_0) = \mathrm{MD}(z_0)$ as before. Therefore under (5) and $\beta_1 > \beta_0$ we evaluate k_0 as in (3) and the Benefit Area $\mathrm{BA}_{\mathrm{QL}}$ can be evaluated as in (4).

Case (c): $\alpha < 0$, $D = \beta^2 - 4\alpha\gamma > 0$. Let us now consider the case $\alpha < 0$. Under this assumption the restriction $D = 0$ is not considered, as the values of $\varphi(z)$ have to be negative.

Under the assumption of Case (c), the value $\varphi(-\frac{\beta}{2\alpha}) = \frac{4\alpha\gamma - \beta^2}{4\alpha}$ corresponds to the maximum value of $\varphi(z)$. We consider the situation where $\rho_1 < 0 < -\frac{\beta}{2\alpha} < \rho_2$ (see Fig. 4) while the case $0 < \rho_1 < -\frac{\beta}{2\alpha} < \rho_2$ has no particular interest (it can be also considered as in Case (b), see Fig. 3).

Proposition 2 *For the Case (c) as above we have:* $\rho_1 < 0 < -\frac{\beta}{2\alpha} < \rho_2$ *when* $\alpha\gamma < 0$.

Proof The imposed assumption is equivalent to $\alpha\varphi(0) < 0 \Leftrightarrow \alpha\gamma < 0$ as $\rho_1\rho_2 < 0$, $\alpha\varphi(-\frac{\beta}{2\alpha}) < 0 \Leftrightarrow \alpha\gamma < (\frac{\beta}{2})^2$. Therefore the imposed restrictions are $\alpha\gamma < 0 < (\frac{\beta}{2})^2$ (compare with (5)). Actually, $\alpha\gamma < 0$.

Case (c) requires that $\beta_0 < \gamma$ and $\beta_1 > 0$. To calculate z_0 we proceed as in (1) and z_0 is evaluated as in (2). Therefore, with $\alpha < 0$ we have $\beta_1 - \beta < 0$, i.e. $\beta_1 < \beta$. Thus for $\beta_1 < \beta$, $\alpha\gamma < 0$, the BA as in (4) is still valid.

3 An Empirical Application

In the empirical application, regression analysis is adopted to estimate the involved parameters. The available data for different European countries are used, as derived and described by [3, 4].

The abatement cost function measures the cost of reducing tonnes of emissions of a pollutant, like sulphur (S), and differs from country to country depending on the local costs of implementing best practice abatement techniques as well as on the existing power generation technology. For abating sulphur emissions various control methods exist with different cost and applicability levels, see [3–6].

Given the generic engineering capital and operating control cost functions for each efficient abatement technology, total and marginal costs of different levels of pollutant's reduction at each individual source and at the national (country) level can be constructed. According to [3, 4, 8], the cost of an emission abatement option is given by its total annualized cost (TAC) calculated by the addition of fixed and variable operating and maintenance costs. For every European country a least cost curve is derived by finding the technology on each pollution source with the lowest marginal cost per tonne of pollutant removed in the country and the amount of pollutant removed by that method on that pollution source.

Specifically the abatement cost curves were derived for all European countries after considering all sectors and all available fuels with their sulphur content for the year 2000. See [2], for technical details on deriving an abatement cost curve and on using pre-during and post–combustion desulphurization techniques. Figure 5 shows the marginal cost curve in the case of Austria and for the year 2000.

For analytical purposes, it is important to approximate the cost curves of each country by adopting a functional form extending the mathematical models described above to stochastic models, [7]. At the same time, the calculation of the damage function $\varphi(z)$ is necessary as proposed in [9, 13, 14]. The only information available is to "calibrate" the damage function, on the assumption that national authorities act independently (as Nash partners in a non-cooperative game with the rest of the world) taking as given deposits originating in the rest of the world, see [10].

The results are presented in Table 1, where Eff is the efficiency of the benefit area, in comparison with the maximum evaluated from the sample of countries under investigation and can be estimated using as measure of efficiency the expression as defined in [9]:

$$\mathrm{Eff} = \left(\frac{\mathrm{BA}}{\max \mathrm{BA}}\right) \times 100.$$

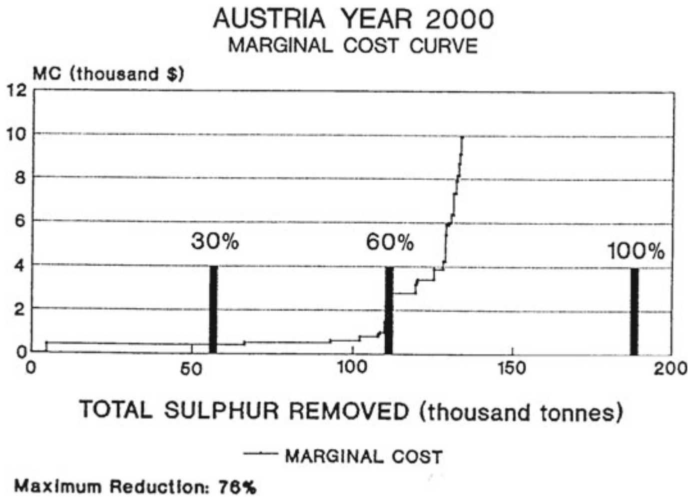

Fig. 5 The marginal abatement cost curve for Austria and for the year 2000 (*Source* Modified from [2])

