Chapter 11
Design of Reactionless Mechanisms Based on Constrained Optimization Procedure

Himanshu Chaudhary and Kailash Chaudhary

Abstract This chapter presents an optimization technique to dynamically balance planar mechanisms by minimizing the shaking forces and shaking moments due to inertia-induced forces. Dynamically equivalent systems of point masses which represent rigid links and counterweights are useful for developing optimization technique. The point-mass parameters are explicitly identified as the design variables. The balancing problem is formulated as both single-objective and multi-objective optimization problem and solved using genetic algorithm which produces better results as compared to the conventional optimization algorithm. Also, for the multi-objective optimization problem, multiple optimal solutions are created as a *Pareto front* using the genetic algorithm. The reduction of shaking force and shaking moment is obtained by optimizing the link mass distribution and counterweight of their point masses. The inertial properties of balanced mechanism are then computed in reverse by applying dynamical equivalent conditions from the optimized design variables. The effectiveness of the methodology is shown by applying it to problems of planar four-bar, slider-crank, and Stephenson six-bar mechanisms.

Keywords Dynamic balancing • Equimomental system • Genetic algorithm • Optimization • Shaking force and shaking moment

The design of reactionless mechanisms is important in order to (1) reduce the amplitude of vibration of the frame on which the mechanism is mounted due to transmission of shaking forces and (2) smoothen highly fluctuating driving torque/force needed to obtain nearly constant drive speed. Since any vibration leads to noise, wear, fatigue, etc., in the mechanism, its reduction improves several aspects of mechanical design as well. Design a reactionless mechanism means the balancing of shaking force, shaking moment, and input-torque fluctuations together. The shaking force can be eliminated completely by attaching counterweights and/or redistributing masses of the moving links. This will increase overall mass

H. Chaudhary (✉) • K. Chaudhary
Department of Mechanical Engineering, Malaviya National Institute of Technology Jaipur, JLN Marg, Jaipur 302017, India
e-mail: hchaudhary.mech@mnit.ac.in

D. Zhang, B. Wei (eds.), *Dynamic Balancing of Mechanisms and Synthesizing of Parallel Robots*, DOI 10.1007/978-3-319-17683-3_11

and moment of inertia of the mechanism. As a result, shaking moment, driving torque, and reactions in the joints will increase significantly. Therefore, to design a mechanism with minimum reaction forces transmitted to the frame, it is required to reduce all the competing dynamic quantities, namely the shaking force, shaking moment, driving torque/force, and bearing reactions simultaneously. This means that design of reactionless mechanism problem can be treated as an optimization problem, whose formulation needs the following:

1. An efficient dynamic algorithm to compute the dynamic quantities
2. Identification of the design variables, and the formulation of the constraints on them that define the design space of the feasible solutions
3. An objective function to evaluate the performance of a mechanism at hand

This chapter presents a constrained optimization procedure to balance the planar mechanisms dynamically. This will minimize the shaking force and shaking moment by optimally distributing the link masses. The concept of equimomental system of point masses is used to identify the design variables and to define the constraints for the optimization problem formulation.

11.1 Equimomental Systems for Planar Motion

A study on an equimomental system of rigidly connected point masses undergoing planar motion is discussed in this section. Inertia-induced dynamic quantities, e.g., shaking force, shaking moment, and input-torque, of a mechanical system depend on the mass and inertia of its each link, and the corresponding mass center location. These inertia properties can be represented more conveniently using the dynamically equivalent system of point masses referred to as *equimomental system* [1–3].

A point mass is an idealized concept, and defined as a mass that is concentrated at a point. Two rigid systems are equimomental if their dynamic behaviors are identical; that is, they have the same mass, the same center of mass, and the same inertia tensor with respect to a common point [1]. Referring to the ith rigid link, Fig. 11.1a, of a planar mechanism, the location of its mass center, C_i, is defined by the vector, $\mathbf{d}_i$, at an angle, θ_i, from the axis O_iX_i of the local frame, $O_iX_iY_i$, fixed to the link. The axis O_iX_i is set along the vector from O_i to O_{i+1}, that is, at an angle α_i from the axis, OX, of the fixed inertial frame, OXY, Fig. 11.1a. The points, O_i and O_{i+1}, on the link are chosen as the points where the ith link is coupled to its neighboring links, whereas link's mass and the mass moment of inertia about O_i are m_i and I_i, respectively. A system of p point masses, which is equimomental to the ith link, is shown in Fig. 11.1b. The point masses, m_{ij}, for $j = 1, \ldots, p$, are fixed in the local frame, $O_iX_iY_i$, and their distances from the origin, O_i, are l_{ij}. The angles, θ_{ij}, are defined between the line joining the point masses from O_i, and the axis, O_iX_i. In this section, all the vectors are represented in the fixed frame, OXY, unless stated otherwise.

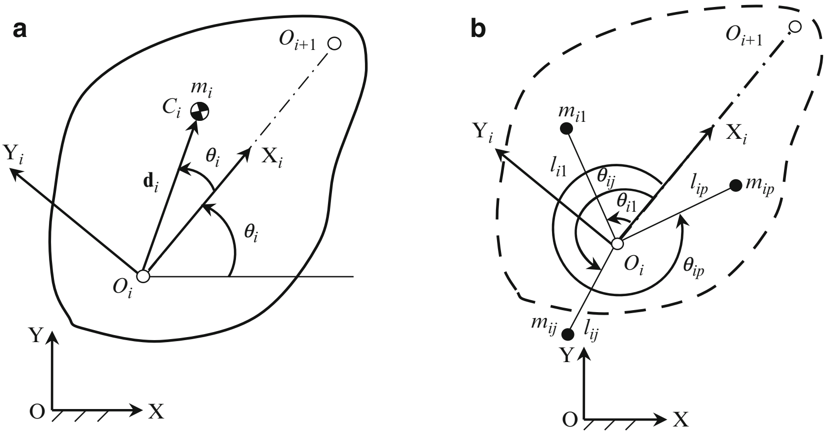


Fig. 11.1 Parameters for rigid link and its equimomental system. (**a**) The *i*th rigid link, (**b**) equimomental system of the *i*th link

