

Chapter 16

Intelligent Systems in Total Quality Management

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Abstract Total Quality Management (TQM) is a widely known and applied concept by organizations for continuous improvement in the workplace. This philosophy is based on eight main principles: customer focus, leadership, involvement of employee, system approach, continuous improvement, process approach, and facts based decision making, and mutually beneficial supplier relationship. TQM includes statistical analysis of data, implementation of corrective and preventive actions, measurement of performance indicators of process and also advancement on actions for continuous improvement. The aim of Statistical Process Control (SPC) is to detect the variation and nonconforming units for improvement in the quality of process. While manually collecting data, the ambiguity or vagueness exists in the data which are called fuzzy data from measurement system or human experts. Fuzzy data can be analyzed by fuzzy control charts in SPC. Fuzzy control charts are implemented for monitoring and analyzing process, and reducing the variability of process. Also, an intelligent system is developed to eliminate or reduce uncertainty on data by using a fuzzy approach.

Keywords Total quality management • Intelligent systems • Fuzzy control charts

16.1 Introduction to Total Quality Management

Quality is one of the most important concepts to meet customer satisfaction. Customers select among competitive product or services according to satisfaction level that not only meet the specifications and functions, but also offer an attractive quality for the product or services.

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The definitions of quality change with regard to the needs and expectations of customers. While quality is defined as “conformance to requirements” by Crosby and “fitness for use” by Juran in early literature of quality history, nowadays quality is seen as “the totality of features and characteristics of a product or service that bear on its ability to satisfy stated or implied needs” by ISO 9000 series, moreover “Quality denotes an excellence in goods and services, especially to the degree they conform to requirements and satisfy customers” by ASQC. As seen in the latest definition of quality, customer satisfaction comes to the fore, especially ISO 9001:2000, Quality Management System—Requirements, standard based on the principles of Total Quality Management approach. In the 1980s, it was accepted that quality is a management system and should be applied to every process and activity of a company.

TQM is a management system to reach successful results through customer satisfaction by achieving cultural change in an organization. Any Organization sets out and manages its processes by defining a customer focused policy and strategies, and by considering partnerships and resources with all employee’s contributions for continual improvement. Thus, the organization would obtain excellent results on key performance, employee, customer and society by implementing the TQM approach. We can easily argue that engineering management is achieved by implementing TQM principles.

For development and deployment of TQM worldwide, several models are provided and prizes are given to organizations. Deming Model in Japan has been applied since 1951, Malcom Baldrige Model in the USA since 1986, European Foundation for Quality Management (EFQM) for Excellent Model in Europe since 1992 are all applied by companies or non-profit organizations. Also EFQM Model has been applied in Turkey since 1993 by KalDer. KalDer is a Turkish Society that aims to establish and develop the concepts of TQM and to boost the competitiveness of organizations. EFQM Model is described as in Fig. 16.1.

EFQM Model presents the framework to the management to apply TQM philosophy. EFQM Model underlines that “Excellent results of organization associated

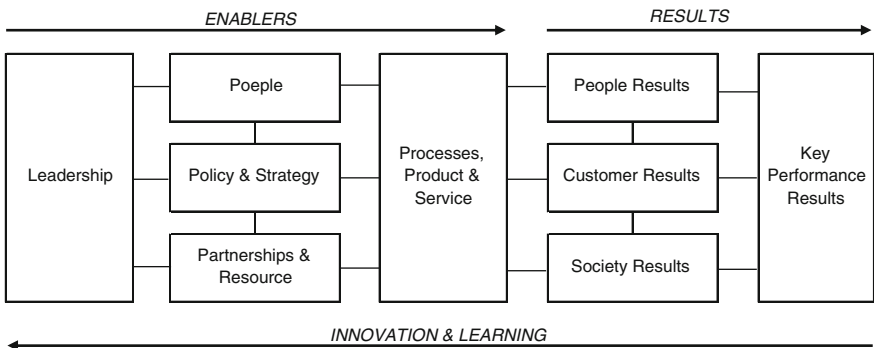


Fig. 16.1 EFQM model (<http://www.efqm.org>)

with people, customer, society and key performance can be achieved through appropriate leadership managing people, policy and strategy, partnership and resource”. TQM states eight main principles for applying the EFQM model.

16.2 Total Quality Management Principles

Total Quality Management has eight principles and ISO 9000 series are based on these principles (Montgomery 2009). The leaders consider these principles to manage organizations according to the TQM philosophy. The eight principles are defined in ISO 9000:2005, Quality Management System—Fundamentals and vocabulary, and in ISO 9004:2009, managing for the sustained success of an organization—A Quality Management Approach.

16.2.1 Leadership

Leaders understand the whole partnership’s needs which come from customers, owners, employees, suppliers etc., organize and establish the vision, mission and strategies of organization and motivate the people in this direction. Leaders also set the personal goals, provide sources and encourage their employees to achieve these goals to reach the organization’s objectives (Kolarik 1995).

16.2.2 Customer Focus

Organizations make research and understand customer’s current and future needs and expectations by defining and using several tools and ways for collecting and analyzing the customer’s data. Organizational plans and actions are described so that they meet these needs which are reflected on the product or services. In addition, organizations establish a systematic approach to manage customer relations.

16.2.3 Involvement of Employee

People at all levels are motivated and committed to achieve organization’s goals for continual improvement. Reachable and accountable personal goals are evaluated periodically by managers. Moreover, personal goals meet at least one of the organization’s strategies. The leader provides the necessary training and resources to achieve employee’s goal.

16.2.4 Process Approach

When organizations define their process and manage them by defining and monitoring process performance indications, organization's outputs can be seen to be more efficient. Indeed, the process approach is provided to see and manage the activities. A continuation improvement, lower cost and shorter production time lead to efficiency. Process responsible people, process performance indicators, process inputs/outputs, sub-process etc. are identified while establishing processes. Organizations also define the main process, support processes, sub-processes, and detail processes. Moreover they can define the critical processes that support their goals and strategies which need to be improved. The process performance indicators are reviewed periodically by the top management.

16.2.5 System Approach

Documentation, procedures, instructions, forms etc. help to set the organization's system. These documents show the organization's way that performs its work. The System approach contributes to the organization's efficiency and effectiveness by achieving its goals and strategies.

16.2.6 Continuous Improvement

Continuous improvement is provided with planning activities and their resources at all levels for achieving the organization's goals. Monitoring performance indicators, reviewing the organization's results by top management, corrective and preventive actions, and employee's activities result in continual improvement. Continuous improvement can be achieved by employee's contributions.

16.2.7 Facts Based Decision Making

The leader makes decisions based on the analysis of data from process. Therefore, leader can take effective decisions. The accuracy and reliability of data should be also ensured. While collecting data, the information's systems are used both to provide the accuracy of data and to store it. The statistical methods are used for analyzing data, such as, hypothesis tests, variance analysis, regression analysis, and so forth.

16.2.8 Mutually Beneficial Supplier Relationship

Both the organizations and their suppliers set the relationships to create value mutually beneficially. Additionally, organizations set long-term relations with their suppliers. They co-operate on training, continuous improvement, setting open and speed communications and answering responses. Furthermore, they can operate to increase the ability to create value.

16.3 Total Quality Management with Intelligent Systems

Intelligent systems connect the engineering tools with human thinking. Therefore, intelligent systems behave as a human decision mechanism by reflecting human sensitivity. Above all, fuzzy systems consider human linguistic expressions. When we look at this perspective, fuzzy systems are thought as an intelligent systems. Linguistic expressions are used frequently in decision making stages in the real world. Linguistic terms like ‘perfect’, ‘good’, ‘medium’ an etc.; are represented by fuzzy numbers in the fuzzy set theory. Many fuzzy statistical tools or fuzzy quality control techniques are developed for analyzing linguistic terms regarding fuzzy rules. In this manner, fuzzy distributions, fuzzy confidence intervals, fuzzy hypotheses, fuzzy control charts exist in the literature. Statistical process control is the main topic of TQM. In fuzzy statistical process control, fuzzy control charts are used for monitoring and analyzing the process. When data collected from process include ambiguities, the classical control charts are not applicable to evaluate the fuzzy data. Hence, fuzzy control charts are required to examine the fuzzy data. Fuzzy control charts play an important role to represent uncertainty that comes from human subjectivity and measurement systems. In addition, ANFIS (Adaptive Neuro Fuzzy Inference Systems), ANN (Artificial Neural Network), fuzzy QFD (Quality Function Deployment) and fuzzy FMEA (Failure Model and Effect Analysis) are intelligent systems in TQM.