Table 1 Coefficient estimates in the case of quadratic MD and MAC functions

Countries	c_0	c_1	c_2	b_0	b_1	b_2	R^2
Albania	0.7071	0.01888	0.0001397	−3.3818	0.015	0.0048	0.819
Austria	8.57143	0.055012	0.0001145	3.274	−0.221	0.004	0.748
Belgium	2.2424	0.03869	0.0001688	0.497	−0.124	0.003	0.851
Former Czech.	37.794	0.100323	0.000059	11.241	0.2358	0.00018	0.723
Denmark	10.0	0.1923	0.0060811	−2.49	0.099	0.0053	0.923
Finland	4.021	0.0781	0.0001459	2.343	−0.098	0.0046	0.583
France	33.158	0.277352	0.000197	42.374	−0.053	0.0018	0.945
Greece	3.7373	0.034133	0.0000491	−1.614	0.342	0.0006	0.998
Hungary	5.101	0.031488	0.0000417	2.506	0.216	0.0004	0.923
Italy	21.01	0.030036	0.0000191	12.5	0.36	0.0003	0.689
Luxembourg	0.421	0.3161	0.0272381	−0.7272	0.01	0.09234	0.883
Netherlands	8.353	0.19513	0.0035144	−6.18	0.41	0.0009	0.794
Norway	1.421	0.07852	0.0001701	0.94	−0.244	0.0164	0.878
Poland	6.212	0.023153	0.000071	−8.023	0.324	0.00009	0.77
Romania	9.091	0.011364	0.0000624	5.502	0.19	0.0001	0.81
Spain	11.7	0.007288	0.0049742	10.21	−0.021	0.00014	0.992
Sweden	2.4	0.06423	0.0000932	4.074	−0.252	0.004	0.854
Switzerland	2.4	0.56027	0.002803	5.7543	−1.6289	0.11203	0.912
Turkey	14.9	0.01781	0.0000122	8.0622	0.011	0.00036	0.932
UK	19.1	0.06879	0.0000467	15.54	0.0264	0.0003	0.884

Table 2 Calculated "calibrated" benefit areas (BA^c)

Countries	Linear–Quadratic					
	D	z_0	$g(z_0)$	$G(z_0)$	BA	Eff
Albania	0.0785	29.594	−3.38	−52.05	81.24	3.5872
Austria	0.1609	84.649	3.3	294.1	628.6	27.756
Belgium	0.0474	63.406	0.5	37.2	182.8	8.0715
Former Czech.	0.0378	160.988	11.24	5119.8	2264.6	100
Denmark	0.2735	58.138	−2.5	369.72	536.7	23.698
Finland	0.0619	46.182	2.4	154.72	114.3	5.0453
France	0.0428	149.22	42.4	7726.3	309.2	13.65
Greece	0.1076	16.83	−1.62	22.23	45.5	2.0095
Hungary	0.0381	13.66	2.51	54.72	17.9	0.7901
Italy	0.1191	25.22	12.5	431.19	108.1	4.7726
Luxembourg	0.5178	5.56	−0.73	1.4	5.8	0.2572
Netherlands	0.0985	54.98	−6.18	329.7	424.4	18.741
Norway	0.1356	21.056	0.94	16.75	30.6	1.3508
Poland	0.0956	46.67	−8.03	−18.57	333.7	14.734
Romania	0.0333	19.87	5.5	147.1	35.8	1.5803
Spain	0.0016	245.43	10.2	2563.2	527.8	23.305
Sweden	0.0732	73.35	4.1	147.1	201.7	8.9075
Switzerland	3.2893	17.87	5.56	55.8	76.5	3.378
Turkey	0.0099	147.82	8.1	1698.5	698.65	30.851
UK	0.0061	200.5	15.6	4452.1	759.9	33.551

Looking at Table 2 is worth mentioning that large industrial upwind counties seem to have a large benefit area. Looking at the European Monitoring and Evaluation Program (EMEP) and the provided transfer coefficients matrices with emissions and depositions between the European countries it can be seen that the countries with large benefit areas are those with large numbers on the diagonal indicating the significance of the domestic sources of pollution [1]. At the same time the large off–diagonal transfer coefficients show the influence of one country on another in terms of the externalities imposed by the Eastern European countries on the others and the transboundary nature of the problem. In the same lines, near to the sea countries may face small benefit areas as the damage caused by acidification depends on where the depositions occur. In the case of occurrence over the sea it is less likely to have much harmful effect, as the sea is naturally alkaline. Similarly if it occurs over sparsely populated areas with acid tolerant soils then the damage is low, [10].

4 Conclusions and Policy Implications

The typical approach defining the optimal pollution level has been to equate the marginal (of an extra unit of pollution) damage cost with the corresponding marginal abatement cost. An efficient level of emissions maximizes the net benefit, that is, the difference between abatement and damage costs. Therefore the identification of this efficient level shows the level of benefits maximization, which is the resulting output level if external costs (damages) are fully internalized.

In this paper the corresponding optimal cost and benefit points were evaluated analytically. We shown that the optimal pollution level can be evaluated only under certain conditions. From the empirical findings is clear that the evaluation of the "calibrated" Benefit Area, as it was developed, provides an index to compare the different policies adopted from different countries. In this way a comparison of different policies can be performed. Certainly the policy with the maximum Benefit Area is the best, and the one with the minimum is the worst. Clearly the index BA^c provides a new measure for comparing the adopted policies.

It is clear that due to the model selection, the regression fit of the model, the undergoing errors and the propagation create a Risk associated with the value of the Benefit Area. This Associated Risk is that we try to reduce, choosing the best model, and collecting the appropriately data.

Policy makers may have multiple objectives with efficiency and sustainability being high priorities. Environmental policies should consider that economic development is not uniform across regions and may differ significantly, [12]. At the same time reforming economic policies to cope with EU enlargement may face problems and this may in turn affect their economic efficiencies, [11].

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An extended version of this presentation with the consideration of all combinations of linear, quadratic and exponential abatement and damage cost functions as well as the relation to the existing relative literature can be fount in [10].

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