If the p point masses are equimomental to the ith link, then they must satisfy the conditions of dynamic equivalence with reference to the fixed frame, OXY, given as

$$\sum_{j=1}^{p} m_{ij} = m_i \tag{11.1}$$

$$\sum_{j=1}^{p} m_{ij} l_{ij} \cos\left(\theta_{ij} + \alpha_i\right) = m_i d_i \cos\left(\theta_i + \alpha_i\right) \tag{11.2}$$

$$\sum_{j=1}^{p} m_{ij} l_{ij} \sin\left(\theta_{ij} + \alpha_i\right) = m_i d_i \sin\left(\theta_i + \alpha_i\right) \tag{11.3}$$

$$\sum_{j=1}^{p} m_{ij} l_{ij}^2 = I_i \tag{11.4}$$

The first subscript i denotes the link number, and the second one, i.e., $j = 1, \ldots, p$, represents the point masses corresponding to the ith link. Since each mass requires three parameters, (m_{ij}, l_{ij}, θ_{ij}), to identify it, a total of $3p$ parameters are necessary to completely define the equimomental system of p point masses. However, there are four constraints, namely Eqs. (11.1–11.4), that need to be satisfied. Hence, an infinite number of solutions exist for $p \geq 2$, as the resultant system of equations is underdeterminate; that is, the number of unknowns is more than the equations [2]. If $p = 1$, there is only one point mass with three unknown parameters, which cannot satisfy all the four conditions, Eqs. (11.1–11.4), unless they are consistent. This is because the resulting system of equations is overdeterminate with more equations than the number of unknowns. Typically, such system of equations does not yield any solution unless the equations are consistent. As a consequence, an equimomental system of a rigid link moving in a plane cannot be represented using one point mass, which is obvious from the fundamental knowledge of mechanics. Clearly, the minimum number of point masses is then two giving six unknown parameters, of which two need to be assigned arbitrarily. If three point masses are taken, five parameters are to be assigned arbitrarily. In general, $(3p-4)$ parameters need to be assigned arbitrarily so that the remaining four are determinate. Note here that it is not always possible to get all the point masses positive. This, however, does not hinder the process of representing the rigid link as long as the total mass and the moment of inertia about the mass center give positive values [4].

11.1.1 Two-Point-Mass Model

As explained in the previous section, an equimomental system of point masses of a rigid link moving in a plane requires at least two point masses. The representation of the link by the equimomental system of two point masses is referred to as two-point-mass model. Similarly, equimomental system of three point masses is called three-point-mass model, and so on. In this section, the conversion of a rigid link into the two-point-mass model is illustrated. Let a two-point-mass model for *i*th rigid link is moving in the XY plane. The polar coordinates of the point masses are (l_{ij}, θ_{ij}), for $j = 1, 2$. Note that the point masses are rigidly fixed in the local frame. The system of two point masses is then equimomental to the rigid link if it satisfies the conditions given by Eqs. (11.1–11.4).

For the rigid link of given mass, its center location, and inertia, one can convert the link into an appropriate two point masses. Since the four equations are nonlinear in six unknown parameters of the two point masses, a judicious selection is required to choose for the arbitrary assigned parameters. Assuming $\theta_{i1} = 0$ and $\theta_{i1} = \pi/2$ [5], the four parameters of the two point masses, namely m_{i1}, m_{i2}, l_{i1}, and l_{i2}, are determined from Eqs. (11.1–11.4) as

$$m_{i1} = \frac{\left(m_i^2\bar{x}_i^2 - m_i^2\bar{y}_i^2 + m_i I_i\right) \pm \sqrt{\left(m_i^2\bar{x}_i^2 - m_i^2\bar{y}_i^2 + m_i I_i\right)^2 - 4m_i I_i \bar{x}_i^2}}{2I_i} \tag{11.5}$$

$$m_{i2} = m_i - m_{i1} \tag{11.6}$$

$$l_{i1} = \frac{m_i\bar{x}_i}{m_{i1}} \tag{11.7}$$

$$l_{i2} = \frac{m_i\bar{y}_i}{m_{i2}} \tag{11.8}$$

where the Cartesian coordinates of the mass center of ith rigid link are $\bar{x}_i = d_i \cos\theta_i$ and $\bar{y}_i = d_i \sin\theta_i$. Hence, each point mass has two solutions. If $\theta_{i1} = 0$, i.e., the mass center of the rigid link lies on the X-axis, two sets of the solutions are as follows:

$$m_{i1} = \begin{cases} \frac{m_i^2 d_i^2}{I_i} \\ m_i \end{cases} ; m_{i2} = \begin{cases} m_i - \frac{m_i^2 d_i^2}{I_i} \\ 0 \end{cases} ; l_{i1} = \begin{cases} \frac{m_i d_i}{m_{i1}} \\ d_i \end{cases} ; \text{ and } l_{i2} = \begin{cases} 0 \\ 0 \end{cases} \tag{11.9}$$

Thus, one can convert a given rigid link into a suitable two-point-mass model assuming any two-point-mass parameters.

11.1.2 Three-Point-Mass Model

In this section, the procedure of finding a three-point-mass model is illustrated. Consider a three-point-mass model for ith rigid link moving in the XY plane. The polar coordinates of the point masses are (l_{ij}, θ_{ij}), for $j = 1, 2,$ and 3. Similar to two-point-mass model, the three-point-mass model would then be equimomental to the original rigid link if Eqs. (11.1–11.4) are satisfied.