Fuzzy control charts for both fuzzy variable and attribute data are introduced in the literature. While some of them use transformation techniques, few are based on fuzzy rules. In the following section, fuzzy control charts for variable and attribute fuzzy data are given in detail with formulations.

16.4 Fuzzy Statistical Process Control

Statistical process control (SPC) is an approach that uses statistical techniques to monitor the process Control charts proposed by W.A. Shewhart in 1920s. It has a widespread application especially in the production processes. These control charts were designed to monitor a process for both shifts in mean and variance of a single

quality characteristic. Data from process may have uncertainty that comes from the measurement system, including operators and gages, and environmental conditions. The fuzzy set theory is a very useful tool to handle this uncertainty. Fuzzy data are analysed by fuzzy control charts because fuzzy data cannot be evaluated by traditional control charts. Published the first papers on fuzzy control charts, Raz and Wang (1990) and Wang and Raz (1990). Whereas Kanagawa et al. (1993) modified Wang and Raz's control charts. The theoretical base of α -cut control chart was firstly introduced by Gülbay et al. (2004). Also, Gülbay and Kahraman (2006a, b) proposed the direct fuzzy approach. Faraz and Moghadam (2007) introduced a fuzzy chart for controlling the process mean. The theoretical structure of fuzzy individual and moving range control charts with α -cuts and fuzzy $\tilde{\bar{X}} - \tilde{R}$ and $\tilde{\bar{X}} - \tilde{S}$ control charts were developed by Erginel (2008), and Şentürk and Erginel (2008), respectively. In addition, fuzzy regression control charts based on α -cut approximation was introduced by Şentürk (2010) and fuzzy \tilde{u} control charts with applications was presented by Şentürk et al. (2011). Fuzzy $\tilde{\bar{X}} - \tilde{S}$ control charts are applied to the packing process of food industry by Erginel et al. (2011). Fuzzy control charts with fuzzy rules were introduced by Kaya and Kahraman (2011) and Erginel (2014). Fuzzy EWMA control chart was introduced by Şentürk et al. (2014).

In fuzzy control charts, control limits can be transformed to fuzzy control limits by using fuzzy numbers and their several types of membership functions. Fuzzy control limits provide a more accurate and flexible evaluation. Some measures of central tendency in descriptive statistics can be used in fuzzy variable control charts and fuzzy attribute control charts for transforming fuzzy data to the crisp data. These measures can be used to convert fuzzy sets into scalars which are fuzzy mode, α -level fuzzy midrange, fuzzy median and fuzzy average. For the selection of these fuzzy measures, there is no theoretical basis. This selection is mainly based on the ease of computation or preference of the user. These fuzzy transformation techniques are introduced by Wang and Raz (1990) and given in the following section.

The fuzzy mode: f_{mode} : The fuzzy mode of a fuzzy set F is the value of the base variable where the membership function equals 1. This is stated as:

$$f_{\text{mode}} = \{x | \mu_F(x) = 1\}, \quad \forall x \in F. \quad (16.1)$$

It is unique if the membership function is unimodal.

The α -level fuzzy midrange: f_{mr}^α : The average of the end points of an α -cut. An α -cut, denoted by F_α , is a non fuzzy subset of the base variable x containing all the values with membership function values greater than or equal to α . Thus $f_\alpha = \{x | \mu_F(x) \geq \alpha\}$. If a_α and c_α are end points of α -cut F_α such that $a_\alpha = \text{Min}\{F_\alpha\}$ and $c_\alpha = \text{Max}\{F_\alpha\}$, then,

$$f_{mr}^{\alpha} = \frac{1}{2}(a_{\alpha} + c_{\alpha}) \quad (16.2)$$

The fuzzy median: f_{med} : This is the point that partitions the curve under the membership function of a fuzzy set into two equal regions satisfying the following equation:

$$\int_a^{f_{med}} \mu_F(x)dx = \int_{f_{med}}^c \mu_F(x)dx = \frac{1}{2} \int_a^c \mu_F(x)dx \quad (16.3)$$

where a and c are the end points in the base variable of the fuzzy set F such that $a < c$.

The fuzzy average: f_{avg} : Based on Zadeh, the fuzzy average is:

$$f_{avg} = Av(x; F) = \frac{\int_{x=0}^1 x\mu_F(x)dx}{\int_{x=0}^1 \mu_F(x)dx} \quad (16.4)$$

In the following section, the theoretical structure of fuzzy control charts for both variable and attribute fuzzy data combined with fuzzy transformation techniques are given.

16.4.1 Fuzzy Variable Control Charts

If fuzzy data are a measurable scale, fuzzy variable control charts are available to use for evaluating the process. For analyzing both the shift in mean and deviation on fuzzy data from process, fuzzy $\tilde{\bar{X}}$ and \tilde{R} control charts, fuzzy $\tilde{\bar{X}}$ and \tilde{S} control charts and fuzzy individual (\tilde{X}) and moving range ($M\tilde{R}$) control chart are introduced in the following sections. Also, fuzzy regression control charts is developed to evaluate the tool wearing problem in process, and fuzzy EWMA control charts is implemented to detect the small shifts in fuzzy data.

16.4.1.1 Fuzzy $\tilde{\bar{X}}$ and \tilde{R} Control Charts

The R chart is a very popular control chart used to monitor the dispersion associated with a quality characteristic. Its simplicity of construction and maintenance makes the R chart very commonly used and the range is a good measure of variation for small subgroup sizes ($n < 10$). Also, fuzzy \tilde{R} control chart are used for evaluating the deviation of process with fuzzy data by Şentürk and Erginel (2009). In the fuzzy

case, triangular fuzzy ranges are represented as (R_{aj}, R_{bj}, R_{cj}) fuzzy numbers for j fuzzy sample. R_{aj}, R_{bj}, R_{cj} are calculated as follows:

$$R_{aj} = X_{\max,aj} - X_{\min,cj} \quad j = 1, 2, \dots, m \tag{16.5}$$

$$R_{bj} = X_{\max,bj} - X_{\min,bj} \quad j = 1, 2, \dots, m \tag{16.6}$$

$$R_{cj} = X_{\max,cj} - X_{\min,aj} \quad j = 1, 2, \dots, m \tag{16.7}$$

where $(X_{\max,aj}, X_{\max,bj}, X_{\max,cj})$ is the maximum fuzzy number in the sample and $(X_{\min,aj}, X_{\min,bj}, X_{\min,cj})$ is the minimum fuzzy number in the sample. Then,

$$\bar{R}_a = \frac{\sum R_{aj}}{m} \quad \bar{R}_b = \frac{\sum R_{bj}}{m} \quad \bar{R}_c = \frac{\sum R_{cj}}{m} \tag{16.8}$$

Also, $\bar{R}_a, \bar{R}_b,$ and \bar{R}_c are the arithmetic means of the least possible values, the most possible values, and the largest possible values, respectively. While using \bar{R}_a, \bar{R}_b and, fuzzy \tilde{R} control chart limits can be obtained but they are represented by triangular fuzzy numbers as follows:

$$U\tilde{C}L_R = D_4(\bar{R}_a, \bar{R}_b, \bar{R}_c) \tag{16.9}$$

$$C\tilde{L}_R = (\bar{R}_a, \bar{R}_b, \bar{R}_c) \tag{16.10}$$

$$L\tilde{C}L_R = D_3(\bar{R}_a, \bar{R}_b, \bar{R}_c) \tag{16.11}$$

where D_3 and D_4 are control chart coefficients. An α -cut is a nonfuzzy set which comprises of all elements whose membership degrees are greater than or equal to α . Applying α -cuts of fuzzy sets, the values of \bar{R}_a^α and \bar{R}_c^α are determined as follows:

$$\bar{R}_a^\alpha = \bar{R}_a + \alpha(\bar{R}_b - \bar{R}_a) \tag{16.12}$$

$$\bar{R}_c^\alpha = \bar{R}_c - \alpha(\bar{R}_c - \bar{R}_b) \tag{16.13}$$

An α -cut fuzzy \tilde{R} control chart can be calculated as follows:

$$U\tilde{C}L_R^\alpha = D_4\bar{R}^\alpha = D_4(\bar{R}_a^\alpha, \bar{R}_b, \bar{R}_c^\alpha) \tag{16.14}$$

$$C\tilde{L}_R^\alpha = \bar{R}^\alpha = (\bar{R}_a^\alpha, \bar{R}_b, \bar{R}_c^\alpha) \tag{16.15}$$

$$L\tilde{C}R_R^\alpha = D_3\bar{R}^\alpha = D_3(\bar{R}_a^\alpha, \bar{R}_b, \bar{R}_c^\alpha) \tag{16.16}$$

When α -level fuzzy midrange transformation techniques are used to obtain the control limits, α -level fuzzy midrange for α -cut fuzzy \tilde{R} control limits can be calculated as follows:

$$UCL_{mr-R}^\alpha = D_3 f_{mr-R}^\alpha(C\tilde{L}) \tag{16.17}$$

$$CL_{mr-R}^\alpha = f_{mr-R}^\alpha(C\tilde{L}) = \frac{\bar{R}_a^\alpha + \bar{R}_c^\alpha}{2} \tag{16.18}$$

$$LCL_{mr-R}^\alpha = D_4 f_{mr-R}^\alpha(C\tilde{L}) \tag{16.19}$$

These control limits are used to give a decision such as “*in-control*” or “*out-of-control*” for a process. The condition of process control for each sample can be defined as,

$$Process\ control = \left\{ \begin{array}{ll} in-control & ,for\ LCL_{mr-R}^\alpha \leq S_{mr-R,j}^\alpha \leq UCL_{mr-R}^\alpha \\ out-of\ control & ,for\ otherwise \end{array} \right\} \tag{16.20}$$

where the definition of α -level fuzzy midrange of sample j for fuzzy \tilde{R} control chart is calculated as follows:

$$S_{mr-R,j}^\alpha = \frac{(R_{a_j} + R_{c_j}) + \alpha[(R_{b_j} - R_{a_j}) - (R_{c_j} - R_{b_j})]}{2} \tag{16.21}$$

The fuzzy \tilde{X} control chart is used to analyze shifts in mean in fuzzy environment. The triangular fuzzy numbers are represented as $(X_{a_j}, X_{b_j}, X_{c_j})$ for j . fuzzy sample. The center line, $C\tilde{L}$ is the arithmetic mean of fuzzy samples, $(\bar{\bar{X}}_a, \bar{\bar{X}}_b, \bar{\bar{X}}_c)$ are called general mean, and calculated as follows:

$$C\tilde{L} = (\bar{\bar{X}}_a, \bar{\bar{X}}_b, \bar{\bar{X}}_c) = \left(\frac{\sum_{j=1}^m \bar{X}_{a_j}}{m}, \frac{\sum_{j=1}^m \bar{X}_{b_j}}{m}, \frac{\sum_{j=1}^m \bar{X}_{c_j}}{m} \right) \tag{16.22}$$

The fuzzy \tilde{X} control limits based on ranges are calculated as following equations:

$$\begin{aligned} U\tilde{C}L_{\bar{x}} &= (\bar{\bar{X}}_a, + A_2\bar{\bar{R}}_a, \bar{\bar{X}}_b + A_2\bar{\bar{R}}_b, \bar{\bar{X}}_c + A_2\bar{\bar{R}}_c) \\ &= (L\tilde{C}L_1, L\tilde{C}L_2, L\tilde{C}L_3) \end{aligned} \tag{16.23}$$

$$C\tilde{L}_{\bar{x}} = (\bar{\bar{X}}_a, \bar{\bar{X}}_b, \bar{\bar{X}}_c) = (C\tilde{L}_1, C\tilde{L}_2, C\tilde{L}_3) \tag{16.24}$$

$$\begin{aligned} L\tilde{C}L_{\bar{x}} &= (\bar{\bar{X}}_a - A_2\bar{\bar{R}}_c, \bar{\bar{X}}_b - A_2\bar{\bar{R}}_b, \bar{\bar{X}}_c - A_2\bar{\bar{R}}_a) \\ &= (L\tilde{C}L_1, L\tilde{C}L_2, L\tilde{C}L_3) \end{aligned} \tag{16.25}$$

where, A_2 is a control chart coefficient and $U\tilde{C}L$ and $L\tilde{C}L$ are the upper and lower control limits, and $C\tilde{L}$ is the center of fuzzy \tilde{X} control chart. Similarly, α -cut fuzzy \tilde{X} control chart limits based on ranges can be stated as follows:

$$\begin{aligned}
 U\tilde{C}L_{\bar{X}}^\alpha &= (\bar{X}_a^\alpha + A_2\bar{R}_a^\alpha, \bar{X}_b + A_2\bar{R}_b, \bar{X}_c^\alpha + A_2\bar{R}_c^\alpha) \\
 &= (U\tilde{C}L_1^\alpha, U\tilde{C}L_2, U\tilde{C}L_3^\alpha)
 \end{aligned}
 \tag{16.26}$$

$$C\tilde{L}_{\bar{X}}^\alpha = (\bar{X}_a^\alpha, \bar{X}_b, \bar{X}_c^\alpha) = (C\tilde{L}_1^\alpha, C\tilde{L}_2, C\tilde{L}_3^\alpha)
 \tag{16.27}$$

$$\begin{aligned}
 L\tilde{C}L_{\bar{X}}^\alpha &= (\bar{X}_a^\alpha - A_2\bar{R}_a^\alpha, \bar{X}_b - A_2\bar{R}_b, \bar{X}_c^\alpha - A_2\bar{R}_c^\alpha) \\
 &= (L\tilde{C}L_1^\alpha, L\tilde{C}L_2, L\tilde{C}L_3^\alpha)
 \end{aligned}
 \tag{16.28}$$

For obtained α -level fuzzy midrange for α -cut fuzzy \tilde{X} control chart, α -level fuzzy midrange transformation techniques are used as follows:

$$UCL_{mr-\bar{X}}^\alpha = CL_{mr-\bar{X}}^\alpha + A_2\left(\frac{\bar{R}_a^\alpha + \bar{R}_c^\alpha}{2}\right)
 \tag{16.29}$$

$$CL_{mr-\bar{X}}^\alpha = f_{mr-\bar{X}}^\alpha(C\tilde{L}) = \frac{CL_{(\bar{X})1}^\alpha + CL_{(\bar{X})3}^\alpha}{2}
 \tag{16.30}$$

$$LCL_{mr-\bar{X}}^\alpha = CL_{mr-\bar{X}}^\alpha - A_2\left(\frac{\bar{R}_a^\alpha + \bar{R}_c^\alpha}{2}\right)
 \tag{16.31}$$

The condition of process control for each sample can be defined as,

$$\text{Process control} = \left\{ \begin{array}{ll} \text{in-control} & , \text{for } LCL_{mr-\bar{X}}^\alpha \leq S_{mr-\bar{X},j}^\alpha \leq UCL_{mr-\bar{X}}^\alpha \\ \text{out-of control} & , \text{for } \text{otherwise} \end{array} \right\}
 \tag{16.32}$$

where the definition of α -level fuzzy midrange of sample j for fuzzy \tilde{X} control chart is

$$S_{mr-\bar{X},j}^\alpha = \frac{(\bar{X}_{a_j} + \bar{X}_{c_j}) + \alpha[(\bar{X}_{b_j} - \bar{X}_{a_j}) - (\bar{X}_{c_j} - \bar{X}_{b_j})]}{2}
 \tag{16.33}$$