Note that there are nine unknown parameters of point masses, namely m_{ij}, l_{ij}, and θ_{ij}, for $j = 1, 2,$ and 3, in the four equimomental equations. Hence, it is important to decide which five parameters should be chosen so that the remaining four become determinate. It is advisable to choose l_{ij} and θ_{ij}, so that the dynamic equivalence conditions become linear in point masses. Assuming $l_{i2} = l_{i3} = l_{i1}$ and substituting them in Eq. (11.4) yield

$$\left(\sum_{j=1}^{3} m_{ij} \right) l_{i1}^2 = m_i k_i^2 \tag{11.10}$$

where $m_i k_i^2 = I_i^c + m_i d_i^2$, k_i being the radius of gyration about the point, O_i. Equation (11.10) gives $l_{i1} = \pm k_i$. Taking the positive value for l_{i1}, which is physically possible, Eqs. (11.1–11.3) are then written in a compact form as

$$\mathbf{Km} = \mathbf{b} \tag{11.11}$$

where the 3×3 matrix, $\mathbf{K}$, and the three vectors, $\mathbf{m}$ and $\mathbf{b}$, are as follows:

$$\mathbf{K} = \begin{bmatrix} 1 & 1 & 1 \\ k_i \cos\theta_{i1} & k_i \cos\theta_{i2} & k_i \cos\theta_{i3} \\ k_i \sin\theta_{i1} & k_i \sin\theta_{i2} & k_i \sin\theta_{i3} \end{bmatrix} ; \mathbf{m} = \begin{bmatrix} m_{i1} \\ m_{i2} \\ m_{i3} \end{bmatrix} ; \mathbf{b} = \begin{bmatrix} m_i \\ m_i d_i \cos\theta_i \\ m_i d_i \sin\theta_i \end{bmatrix} \tag{11.12}$$

The magnitudes of three point masses are then solved from Eq. (11.11) by assuming suitable values for θ_{ij}, $j = 1, 2,$ and 3. It is clear that the solution for $\mathbf{m}$

exists if $\det(\mathbf{K}) \neq 0$, i.e., $\theta_{i1} \neq \theta_{i2}$, $\theta_{i1} \neq \theta_{i3}$, and $\theta_{i2} \neq \theta_{i3}$. It means that any two point masses should not lie on the same radial line emanating from the origin, O_i. The vector $\mathbf{m}$ is obtained as

$$\mathbf{m} = \mathbf{K}^{-1}\mathbf{b} \tag{11.13}$$

where $\mathbf{K}^{-1}$ is evaluated as

$$\mathbf{K}^{-1} = \frac{k_i}{\det(\mathbf{K})}\begin{bmatrix} k_i \sin(\theta_{i3} - \theta_{i2}) & (\sin\theta_{i2} - \sin\theta_{i3}) & (\cos\theta_{i3} - \cos\theta_{i2}) \\ -k_i \sin(\theta_{i3} - \theta_{i1}) & (\sin\theta_{i3} - \sin\theta_{i1}) & (\cos\theta_{i1} - \cos\theta_{i3}) \\ -k_i \sin(\theta_{i1} - \theta_{i2}) & (\sin\theta_{i1} - \sin\theta_{i2}) & (\cos\theta_{i2} - \cos\theta_{i1}) \end{bmatrix}$$

in which $\det(\mathbf{K}) = k_i^2\,[\sin(\theta_{i3} - \theta_{i2}) + \sin(\theta_{i2} - \theta_{i1}) + \sin(\theta_{i1} - \theta_{i3})]$. It is evident from the solution, Eq. (11.13), that the sum of the point masses is equal to mass of the link for any values of angles except $\theta_{i1} \neq \theta_{i2}$, $\theta_{i1} \neq \theta_{i3}$, and $\theta_{i2} \neq \theta_{i3}$. Note here that there is a possibility that some point masses are negative. It does not hinder the process of representing the rigid link as long as its mass, m, and inertia, I^c, are positive and real, as pointed out earlier. As an example, if $\theta_{i1} = 0$, $\theta_{i2} = 2\pi/3$, and $\theta_{i3} = 4\pi/3$, the point masses are calculated as

$$m_{i1} = \frac{m_i}{3}\left(1 + \frac{2d_i\cos\theta_i}{k_i}\right) \tag{11.14}$$

$$m_{i2} = \frac{m_i}{3}\left(1 - \frac{d_i\cos\theta_i}{k_i} + \frac{\sqrt{3}d_i\sin\theta_i}{k_i}\right) \tag{11.15}$$

$$m_{i3} = \frac{m_i}{3}\left(1 - \frac{d_i\cos\theta_i}{k_i} - \frac{\sqrt{3}d_i\sin\theta_i}{k_i}\right) \tag{11.16}$$

which take simpler form if the origin, O_i, coincides with the mass center of the link, C_i; that is, $d_i = 0$. Substituting $d_i = 0$ in Eqs. (11.14–11.16), one obtains $m_{i1} = m_{i2} = m_{i3} = m_i/3$. It means that the point masses of the link are distributed equally, and located on the circumference of a circle having radius k_i.

It is pointed out here that in mechanism analysis, links are often considered as one dimensional, e.g., a straight rod, in which its diameter or width and thickness are very small in comparison to the length. Considering that the mass lying along the X-axis of the local frame, the dynamical equivalence conditions, Eqs. (11.1–11.4), reduce to

$$\sum_{j=1}^{p} m_{ij} = m_i \tag{11.17}$$

$$\sum_{j=1}^{p} m_{ij} x_{ij} = m_i \bar{x}_i \tag{11.18}$$

$$\sum_{j=1}^{p} m_{ij} x_{ij}^2 = I_i^c + m_i \bar{x}_i^2 \tag{11.19}$$

It is evident from Eqs. (11.17–11.19) that a minimum of two point masses is also required to represent a one-dimensional link, introducing a total of four variables, i.e., m_{i1}, m_{i2}, x_{i1}, and x_{i2}. Specifying any one of the variables, the other three variables can be found uniquely. It is pointed out here that a common practice in the dynamics study of reciprocating engine is to replace the connecting rod by two point masses, where the masses are placed at the ends of the connecting rod. This does not provide a true equivalent system unless the three equations, Eqs. (11.17–11.19), in two unknowns, m_{i1} and m_{i2}, leading to an overdetermined system are consistent.