16.4.1.2 Fuzzy \tilde{X} and \tilde{S} Control Charts

When the sample size increases ($n > 10$), the utility of the range measure as a yardstick of dispersion decreases and as a result the standard deviation measure is preferred. Also, for evaluating the deviation of process with fuzzy numbers, fuzzy \tilde{S} control chart is used by Şentürk and Erginel (2009). Fuzzy \tilde{S} control chart limits can be obtained as follows:

$$U\tilde{C}L_S = B_4(\bar{S}_a, \bar{S}_b, \bar{S}_c) \quad (16.34)$$

$$C\tilde{L}_S = (\bar{S}_a, \bar{S}_b, \bar{S}_c) \quad (16.35)$$

$$L\tilde{C}L_S = B_3\bar{S} = B_3(\bar{S}_a, \bar{S}_b, \bar{S}_c) \quad (16.36)$$

where the fuzzy \tilde{S}_j is a standard deviation of sample j and B_3 and B_4 are control chart coefficients. The fuzzy average $\tilde{\bar{S}}$ is calculated as follows:

$$\tilde{S}_j = \sqrt{\frac{\sum_{i=1}^n [(X_a, X_b, X_c)_{ij} - (\bar{X}_a, \bar{X}_b, \bar{X}_c)_j]^2}{n-1}} \quad (16.37)$$

$$\tilde{\bar{S}} = \left(\frac{\sum_{j=1}^m S_{aj}}{m}, \frac{\sum_{j=1}^m S_{bj}}{m}, \frac{\sum_{j=1}^m S_{cj}}{m} \right) = (\bar{S}_a, \bar{S}_b, \bar{S}_c) \quad (16.38)$$

The α -cut fuzzy \tilde{S} control chart limits are managed as follows:

$$U\tilde{C}L_S^\alpha = B_4\bar{S}^\alpha = B_4(\bar{S}_a^\alpha, \bar{S}_b^\alpha, \bar{S}_c^\alpha) \quad (16.39)$$

$$C\tilde{L}_S^\alpha = \bar{S}^\alpha = (\bar{S}_a^\alpha, \bar{S}_b^\alpha, \bar{S}_c^\alpha) \quad (16.40)$$

$$L\tilde{C}R_S^\alpha = B_3\bar{S}^\alpha = B_3(\bar{S}_a^\alpha, \bar{S}_b^\alpha, \bar{S}_c^\alpha) \quad (16.41)$$

where

$$\bar{S}_a^\alpha = \bar{S}_a + \alpha(\bar{S}_b - \bar{S}_a) \quad (16.42)$$

$$\bar{S}_c^\alpha = \bar{S}_c - \alpha(\bar{S}_c - \bar{S}_b) \quad (16.43)$$

The control limits of α -level fuzzy midrange for α -cut fuzzy \tilde{S} control chart can be obtained in a similar way to α -cut fuzzy \tilde{R} control chart.

$$UCL_{mr-s}^\alpha = B_4 f_{mr-s}^\alpha(C\tilde{L}) \quad (16.44)$$

$$CL_{mr-s}^\alpha = f_{mr-s}^\alpha(C\tilde{L}) = \frac{\bar{S}_a^\alpha + \bar{S}_c^\alpha}{2} \quad (16.45)$$

$$LCL_{mr-s}^\alpha = B_3 f_{mr-s}^\alpha(C\tilde{L}) \quad (16.46)$$

The condition of process control for each sample can be defined as,

$$Process\ control = \left\{ \begin{array}{l} in-control \quad ,for \quad LCL_{mr-s}^\alpha \leq S_{mr-s,j}^\alpha \leq UCL_{mr-s}^\alpha \\ out-of\ control \quad ,for \quad otherwise \end{array} \right\} \tag{16.47}$$

where the definition of α -level fuzzy midrange of sample j for fuzzy \tilde{S} control chart is

$$S_{mr-s,j}^\alpha = \frac{(S_{aj} + S_{cj}) + \alpha[(S_{bj} - S_{aj}) - (S_{cj} - S_{bj})]}{2} \tag{16.48}$$

$(\bar{X}_a, \bar{X}_b, \bar{X}_c)$ and $(\bar{S}_a, \bar{S}_b, \bar{S}_c)$ are used for analyzing the shift in mean of fuzzy numbers. The control limits of fuzzy \tilde{X} Control Chart based on standard deviation are obtained as follows:

$$\begin{aligned} U\tilde{C}L_{\tilde{x}} &= (\bar{X}_a + A_3\bar{S}_a, \bar{X}_b + A_3\bar{S}_b, \bar{X}_c + A_3\bar{S}_c) \\ &= (L\tilde{C}L_1, LC\tilde{L}_2, L\tilde{C}L_3) \end{aligned} \tag{16.49}$$

$$C\tilde{L}_{\tilde{x}} = (\bar{X}_a, \bar{X}_b, \bar{X}_c) = (C\tilde{L}_1, C\tilde{L}_2, C\tilde{L}_3) \tag{16.50}$$

$$\begin{aligned} L\tilde{C}L_{\tilde{x}} &= (\bar{X}_a - A_3\bar{S}_a, \bar{X}_b - A_3\bar{S}_b, \bar{X}_c - A_3\bar{S}_c) \\ &= (L\tilde{C}L_1, LC\tilde{L}_2, L\tilde{C}L_3) \end{aligned} \tag{16.51}$$

where, A_3 is a control chart coefficient. The α -cut fuzzy \tilde{X} control chart limits based on standard deviation can be obtained as follows:

$$\begin{aligned} U\tilde{C}L_{\tilde{x}}^\alpha &= (\bar{X}_a^\alpha + A_3\bar{S}_a^\alpha, \bar{X}_b^\alpha + A_3\bar{S}_b^\alpha, \bar{X}_c^\alpha + A_3\bar{S}_c^\alpha) \\ &= (U\tilde{C}L_1^\alpha, U\tilde{C}L_2^\alpha, U\tilde{C}L_3^\alpha) \end{aligned} \tag{16.52}$$

$$C\tilde{L}_{\tilde{x}}^\alpha = (\bar{X}_a^\alpha, \bar{X}_b^\alpha, \bar{X}_c^\alpha) = (C\tilde{L}_1^\alpha, C\tilde{L}_2^\alpha, C\tilde{L}_3^\alpha) \tag{16.53}$$

$$\begin{aligned} L\tilde{C}L_{\tilde{x}}^\alpha &= (\bar{X}_a^\alpha - A_3\bar{S}_a^\alpha, \bar{X}_b^\alpha - A_3\bar{S}_b^\alpha, \bar{X}_c^\alpha - A_3\bar{S}_c^\alpha) \\ &= (L\tilde{C}L_1^\alpha, L\tilde{C}L_2^\alpha, L\tilde{C}L_3^\alpha) \end{aligned} \tag{16.54}$$

The control limits and center line for α -cut fuzzy \tilde{X} control chart based on standard deviation using α -level fuzzy midrange are:

$$UCL_{mr-\bar{X}}^\alpha = CL_{mr-\bar{X}}^\alpha + A_3 \left(\frac{\bar{S}_a^\alpha + \bar{S}_c^\alpha}{2} \right) \tag{16.55}$$

$$CL_{mr-\bar{X}}^\alpha = f_{mr-\bar{X}}^\alpha (C\tilde{L}) = \frac{CL_{(\bar{X})1}^\alpha + CL_{(\bar{X})3}^\alpha}{2} \tag{16.56}$$

$$LCL_{mr-\bar{X}}^\alpha = CL_{mr-\bar{X}}^\alpha - A_3 \left(\frac{\bar{S}_a^\alpha + \bar{S}_c^\alpha}{2} \right) \tag{16.57}$$

The condition of process control for each sample can be defined as,

$$Process\ control = \left\{ \begin{array}{ll} in-control & ,for\ LCL_{mr-\bar{X}}^\alpha \leq S_{mr-\bar{X},j}^\alpha \leq UCL_{mr-\bar{X}}^\alpha \\ out-of\ control & ,for\ otherwise \end{array} \right\} \tag{16.58}$$

16.4.1.3 Fuzzy Individual (\tilde{X}) and Moving Range (\tilde{MR}) Control Chart

When only one observation is sampled, the individual control chart is used for analyzing the shift in mean of process. In this case, moving ranges are used for evaluating the deviation of process. While using triangular fuzzy numbers, fuzzy moving ranges are calculated by Erginel (2008) with using fuzzy sample observations as follows:

$$MR_{aj} = X_{\max,a_j} - X_{\min,c_j} \tag{16.59}$$

$$MR_{bj} = X_{\max,b_j} - X_{\min,b_j} \tag{16.60}$$

$$MR_{cj} = X_{\max,c_j} - X_{\min,a_j} \tag{16.61}$$

After calculating $(MR_{aj}, MR_{bj}, MR_{cj})$, for $j = 2, 3, \dots, m - 1$, the mean of fuzzy moving range for fuzzy samples is given in the following.

$$\overline{MR}_a = \frac{\sum_{j=2}^{m-1} R_{aj}}{m-2} \quad \overline{MR}_b = \frac{\sum_{j=2}^{m-1} R_{bj}}{m-2} \quad \overline{MR}_c = \frac{\sum_{j=2}^{m-1} R_{cj}}{m-2} \tag{16.62}$$

Fuzzy moving range control charts limits:

$$U\tilde{C}L_{MR} = D_4(\overline{MR}_a, \overline{MR}_b, \overline{MR}_c) \tag{16.63}$$

$$C\tilde{L}_{MR} = (\overline{MR}_a, \overline{MR}_b, \overline{MR}_c) \tag{16.64}$$

$$\tilde{L}\tilde{C}L_{MR} = D_3(\overline{MR}_a, \overline{MR}_b, \overline{MR}_c) \tag{16.65}$$

α -cut fuzzy moving range control chart:

$$U\tilde{C}L_{MR}^\alpha = D_4(\overline{MR}_a^\alpha, \overline{MR}_b^\alpha, \overline{MR}_c^\alpha) \tag{16.66}$$

$$\tilde{C}\tilde{L}_{MR}^\alpha = (\overline{MR}_a^\alpha, \overline{MR}_b^\alpha, \overline{MR}_c^\alpha) \tag{16.67}$$

$$\tilde{L}\tilde{C}L_{MR}^\alpha = D_3(\overline{MR}_a^\alpha, \overline{MR}_b^\alpha, \overline{MR}_c^\alpha) \tag{16.68}$$

where

$$\overline{MR}_a^\alpha = \overline{MR}_a + \alpha(\overline{MR}_b - \overline{MR}_a) \tag{16.69}$$

$$\overline{MR}_c^\alpha = \overline{MR}_c - \alpha(\overline{MR}_c - \overline{MR}_b) \tag{16.70}$$

The control limits of α -level fuzzy median for α -cut fuzzy $M\tilde{R}$ control chart are calculated based on α -level fuzzy median transformation techniques as follows:

$$UCL_{med-MR}^\alpha = D_4 \left[\frac{1}{3}(\overline{MR}_a^\alpha + \overline{MR}_b^\alpha + \overline{MR}_c^\alpha) \right] \tag{16.71}$$

$$CL_{med-MR}^\alpha = f_{med-MR}^\alpha(\tilde{C}\tilde{L}) = \left[\frac{1}{3}(\overline{MR}_a^\alpha + \overline{MR}_b^\alpha + \overline{MR}_c^\alpha) \right] \tag{16.72}$$

$$LCL_{med-MR}^\alpha = D_3 \left[\frac{1}{3}(\overline{MR}_a^\alpha + \overline{MR}_b^\alpha + \overline{MR}_c^\alpha) \right] \tag{16.73}$$

The condition of process control for each sample can be defined as in Eq. (16.74).

$$Process\ control = \begin{cases} in-control & ,for\ LCL_{med-MR}^\alpha \leq S_{med,j-MR}^\alpha \leq UCL_{med-MR}^\alpha \\ out-of\ control & ,for\ otherwise \end{cases} \tag{16.74}$$

For a sample j , α -level fuzzy median $S_{med-MR,j}^\alpha$ can be calculated and provided as follows:

$$S_{med,j-MR}^\alpha = \frac{1}{3}(\overline{MR}_{a,j}^\alpha + \overline{MR}_{b,j}^\alpha + \overline{MR}_{c,j}^\alpha) \tag{16.75}$$

The fuzzy \tilde{X} control chart with moving ranges is used for evaluating the shifts in mean of process with fuzzy data. Fuzzy center line, fuzzy upper and fuzzy lower limits of fuzzy \tilde{X} control chart are obtained as follows:

$$U\tilde{C}L_x = \left(\bar{X}_a + \frac{3}{d_2} \overline{MR}_a, \bar{X}_b + \frac{3}{d_2} \overline{MR}_b, \bar{X}_c + \frac{3}{d_2} \overline{MR}_c \right) \quad (16.76)$$

$$C\tilde{L}_x = (\bar{X}_a, \bar{X}_b, \bar{X}_c) = (C\tilde{L}_1, C\tilde{L}_2, C\tilde{L}_3) \quad (16.77)$$

$$L\tilde{C}L_x = \left(\bar{X}_a - \frac{3}{d_2} \overline{MR}_c, \bar{X}_b - \frac{3}{d_2} \overline{MR}_b, \bar{X}_c - \frac{3}{d_2} \overline{MR}_a \right) \quad (16.78)$$

An α -cut fuzzy \tilde{X} control chart is obtained by integrating fuzzy moving range and α -cuts as in the following:

$$U\tilde{C}L_x^\alpha = \left(\bar{X}_a^\alpha + \frac{3}{d_2} \overline{MR}_a^\alpha, \bar{X}_b + \frac{3}{d_2} \overline{MR}_b, \bar{X}_c^\alpha + \frac{3}{d_2} \overline{MR}_c^\alpha \right) \quad (16.79)$$

$$C\tilde{L}_x^\alpha = (\bar{X}_a^\alpha, \bar{X}_b, \bar{X}_c^\alpha) = (C\tilde{L}_1^\alpha, C\tilde{L}_2, C\tilde{L}_3^\alpha) \quad (16.80)$$

$$L\tilde{C}L_x^\alpha = \left(\bar{X}_a^\alpha - \frac{3}{d_2} \overline{MR}_c^\alpha, \bar{X}_b - \frac{3}{d_2} \overline{MR}_b, \bar{X}_c^\alpha - \frac{3}{d_2} \overline{MR}_a^\alpha \right) \quad (16.81)$$

α -level fuzzy median for α -cut fuzzy \tilde{X} control chart can be also calculated as follows:

$$UCL_{med-x}^\alpha = CL_{med-x}^\alpha + \frac{3}{d_2} \left[\frac{1}{3} (\overline{MR}_a^\alpha + \overline{MR}_b + \overline{MR}_c^\alpha) \right] \quad (16.82)$$

$$CL_{med-x}^\alpha = f_{med-x}^\alpha(C\tilde{L}) = \frac{1}{3} (CL_{(x)1}^\alpha + CL_{(x)2} + CL_{(x)3}^\alpha) \quad (16.83)$$

$$LCL_{med-x}^\alpha = CL_{med-x}^\alpha - \frac{3}{d_2} \left[\frac{1}{3} (\overline{MR}_a^\alpha + \overline{MR}_b + \overline{MR}_c^\alpha) \right] \quad (16.84)$$

where d_2 is the control chart coefficient. The condition of process control for each sample can be defined as,

$$Process\ control = \left\{ \begin{array}{ll} in-control & ,for\ LCL_{med-x}^\alpha \leq S_{med,j-x}^\alpha \leq UCL_{med-x}^\alpha \\ out-of\ control & ,for\ otherwise \end{array} \right\} \quad (16.85)$$

For a sample j , α -level fuzzy median $S_{med-x,j}^\alpha$ can be calculated as follows:

$$S_{med-x,j}^\alpha = \frac{1}{3} (X_{aj}^\alpha + X_{bj} + X_{cj}^\alpha) \quad (16.86)$$

16.4.2 Fuzzy Attribute Control Charts

If the quality characteristics are represented by attributes, fuzzy attribute control charts are used to evaluate the process such as fuzzy p for the fuzzy fraction of nonconforming, fuzzy np for fuzzy nonconforming units, fuzzy c for fuzzy number of nonconformities and fuzzy u control charts for fuzzy nonconformities per unit.