Using the concept of equimomental system, Sherwood and Hokey [4] presented the optimization of mass distribution in mechanisms. Hockey [6] discussed the input-torque fluctuations of mechanisms subject to external loads by means of properly distributing the link masses. Using the two-point-mass model, momentum balancing of four-bar linkages was presented in [7]. Optimum balancing of combined shaking force, shaking moment, and torque fluctuations in high-speed linkages was reported in Lee and Cheng [5] where a two-point-mass model was used. The concept can also be applied for the kinematic and dynamic analyses of mechanisms [8]. Simultaneous minimization of shaking force, shaking moment, and other quantities using the dynamical equivalent system of point masses and optimum mass distribution has been attempted in [9, 10]. However, the results do not show significant improvement in the performances.

11.2 Balancing of Planar Mechanisms

Balancing of shaking force and shaking moment in the mechanisms is important in order to obtain reactionless mechanisms. Several methods are developed to eliminate the shaking force and shaking moment in planar mechanisms. The methods to completely eliminate the shaking force are generally based on two principles: (1) making the total potential energy of a mechanism constant [11], and (2) making the total mass center of a mechanism stationary [12, 13]. Studies based on potential energy use elastic elements like springs to balance the force. On the other hand, the methods based on making total mass center stationary use mass redistribution/counterweights. Different techniques are used for tracing and making the total mass center stationary. For example, the method of *principal vectors* [14] describes the position of the mass center by a series of vectors that are directed along the links. These vectors trace the mass center of the mechanism at hand, and the conditions are derived to make the system mass center stationary. A more referred method in the literature is the method of *linearly independent vectors* [12] where the stationary condition was achieved by redistributing the link masses in such a manner that the coefficients of the time-dependent terms of the equations describing the total center of the mass trajectory vanish. Kochev [15] presented a general method using ordinary vector algebra instead of the complex number representation of the vectors [12] for full force balance of the planar linkages. One of the attractive features of a force-balanced linkage is that the shaking force vanishes, and the shaking moment reduces to a pure torque which is independent of reference point. However, only shaking force balancing is not effective in the balancing of mechanisms, as (1) it mostly increases the total mass of the mechanism, (2) it needs some arrangement like counterweights that increase the total mass, and (3) it increases the other dynamic characteristics, like shaking moment, driving torque, and bearing reactions. The influence of the complete shaking force balancing is thoughtfully investigated by Lowen et al. [16] on the bearing reactions, input-torque, and shaking moment for a family of crank-rocker four-bar linkages. This study shows that these dynamic quantities increase, and in some cases their values rise up to five times.

Several authors attempted to treat the balancing problem as a complete shaking force and shaking moment balancing. Elliot et al. [17] developed a theory to balance torque, shaking force, and shaking moment by extending the method of linearly independent vectors. Similarly, the analytical conditions are presented for complete balancing of shaking force and shaking moment in [18]. Complete moment balancing is also achieved by a cam-actuated oscillating counterweight [19], inertia counterweight [20], physical pendulum [21], geared counterweights, and inertia flywheel [22, 23]. More information on complete shaking moment balancing can be obtained in a critical review by Lowen et al. [24], Kochev [25], and Arakelian and Smith [26]. Practically, these methods not only increase the mass of the system but also increase its complexity.

An alternate way to reduce the shaking force and shaking moment along with other dynamic quantities such as input-torque and bearing reactions is to optimize all the dynamic quantities. Since shaking moment reduces to a pure torque in a force-balanced linkage, many researchers used the fact to develop their theory of shaking moment optimization. Berkof and Lowen [27] proposed an optimization method to minimize the root-mean-square (RMS) value of the shaking moment in a fully force-balanced in-line four-bar linkage whose input link rotates at a constant speed. As an extension of this method, Carson and Stephens [7] highlighted the need to consider feasibility limits of the link parameters. A different approach for the optimization of shaking moment in a force-balanced four-bar linkage is proposed by Hains [28]. Using the principle of the independence of the static balancing properties of a linkage from the axis of rotation of the counterweights, partial shaking moment balancing is suggested by Arakelian and Dahan [29]. The principle of momentum conservation is also used by Wiederich and Roth [30] to reduce the shaking moment in a fully force-balanced four-bar linkage.

11.2.1 Problem Formulation

The problem of mechanism balancing is formulated here as an optimization problem. In order to identify the design variables and the associated constraints, a set of equimomental point masses is defined for each link of a mechanism at hand. To calculate the shaking force and shaking moment dynamic equations of motion in the minimal set are derived in the parameters of the point masses. These parameters are then treated as design variables to redistribute the link masses to minimize the force transmitted to the frame.

11.2.2 Equations of Motion in Terms of Point-Mass System

The Newton-Euler (NE) equations of motion for the ith rigid link moving in a plane (Fig. 11.1) are given as [31]

$$\mathbf{M}_i\dot{\mathbf{t}}_i + \mathbf{C}_i\mathbf{t}_i = \mathbf{w}_i \tag{11.20}$$

where the three vectors, $\mathbf{t}_i$, $\dot{\mathbf{t}}_i$, and $\mathbf{w}_i$, are defined as the twist, twist rate, and wrench of the ith link with respect to the origin, O_i; that is,

$$\mathbf{t}_i = \begin{bmatrix} \omega_i \\ \mathbf{v}_i \end{bmatrix}; \dot{\mathbf{t}}_i = \begin{bmatrix} \dot{\omega}_i \\ \dot{\mathbf{v}}_i \end{bmatrix} \text{ and } \mathbf{w}_i = \begin{bmatrix} n_i \\ \mathbf{f}_i \end{bmatrix} \tag{11.21}$$

in which ω_i and $\mathbf{v}_i$ are the scalar angular velocity about the axis perpendicular to the plane of motion, and the two-vector of linear velocity of point O_i of the ith link, respectively. Accordingly, $\dot{\omega}_i$ and $\dot{\mathbf{v}}_i$ are the time derivatives of ω_i and $\mathbf{v}_i$, respectively. Also, the scalar, n_i, and the two-vector, $\mathbf{f}_i$, are the resultant moment about O_i and the resultant force at O_i, respectively. Moreover, the 3×3 matrices, $\mathbf{M}_i$ and $\mathbf{C}_i$, are given as