16.4.2.1 Fuzzy \tilde{p} Control Charts

Fuzzy \tilde{p} -control chart based on constant sample size:

In the fuzzy case, the fuzzy number of nonconforming units is stated by triangular fuzzy numbers $(d_{a_j}, d_{b_j}, d_{c_j})$. In this case, the fuzzy fraction nonconforming can be expressed by triangular fuzzy numbers $(p_{a_j}, p_{b_j}, p_{c_j})$. $(\bar{p}_a, \bar{p}_b, \bar{p}_c)$ is the fuzzy average of the fractions nonconforming, where $j = 1, 2, \dots, m$.

$$p_{a_j} = \frac{d_{a_j}}{n} \quad p_{b_j} = \frac{d_{b_j}}{n} \quad p_{c_j} = \frac{d_{c_j}}{n} \tag{16.87}$$

$$\bar{p}_a = \frac{\sum p_{a_j}}{m} \quad \bar{p}_b = \frac{\sum p_{b_j}}{m} \quad \bar{p}_c = \frac{\sum p_{c_j}}{m} \tag{16.88}$$

Fuzzy center line, fuzzy upper and fuzzy lower limits of fuzzy \tilde{p} -control chart are obtained as follows, that is to say by using fuzzy averages the fraction nonconforming $(\bar{p}_a, \bar{p}_b, \bar{p}_c)$ and the rules of fuzzy arithmetic are described by Kahraman and Yavuz (2010).

$$U\tilde{C}L_p = \left(\bar{p}_a + 3\sqrt{\frac{\bar{p}_a(1-\bar{p}_a)}{n}}, \bar{p}_b + 3\sqrt{\frac{\bar{p}_b(1-\bar{p}_b)}{n}}, \bar{p}_c + 3\sqrt{\frac{\bar{p}_c(1-\bar{p}_c)}{n}} \right) \tag{16.89}$$

$$\tilde{C}L_p = (\bar{p}_a, \bar{p}_b, \bar{p}_c) \tag{16.90}$$

$$L\tilde{C}L_p = \left(\bar{p}_a - 3\sqrt{\frac{\bar{p}_a(1-\bar{p}_a)}{n}}, \bar{p}_b - 3\sqrt{\frac{\bar{p}_b(1-\bar{p}_b)}{n}}, \bar{p}_c - 3\sqrt{\frac{\bar{p}_c(1-\bar{p}_c)}{n}} \right) \tag{16.91}$$

The α -cut fuzzy \tilde{p} -control chart is obtained as follows:

$$U\tilde{C}L_p^\alpha = \left(\bar{p}_a^\alpha + 3\sqrt{\frac{\bar{p}_a^\alpha(1-\bar{p}_a^\alpha)}{n}}, \bar{p}_b + 3\sqrt{\frac{\bar{p}_b(1-\bar{p}_b)}{n}}, \bar{p}_c + 3\sqrt{\frac{\bar{p}_c(1-\bar{p}_c)}{n}} \right) \tag{16.92}$$

$$\tilde{CL}_p^\alpha = (\bar{p}_a^\alpha, \bar{p}_b, \bar{p}_c^\alpha) \tag{16.93}$$

$$L\tilde{CL}_p^\alpha = \left(\bar{p}_a^\alpha - 3\sqrt{\frac{\bar{p}_c^\alpha(1-\bar{p}_c^\alpha)}{n}}, \bar{p}_b - 3\sqrt{\frac{\bar{p}_b(1-\bar{p}_b)}{n}}, \bar{p}_c - 3\sqrt{\frac{\bar{p}_a^\alpha(1-\bar{p}_a^\alpha)}{n}} \right) \tag{16.94}$$

where;

$$\bar{p}_a^\alpha = \bar{p}_a + \alpha(\bar{p}_b - \bar{p}_a) \tag{16.95}$$

$$\bar{p}_c^\alpha = \bar{p}_c - \alpha(\bar{p}_c - \bar{p}_b) \tag{16.96}$$

By using these formulations, the fuzzy center line, fuzzy upper and fuzzy lower limits of α -level fuzzy median for α -cut fuzzy \tilde{p} -control chart can be calculated as below;

$$UCL_{med-p}^\alpha = CL_{med-p}^\alpha + 3\sqrt{\frac{CL_{med-p}^\alpha(1-CL_{med-p}^\alpha)}{n}} \tag{16.97}$$

$$CL_{med-p}^\alpha = \frac{1}{3}(\bar{p}_a^\alpha + \bar{p}_b + \bar{p}_c^\alpha) \tag{16.98}$$

$$LCL_{med-p}^\alpha = CL_{med-p}^\alpha - 3\sqrt{\frac{CL_{med-p}^\alpha(1-CL_{med-p}^\alpha)}{n}} \tag{16.99}$$

The condition of process control for each sample can be defined as;

$$Process\ control = \left\{ \begin{array}{ll} in-control & ,for\ LCL_{med-p}^\alpha \leq S_{med-p,j}^\alpha \leq UCL_{med-p}^\alpha \\ out-of\ control & ,for\ otherwise \end{array} \right\} \tag{16.100}$$

Fuzzy \tilde{p} -control chart based on variable sample size:

When the sample size is not constant, variable sample size should be used in fuzzy p -control chart. In this case, control limits are calculated using each individual sample size. The fuzzy fraction nonconforming for each sample and their fuzzy averages are calculated in line with the following equations, respectively;

$$p_{a_j} = \frac{d_{a_j}}{n_j} \quad p_{b_j} = \frac{d_{b_j}}{n_j} \quad p_{c_j} = \frac{d_{c_j}}{n_j} \tag{16.101}$$

$$\bar{p}_a = \frac{\sum d_{a_j}}{\sum n_j} \quad \bar{p}_b = \frac{\sum d_{b_j}}{\sum n_j} \quad \bar{p}_c = \frac{\sum d_{c_j}}{\sum n_j} \tag{16.102}$$

The control limits can be calculated in fuzzy \tilde{p} -control chart for each n_j by using triangular membership function and fuzzy average of sample fraction nonconforming such as;

$$U\tilde{C}L_{p,j} = \left(\bar{p}_a + 3\sqrt{\frac{\bar{p}_a(1-\bar{p}_a)}{n_j}}, \bar{p}_b + 3\sqrt{\frac{\bar{p}_b(1-\bar{p}_b)}{n_j}}, \bar{p}_c + 3\sqrt{\frac{\bar{p}_c(1-\bar{p}_c)}{n_j}} \right) \quad (16.103)$$

$$\tilde{C}L_{p,j} = (\bar{p}_a, \bar{p}_b, \bar{p}_c) \quad (16.104)$$

$$L\tilde{C}L_{p,j} = \left(\bar{p}_a - 3\sqrt{\frac{\bar{p}_a(1-\bar{p}_a)}{n_j}}, \bar{p}_b - 3\sqrt{\frac{\bar{p}_b(1-\bar{p}_b)}{n_j}}, \bar{p}_c - 3\sqrt{\frac{\bar{p}_c(1-\bar{p}_c)}{n_j}} \right) \quad (16.105)$$

α -cut control limits for fuzzy \tilde{p} -control chart based on variable sample size is presented in the following equations:

$$U\tilde{C}L_{p,j}^\alpha = \left(\bar{p}_a^\alpha + 3\sqrt{\frac{\bar{p}_a^\alpha(1-\bar{p}_a^\alpha)}{n_j}}, \bar{p}_b + 3\sqrt{\frac{\bar{p}_b(1-\bar{p}_b)}{n_j}}, \bar{p}_c + 3\sqrt{\frac{\bar{p}_c^\alpha(1-\bar{p}_c^\alpha)}{n_j}} \right) \quad (16.106)$$

$$\tilde{C}L_{p,j}^\alpha = (\bar{p}_a^\alpha, \bar{p}_b, \bar{p}_c^\alpha) \quad (16.107)$$

$$L\tilde{C}L_{p,j}^\alpha = \left(\bar{p}_a^\alpha - 3\sqrt{\frac{\bar{p}_a^\alpha(1-\bar{p}_a^\alpha)}{n_j}}, \bar{p}_b - 3\sqrt{\frac{\bar{p}_b(1-\bar{p}_b)}{n_j}}, \bar{p}_c - 3\sqrt{\frac{\bar{p}_c^\alpha(1-\bar{p}_c^\alpha)}{n_j}} \right) \quad (16.108)$$

Control limits of α -level fuzzy median for α -cut fuzzy \tilde{p} -control chart based on variable sample size can be calculated by considering fuzzy median transformation technique as follows;

$$UCI_{med-p,j}^\alpha = CL_{med-p}^\alpha + 3\sqrt{\frac{CL_{med-p}^\alpha(1-CL_{med-p}^\alpha)}{n_j}} \quad (16.109)$$

$$CL_{med-p}^\alpha = \frac{1}{3}(\bar{p}_a^\alpha + \bar{p}_b + \bar{p}_c^\alpha) \quad (16.110)$$

$$LCL_{med-p,j}^\alpha = CL_{med-p}^\alpha - 3\sqrt{\frac{CL_{med-p}^\alpha(1-CL_{med-p}^\alpha)}{n_j}} \quad (16.111)$$

The condition of process control for each sample is

$$Process\ control = \left\{ \begin{array}{l} in-control \quad ,for \quad LCL_{med-p,j}^\alpha \leq S_{med-p,j}^\alpha \leq UCL_{med-p,j}^\alpha \\ out-of\ control \quad ,for \quad otherwise \end{array} \right\} \tag{16.112}$$

α -level fuzzy median value for each sample can be stated as follows;

$$S_{med-p,j}^\alpha = \frac{1}{3} \left(p_{a,j}^\alpha + p_{b,j} + p_{c,j}^\alpha \right) j = 1, 2, \dots, m \tag{16.113}$$

16.4.2.2 Fuzzy $n\tilde{p}$ Control Charts

When sample size is constant, fuzzy $n\tilde{p}$ -control chart is more accessible to deal with *the number of nonconforming units*. In many situations, observation of the number of nonconforming units is easier to interpret than the usual fraction nonconforming control chart. The average of the sample number of nonconforming units can be expressed in triangular fuzzy numbers $(n\bar{p}_a, n\bar{p}_b, n\bar{p}_c)$ as follows;

$$n\bar{p}_a = \frac{\sum_{j=1}^m d_{a_j}}{m} \quad n\bar{p}_b = \frac{\sum_{j=1}^m d_{b_j}}{m} \quad n\bar{p}_c = \frac{\sum_{j=1}^m d_{c_j}}{m} \tag{16.114}$$

The limits of fuzzy $n\tilde{p}$ -control chart can be calculated with the following equations:

$$U\tilde{C}L_{np} = \left(n\bar{p}_a + 3\sqrt{n\bar{p}_a(1 - \bar{p}_a)}, n\bar{p}_b + 3\sqrt{n\bar{p}_b(1 - \bar{p}_b)}, n\bar{p}_c + 3\sqrt{n\bar{p}_c(1 - \bar{p}_c)} \right) \tag{16.115}$$

$$\tilde{C}L_{np} = (n\bar{p}_a, n\bar{p}_b, n\bar{p}_c) \tag{16.116}$$

$$L\tilde{C}L_{np} = \left(n\bar{p}_a - 3\sqrt{n\bar{p}_a(1 - \bar{p}_a)}, n\bar{p}_b - 3\sqrt{n\bar{p}_b(1 - \bar{p}_b)}, n\bar{p}_c - 3\sqrt{n\bar{p}_c(1 - \bar{p}_c)} \right) \tag{16.117}$$

When applying α -cut on fuzzy $n\tilde{p}$ -control chart, the limits of α -cut fuzzy $n\tilde{p}$ -control chart are obtained as follows:

$$U\tilde{C}L_{np}^\alpha = \left(n\bar{p}_a^\alpha + 3\sqrt{n\bar{p}_a^\alpha(1 - \bar{p}_a^\alpha)}, n\bar{p}_b + 3\sqrt{n\bar{p}_b(1 - \bar{p}_b)}, n\bar{p}_c^\alpha + 3\sqrt{n\bar{p}_c^\alpha(1 - \bar{p}_c^\alpha)} \right) \tag{16.118}$$

$$\tilde{C}L_{np}^\alpha = (n\bar{p}_a^\alpha, n\bar{p}_b, n\bar{p}_c^\alpha) \tag{16.119}$$

$$L\tilde{C}L_{np}^\alpha = \left(n\bar{p}_a^\alpha - 3\sqrt{n\bar{p}_a^\alpha(1-\bar{p}_a^\alpha)}, n\bar{p}_b + 3\sqrt{n\bar{p}_b(1-\bar{p}_b)}, n\bar{p}_c^\alpha - 3\sqrt{n\bar{p}_c^\alpha(1-\bar{p}_c^\alpha)} \right) \tag{16.120}$$

By incorporating fuzzy median transformation technique with α -cut fuzzy $n\bar{p}$ -control chart, the following equations can be expressed for α -level fuzzy median for α -cut fuzzy $n\bar{p}$ -control chart;

$$UCL_{med-np}^\alpha = CL_{med-np}^\alpha + 3\sqrt{CL_{med-np}^\alpha \left(1 - \frac{CL_{med-np}^\alpha}{n}\right)} \tag{16.121}$$

$$CL_{med-np}^\alpha = \frac{1}{3} (n\bar{p}_a^\alpha + n\bar{p}_b^\alpha + n\bar{p}_c^\alpha) \tag{16.122}$$

$$LCL_{med-np}^\alpha = CL_{med-np}^\alpha - 3\sqrt{CL_{med-np}^\alpha \left(1 - \frac{CL_{med-np}^\alpha}{n}\right)} \tag{16.123}$$

The condition of process control for each sample can be defined as;

$$Process\ control = \left\{ \begin{array}{ll} in-control & ,for\ LCL_{med-np}^\alpha \leq S_{med-np,j}^\alpha \leq UCL_{med-np}^\alpha \\ out-of\ control & ,for\ otherwise \end{array} \right\} \tag{16.124}$$

where

$$S_{med-np,j}^\alpha = \frac{1}{3} (np_{a,j}^\alpha + np_{b,j} + np_{c,j}^\alpha) \tag{16.125}$$

16.4.2.3 Fuzzy \tilde{c} Control Chart

When we are interested in fuzzy number of nonconformities, fuzzy \tilde{c} control chart is used to evaluate the process. Center line $\tilde{C}L$ is the mean of fuzzy samples and can be represented by triangular fuzzy numbers (Gülbay and Kahraman 2006a, b):

$$\tilde{C}L = \left(\frac{\sum c_{a_i}}{n}, \frac{\sum c_{b_i}}{n}, \frac{\sum c_{c_j}}{n} \right) \tag{16.126}$$

$$U\tilde{C}L_c = (\bar{c}_a + 3\sqrt{\bar{c}_a}, \bar{c}_b + 3\sqrt{\bar{c}_b}, \bar{c}_c + 3\sqrt{\bar{c}_c}) \tag{16.127}$$

$$\tilde{C}L_c = (\bar{c}_a, \bar{c}_b, \bar{c}_c) \tag{16.128}$$

$$\tilde{LCL}_c = (\bar{c}_a - 3\sqrt{\bar{c}_a}, \bar{c}_b - 3\sqrt{\bar{c}_b}, \bar{c}_c - 3\sqrt{\bar{c}_c}) \quad (16.129)$$