$$\mathbf{M}_i = \begin{bmatrix} I_i & -m_i\mathbf{d}_i^{\mathrm{T}}\bar{\mathbf{E}} \\ m_i\bar{\mathbf{E}}\mathbf{d}_i & m_i\mathbf{1} \end{bmatrix} \text{ and } \mathbf{C}_i = \begin{bmatrix} 0 & \mathbf{0}^{\mathrm{T}} \\ -m_i\omega_i\mathbf{d}_i & \mathbf{O} \end{bmatrix} \tag{11.22}$$

where $\mathbf{1}$ and $\mathbf{O}$ are the 2×2 identity and zero matrices, respectively, and $\mathbf{0}$ is the two-vector of zeros, and the 2×2 matrix, $\bar{\mathbf{E}}$, is defined by

$$\bar{\mathbf{E}} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

Upon substitution of the expressions for the scalar, I_i, and the two-vector, $m_i\mathbf{d}_i$, from Eqs. (11.1–11.4), the 3×3 matrices, $\mathbf{M}_i$ and $\mathbf{C}_i$, of Eq. (11.22) are obtained as

$$\mathbf{M}_i = \begin{bmatrix} \sum_j m_{ij} l_{ij}^2 & -\sum_j m_{ij} l_{ij} \sin\left(\theta_{ij} + \alpha_i\right) & \sum_j m_{ij} l_{ij} \cos\left(\theta_{ij} + \alpha_i\right) \\ -\sum_j m_{ij} l_{ij} \cos\left(\theta_{ij} + \alpha_i\right) & \sum_j m_{ij} & 0 \\ \sum_j m_{ij} l_{ij} \sin\left(\theta_{ij} + \alpha_i\right) & 0 & \sum_j m_{ij} \end{bmatrix}$$

$$\mathbf{C}_i = \begin{bmatrix} 0 & 0 & 0 \\ -\omega_i \sum_j m_{ij} l_{ij} \cos\left(\theta_{ij} + \alpha_i\right) & 0 & 0 \\ -\omega_i \sum_j m_{ij} l_{ij} \sin\left(\theta_{ij} + \alpha_i\right) & 0 & 0 \end{bmatrix} \tag{11.23}$$

Equations (11.20) and (11.23) are the equations of motion for the ith link in terms of its $3p$ point-mass parameters, namely m_{ij}, θ_{ij}, and l_{ij}, for $j = 1, \ldots, p$. Now, all or some of the point-mass parameters can be used as design variables based on their influence on the objective function of an optimization problem.

In some research papers, namely by Lee and Cheng [5] and Wiederrich and Roth [30], two-point-mass model was considered to represent the mass and inertia of the links. They assumed that $\theta_{i1} = 0$ and $\theta_{i2} = \pi/2$, amongst the six parameters m_{i1}, m_{i2}, θ_{i1}, θ_{i2}, l_{i1}, and l_{i2}. The remaining parameters were then considered as design variables, and used for the optimization of four-bar mechanisms. In three-point-mass model the following five parameters can be assigned arbitrarily:

$$\theta_{i1} = 0; \theta_{i2} = 2\pi/3; \theta_{i3} = 4\pi/3; \text{ and } l_{i2} = l_{i3} = l_{i1} \tag{11.24}$$

The other four parameters, namely m_{i1}, m_{i2}, m_{i3}, and l_{i1}, are then treated as the design variables for each link.

11.2.3 Definition of Shaking Force and Shaking Moment

Figure 11.2 shows n moving links in a multiloop mechanism where the fixed link, #0, is detached from the other links. The appropriate reaction forces and moments due to the fixed link are indicated on the moving links to maintain the dynamic equilibrium.

The shaking force is now defined as the reaction of the vector sum of all the inertia forces of moving links associated with the mechanism, and the shaking moment is the reaction of the resultant of the inertia moment and the moment of the inertia forces [5]. By the above definitions, the shaking force and the shaking moment with respect to O_1, transmitted to the fixed link, are given by

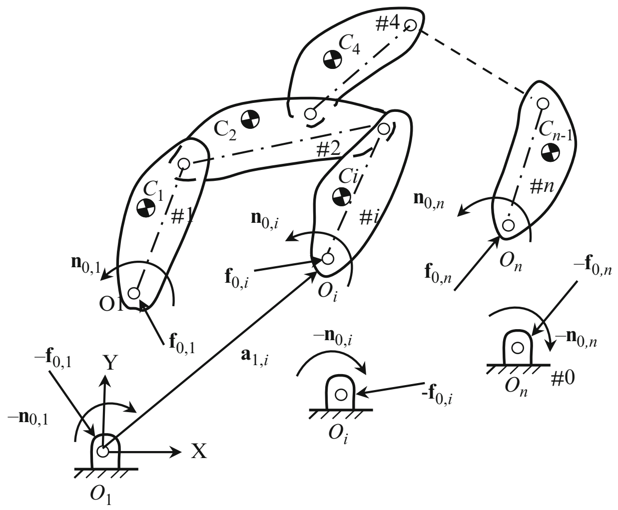


Fig. 11.2 A multiloop mechanism

$$\mathbf{f}_{\text{sh}} = -\sum_{i=1}^{n} \mathbf{f}_i^* \tag{11.25}$$

$$n_{\text{sh}} = -\sum_{i=1}^{n} \left(n_i^* - \mathbf{a}_{1,i}^{\text{T}} \overline{\mathbf{E}} \mathbf{f}_i^* \right) \tag{11.26}$$

where n_i^* and $\mathbf{f}_i^*$ are the inertia moment and the two-vector of inertia force, respectively, acting at and about the origin, O_i, of the ith link. Moreover, the two-vector, $\mathbf{a}_{1,i}$, is defined from O_1 to the origin of the ith link, as shown in Fig. 11.2. Substituting the resultant force and moment in terms of the external force and moment, and the reactions due to the adjoining joints, the force and moment balance expressions for the ith link are written as