α -cut control limits for fuzzy \tilde{c} -control chart are presented in the following equations:

$$U\tilde{CL}_c^\alpha = (\bar{c}_a^\alpha + 3\sqrt{\bar{c}_a^\alpha}, \bar{c}_b^\alpha + 3\sqrt{\bar{c}_b^\alpha}, \bar{c}_c^\alpha + 3\sqrt{\bar{c}_c^\alpha}) \quad (16.130)$$

$$\tilde{CL}_c^\alpha = (\bar{c}_a^\alpha, \bar{c}_b, \bar{c}_c^\alpha) \quad (16.131)$$

$$L\tilde{CL}_c^\alpha = (\bar{c}_a^\alpha - 3\sqrt{\bar{c}_a^\alpha}, \bar{c}_b^\alpha - 3\sqrt{\bar{c}_b^\alpha}, \bar{c}_c^\alpha - 3\sqrt{\bar{c}_c^\alpha}) \quad (16.132)$$

α -level fuzzy median for an α -cut fuzzy- \tilde{c} control chart can be calculated by transforming α -level fuzzy median transformation technique as below:

$$UCL_{med-c}^\alpha = CL_{med-c}^\alpha + 3\sqrt{CL_{med-c}^\alpha} \quad (16.133)$$

$$CL_{med-c}^\alpha = \frac{1}{3}(\bar{c}_a^\alpha, \bar{c}_b, \bar{c}_c^\alpha) \quad (16.134)$$

$$LCL_{med-c}^\alpha = CL_{med-c}^\alpha - 3\sqrt{CL_{med-c}^\alpha} \quad (16.135)$$

The condition of process control for each sample can be defined as,

$$\text{Process control} = \left\{ \begin{array}{ll} \text{in-control} & , \text{for } LCL_{med-c}^\alpha \leq S_{med-cj}^\alpha \leq UCL_{med-c}^\alpha \\ \text{out-of control} & , \text{for } \text{otherwise} \end{array} \right\} \quad (16.136)$$

where

$$S_{med-cj}^\alpha = \frac{1}{3}(c_{a,j}^\alpha, c_{b,j}, c_{c,j}^\alpha) \quad (16.137)$$

16.4.2.4 Fuzzy \tilde{u} Control Chart

If we relate to the fuzzy number of nonconformities on one product, fuzzy \tilde{u} -control chart is used by Şentürk et al. (2011). In this case, the fuzzy number of nonconforming can be expressed by, triangular fuzzy numbers such as $(u_{a_j}, u_{b_j}, u_{c_j})$. Here $(\bar{u}_a, \bar{u}_b, \bar{u}_c)$ are the fuzzy averages of the number of nonconforming $(u_{a_j}, u_{b_j}, u_{c_j})$, respectively.

$$\bar{u}_a = \frac{\sum u_{a_j}}{m}, \bar{u}_b = \frac{\sum u_{b_j}}{m}, \bar{u}_c = \frac{\sum u_{c_j}}{m} \quad (16.138)$$

Fuzzy \tilde{u} -control chart are obtained by using fuzzy numbers and equations as follows:

$$U\tilde{C}L_u = \left(\bar{u}_a + 3\sqrt{\frac{\bar{u}_a}{n_j}}, \bar{u}_b + 3\sqrt{\frac{\bar{u}_b}{n_j}}, \bar{u}_c + 3\sqrt{\frac{\bar{u}_c}{n_j}} \right) \quad (16.139)$$

$$\tilde{C}L_u = (\bar{u}_a, \bar{u}_b, \bar{u}_c) \quad (16.140)$$

$$L\tilde{C}L_u = \left(\bar{u}_a - 3\sqrt{\frac{\bar{u}_a}{n_j}}, \bar{u}_b - 3\sqrt{\frac{\bar{u}_b}{n_j}}, \bar{u}_c - 3\sqrt{\frac{\bar{u}_c}{n_j}} \right) \quad (16.141)$$

Applying α -cut of a fuzzy set, the values of \bar{u}_a^α and \bar{u}_c^α , which are the α -cut representation of average numbers of nonconformities, respectively are determined as follows:

$$\bar{u}_a^\alpha = \bar{u}_a + \alpha(\bar{u}_b - \bar{u}_a) \quad (16.142)$$

$$\bar{u}_c^\alpha = \bar{u}_c - \alpha(\bar{u}_c - \bar{u}_b) \quad (16.143)$$

α -cut fuzzy \tilde{u} -control chart is obtained by integrating fuzzy \tilde{u} -control chart and α -cut as in the following equations:

$$U\tilde{C}L_u^\alpha = \left(\bar{u}_a^\alpha + 3\sqrt{\frac{\bar{u}_a^\alpha}{n_j}}, \bar{u}_b + 3\sqrt{\frac{\bar{u}_b}{n_j}}, \bar{u}_c + 3\sqrt{\frac{\bar{u}_c^\alpha}{n_j}} \right) \quad (16.144)$$

$$\tilde{C}L_u^\alpha = (\bar{u}_a^\alpha, \bar{u}_b, \bar{u}_c^\alpha) \quad (16.145)$$

$$L\tilde{C}L_u^\alpha = \left(\bar{u}_a^\alpha - 3\sqrt{\frac{\bar{u}_a^\alpha}{n_j}}, \bar{u}_b - 3\sqrt{\frac{\bar{u}_b}{n_j}}, \bar{u}_c - 3\sqrt{\frac{\bar{u}_c^\alpha}{n_j}} \right) \quad (16.146)$$

α -level fuzzy median for an α -cut fuzzy \tilde{u} -control chart can be calculated as follows:

$$UCL_{med-u}^\alpha = CL_{med-u}^\alpha + 3\sqrt{\frac{CL_{med-u}^\alpha}{n_j}} \quad (16.147)$$

$$CL_{med-u}^\alpha = \frac{1}{3}(\bar{u}_a^\alpha, \bar{u}_b, \bar{u}_c^\alpha) \quad (16.148)$$

$$LCL_{med-u}^{\alpha} = CL_{med-u}^{\alpha} - 3\sqrt{\frac{CL_{med-u}^{\alpha}}{n_j}} \quad (16.149)$$

The condition of process control for each sample can be defined as,

$$Process\ control = \left\{ \begin{array}{ll} in-control & ,for\ LCL_{med-u}^{\alpha} \leq S_{med-uj}^{\alpha} \leq UCL_{med-u}^{\alpha} \\ out-of\ control & ,for\ otherwise \end{array} \right\} \quad (16.150)$$

where

$$S_{med-uj}^{\alpha} = \frac{1}{3} \left(u_{a,j}^{\alpha}, u_{b,j}, u_{c,j}^{\alpha} \right) \quad (16.151)$$

16.5 Conclusions

TQM is a philosophy which incorporates an improvement for both management quality and production quality. TQM provides continuous improvement, customer satisfaction and success of key performance results. TQM organizes employees, process, product and services through the policy and strategy with strong leadership by using their resource to achieve their target defining policies and strategies. TQM is a root for engineering management through policies, strategies, and partnerships. TQM considers product and process quality using Statistical Control Charts. When any vagueness in the data collected from the process exists, fuzzy control charts are appropriate to monitor and evaluate the process. In this chapter, theoretical bases of both fuzzy variable and fuzzy attribute control charts are introduced as intelligent techniques to implement TQM principles. For further research, Type 2 fuzzy control charts can be developed and their performance can be compared with our study.

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