$$\mathbf{f}_i^* = \mathbf{f}_i^{\text{e}} + \sum_{k=0}^{n} \mathbf{f}_{k,\,i} \tag{11.27}$$

$$n_i^* = n_i^{\text{e}} + \sum_{k=0}^{n} \left(n_{k,\,i} - a_{i,k}^{\text{T}} \overline{\mathbf{E}} \mathbf{f}_{k,\,i} \right) \tag{11.28}$$

where $\mathbf{f}_{k,i}$ and $n_{k,i}$ are the bearing reaction force and moment on the ith link by the kth link, respectively. Note that $\mathbf{f}_{k,i} = \mathbf{0}$ and $n_{k,i} = 0$ if kth link is not directly connected to the ith link. Furthermore, $\mathbf{f}_i^{\text{e}}$ and n_i^{e} are the external force and moment acting at and about the origin, O_i, respectively. Note that the origin for the ith link is defined at the joint where it is coupled with previous link, whereas vector, $\mathbf{a}_{i,k}$, is defined from the origin of the ith link to the joint where the kth link is connected. Upon substitution of Eqs. (11.27 and 11.28) into Eqs. (11.25 and 11.26), the shaking force and the shaking moment with respect to O_1 transmitted to the fixed link, #0, are obtained as

$$\mathbf{f}_{\mathrm{sh}} = -\sum_{j=1}^{n_{\mathrm{f}}} \mathbf{f}_{0,j} - \sum_{i=1}^{n} \mathbf{f}_i^{\mathrm{e}} \tag{11.29}$$

$$n_{\mathrm{sh}} = -\sum_{j=1}^{n_{\mathrm{f}}} \left(n_{0,j} - \mathbf{a}_{1,j}^{\mathrm{T}} \overline{\mathbf{E}} \mathbf{f}_{0,j} \right) - \sum_{i=1}^{n} \left(n_i^{\mathrm{e}} - \mathbf{a}_{1,i}^{\mathrm{T}} \overline{\mathbf{E}} \mathbf{f}_i^{\mathrm{e}} \right) \tag{11.30}$$

where $\mathbf{f}_{0,j}$ represents the reaction force on the jth link by the fixed link, for $j = 1, \ldots, n_{\mathrm{f}}$, n_{f} being the number of links connected to the fixed link. Hence, using Eqs. (11.29 and 11.30), the computation of the reactions at all the joints is not necessary to compute the shaking force and shaking moment. Note that the dynamic quantities, e.g., the shaking force, shaking moment, and bearing reactions, have different units and magnitudes. In order to harmonize them, the force and moment are normalized as [32]

$$\bar{f} = |\mathbf{f}| \,/\, \left(m_{\mathrm{m}}^{\mathrm{o}} a_{\mathrm{m}} \omega_{\mathrm{in}}^2 \right) \tag{11.31}$$

$$\bar{n} = n / \left(m_{\mathrm{m}}^{o} a_{\mathrm{m}}^2 \omega_{\mathrm{in}}^2 \right) \tag{11.32}$$

where a_{m} and $m_{\mathrm{m}}^{\mathrm{o}}$ are the length and mass of the reference link for the normalization, whereas ω_{in} is any input angular velocity. Superscript "o" is used for those parameters of the original mechanism, which will be changing during the optimization.

11.2.4 Optimality Criterion

There are many possible criteria by which the shaking force and shaking moment transmitted to the fixed link of the mechanism can be minimized. For example, one criterion could be based on the RMS values of the shaking force, shaking moment, and required driving torque for a given motion, and/or the combination of these. Besides the RMS values, there are other ways to specify the dynamic quantities also, namely by the maximum values, by the amplitude of the specified harmonics, or by the amplitudes at certain point during the motion cycle. Here, the RMS value is preferred over others as it gives equal emphasis on the results of every time instances, and every harmonic component. The RMS values of the normalized shaking force, $\bar{f}_{\mathrm{sh}}$, and the normalized shaking moment, $\bar{n}_{\mathrm{sh}}$, at δ discrete positions of the mechanism are defined as

$$\tilde{f}_{\mathrm{sh}} = \sqrt{\sum \bar{f}^2_{\mathrm{sh}}/\delta};\ \text{and}\ \tilde{n}_{\mathrm{sh}} = \sqrt{\sum \bar{n}^2_{\mathrm{sh}}/\delta} \tag{11.33}$$

where $\tilde{f}_{\mathrm{sh}}$ and $\tilde{n}_{\mathrm{sh}}$ are the RMS values of the normalized shaking force and the normalized shaking moment, respectively. Considering the RMS values, $\tilde{f}_{\mathrm{sh}}$ and $\tilde{n}_{\mathrm{sh}}$, an optimality criterion can be posed as

$$z = w_1\tilde{f}_{\mathrm{sh}} + w_2\tilde{n}_{\mathrm{sh}} \tag{11.34}$$

where w_1 and w_2 are the weighting factors whose values may vary depending on an application. For example, $w_1 = 1.0$ and $w_2 = 0$ if the objective is to minimize the shaking force only. The design variables and constraints depend upon whether the balancing is done through the redistribution of link masses or counterweighting the links.

11.2.4.1 Mass Redistribution Method

Consider a mechanism having n moving links, i.e., $i = 1, \ldots, n$, and each link is modeled by a system of p equimomental point masses; then the $3p$-vector of point-mass parameters for the ith link is defined as

$$\mathbf{x}_i = \left[m_{i1} \ldots m_{ip}\ l_{i1} \ldots l_{ip}\ \theta_{i1} \ldots \theta_{ip} \right]^{\mathrm{T}} \tag{11.35}$$

Accordingly, the $3np$-vector of the point-mass parameters for the whole mechanism is given by

$$\mathbf{x} = \left[\mathbf{x}_i^{\mathrm{T}} \ldots \mathbf{x}_n^{\mathrm{T}} \right]^{\mathrm{T}} \tag{11.36}$$

If three-point-mass model is used then the dimensions of the vectors, $\mathbf{x}_i$ and $\mathbf{x}$, are 9 and $9n$, respectively. If five parameters per link are assigned arbitrarily according to Eq. (11.24), the remaining four parameters, namely m_{i1}, m_{i2}, m_{i3}, and l_{i1}, per link can be treated as the *design variables* (DV). Finally the $4n$-vector, $\mathbf{x}$, of the DVs using three-point-mass model is defined as

$$\mathbf{x} = [m_{11},\ m_{12},\ m_{13},\ l_{11},\ \ldots,\ m_{n1},\ m_{n2},\ m_{n3},\ l_{n1}]^{\mathrm{T}} \tag{11.37}$$

The constraints on the DVs depend on the allowable minimum and maximum values of the DVs, say, mass and inertia, etc. The minimum mass, $m_{i,\min}$, of the ith link and its mass distribution can be decided by the strength of its material. Furthermore, the maximum mass, $m_{i,\max}$, can be taken into account according to what extent the shaking force and shaking moment are eliminated. Similarly, the limits on parameters, l_{i1}, can be determined based on the limiting values of the moment of inertia. The optimization problem is finally posed as

$$\text{Minimize } z(\mathbf{x}) = w_1\tilde{f}_{\text{sh}} + w_2\tilde{n}_{\text{sh}} \text{ (single objective) OR}$$

$$\text{Minimize } z(\mathbf{x}) = \left[\tilde{f}_{\text{sh}},\ \tilde{n}_{\text{sh}}\right] \text{ (multi-objective)} \tag{11.38a}$$

$$\text{Subject to } m_{i,\min} \le m_i \le m_{i,\max} \tag{11.38b}$$

$$l_{i1,\min} \le l_{i1} \le l_{i1,\max} \tag{11.38c}$$

$$d_{i,\min} \le d_i \le d_{i,\max} \tag{11.38d}$$

$$m_i d_i^2 \le I_i \tag{11.38e}$$

for $i = 1, \ldots, n$, where $m_{i,\min}$, $m_{i,\max}$, $l_{i1,\min}$, and $l_{i1,\max}$ are the lower and upper bounds on m_i and l_{i1}, respectively, and $m_i = m_{i1} + m_{i2} + m_{i3}$. The feasibility of the mass center location and the moment of inertia of the ith link can be achieved using constraints, Eqs. (11.38d and 11.38e), where $I_i = I_i^c + m_i d_i^2$, which implies that the term $m_i d_i^2$ must be less than or equal to the moment of inertia, I_i.

11.2.4.2 Counterweighting Method

In the case of counterweight balancing, counterweights are attached to the moving links such that the shaking force and shaking moment transmitted to the frame of the mechanism are minimum. Assume that the counterweight of mass, m_i^b, with its mass center location, $\left(\bar{x}_i^b, \bar{y}_i^b\right)$, is attached to the ith link as shown in Fig. 11.3a. The equimomental system of the resulting link is shown in Fig. 11.3b, where it is assumed that the point masses of the counterweight mass, m_{ij}^b, are placed at the location of the point masses of original link, m_{ij}^o. Then the counterweight mass, its mass center location, and inertia are defined as

$$m_i^b = \sum_{j=1}^{3} m_{ij}^b \tag{11.39}$$

$$m_i^b \mathbf{d}_i^b = \sum_{j=1}^{3} m_{ij}^b l_{ij}^o \tag{11.40}$$

$$I_i^b = \sum_{j=1}^{3} m_{ij}^b \left(l_{ij}^o\right)^2 \tag{11.41}$$

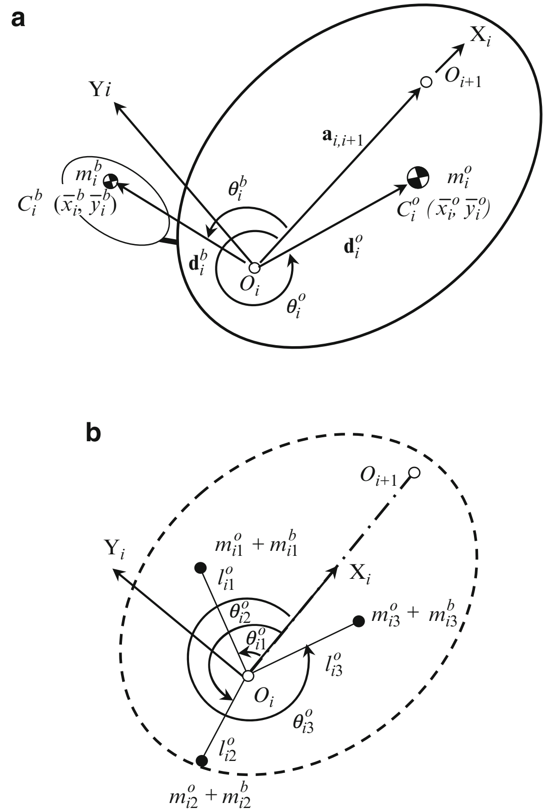


Fig. 11.3 Counterweight balancing. (**a**) Counterweight to the *i*th link, (**b**) equimomental point masses of the counterweight mass

Now, for a mechanism having n moving links, the $3n$-vector of the design variables, $\mathbf{x}^{\mathrm{b}}$, is

$$\mathbf{x}^{\mathrm{b}} = \left[\mathbf{m}_1^{\mathrm{b}^{\mathrm{T}}},\ \ldots,\ \mathbf{m}_n^{\mathrm{b}^{\mathrm{T}}}\right]^{\mathrm{T}} \tag{11.42}$$

where the three-vector, $\mathbf{m}_i^{\mathrm{b}}$, is as follows:

$$\mathbf{m}_i^{\mathrm{b}} = \left[m_{i1}^{\mathrm{b}}\ m_{i2}^{\mathrm{b}}\ m_{i3}^{\mathrm{b}}\right]^{\mathrm{T}},\ \text{for}\ i = 1, \ldots, n$$

Note that m_{ij}^{b} is the jth point mass of the counterweight attached to the ith link. The minimum and maximum mass of counterweight, $m_{i,\min}^{\mathrm{b}}$ and $m_{i,\max}^{\mathrm{b}}$, their locations, and the moment of inertia depend on an application. However, the counterweight balancing problem is stated to determine the mass, m_i^{b}, its mass center location, $\left(\bar{x}_i^{\mathrm{b}}, \bar{y}_i^{\mathrm{b}}\right)$, and the inertia, I_i^{b}, such that the combined effect of shaking force and shaking moment is going to be minimum; that is,

Table 11.1 Definition of normalized parameters

$a_{ij} = \lvert \mathbf{a}_{ij} \rvert / a_m$	Normalized distance between joints i and j
$d_i = \lvert \mathbf{d}_i \rvert / a_m$	Normalized distance of the mass center
$m_i = m_i / m_m^{\mathrm{o}}$	Normalized mass of the ith link
$I_i = I_i / \left(m_m^{\mathrm{o}} a_m^2 \right)$	Normalized moment of inertia of the ith link

The variables, a_m and m_m^{o}, are defined after Eq. (11.32)

$$\text{Minimize } \; z\left(\mathbf{x}^{\mathrm{b}}\right) = w_1 \tilde{f}_{\mathrm{sh}} + w_2 \tilde{n}_{\mathrm{sh}} \tag{11.43a}$$

$$\text{Subject to } \; m_{i,\min}^{\mathrm{b}} \le m_i^{\mathrm{b}} \le m_{i,\max}^{\mathrm{b}} \tag{11.43b}$$

$$d_{i,\min}^{\mathrm{b}} \le d_i^{\mathrm{b}} \le d_{i,\max}^{\mathrm{b}} \tag{11.43c}$$

$$m_i^{\mathrm{b}} \left(d_i^{\mathrm{b}} \right)^2 \le I_i^{\mathrm{b}} \tag{11.43d}$$

for $i = 1, \ldots, n$, where $d_i^{\mathrm{b}} = \sqrt{\bar{x}_i^{\mathrm{b}2} + \bar{y}_i^{\mathrm{b}2}}$. Similar to the constraints in the mass redistribution method, the mass center location and the moment of inertia of the counterweight attached to the ith link are constraints using inequalities of Eqs. (11.43c and 11.43d), respectively.

The optimization methodology using either the mass redistribution or counterweight methods is summarized in the following steps:

1. To harmonize the values of the link parameters, the parameters of the unbalanced mechanism are made dimensionless as explained in Table 11.1.
2. Given mass, its mass center location, and the inertia of each link: m_i, $\bar{x}_i$, $\bar{y}_i$, I_i, of the normalized unbalanced mechanism, find the set of equimomental point masses for each rigid link.
3. Define design variable for the mechanism having n moving links, as in Eqs. (11.37) and (11.42), for the redistribution and counterweight balancing methods, respectively.
4. Define objective function and constraints on the link masses and inertias, i.e., Eqs. (11.38a–11.38e) or (11.43a–11.43d), where the normalized shaking force and shaking moment are defined according to Eqs. (11.31) and (11.32), respectively. For the normalized mechanism operating at $\omega_{\mathrm{in}} = 1$ rad/s, the shaking force and shaking moment are the normalized shaking force and shaking moment.
5. Solve the optimization problem posed in the above step (4) using any standard optimization solver, say, the optimization toolbox of MATLAB [33]. The optimization process can be started with the parameters of the given unbalanced mechanism as the initial design vector.
6. From the optimized parameters, m_{i1}^*, m_{i2}^*, m_{i3}^*, l_{i1}^*, in redistribution method, the optimized mass, m_i^*, the location of the mass center, $(\bar{x}_i^*, \bar{y}_i^*)$, and the

inertia of each link, I_i^*, of the balanced mechanism are determined using the equimomental conditions, i.e., Eqs. (11.1–11.4). Similarly, in counterweight method the optimized total mass, m_i^{b*}, the location of the mass center ($\bar{x}_i^{b*}$, $\bar{y}_i^{b*}$), and the inertia of counterweight attached to each link, I_i^{b*}, of the balanced mechanism are determined using the equimomental conditions, i.e., Eqs. (11.39–11.41), from optimized point masses, m_{i1}^{b*}, m_{i2}^{b*}, and m_{i3}^{b*}.

7. Actual values of link masses or counterweights, their mass center location, and moments of inertia are obtained by multiplying the optimized values with the corresponding normalizing factors, namely m_m^o, a_m, and $m_\mathrm{m}^\mathrm{o} a_\mathrm{m}^2$, respectively.

11.3 Numerical Examples

In this section, the effectiveness of the optimization methodology is shown by applying it to some planar mechanisms. The balancing problems can be framed as single-objective or multi-objective optimization problems to simultaneously minimize the shaking force and shaking moment. To solve these problems using conventional optimization algorithms, "*fmincon*" function in *Optimization Toolbox* of MATLAB is used. Alternatively, the genetic algorithm is also used as solver. Two functions "*ga*" and "*gamultiobj*" in *Genetic Algorithm and Direct Search Toolbox* of MATLAB are used for this purpose. It was observed that GA produces better results as compared to conventional optimization algorithms.

11.3.1 Planar Four-Bar Mechanism

A numerical example of standard four-bar mechanism [5, 31, 34] is solved using the methodology developed in this chapter. The minimization of inertia forces is obtained by redistributing the link masses [35]. The parameters of standard and balanced mechanisms are shown in Table 11.2 whereas the variation of shaking force, shaking moment, and driving torque for complete cycle is shown in Figs. 11.4 and 11.5.

Table 11.2 Dimensionless parameters of standard and balanced mechanism

		Standard mechanism				Balanced mechanism			
Link	Length a_i	Mass m_i	Moment of inertia $I_{i,zz}^c$	d_i	θ_i	Mass m_i	Moment of inertia $I_{i,zz}^c$	d_i	θ_i
1	1	1.0000	0.3300	0.5	0	2.0725	6.9383	0.6688	187.50
2	2	1.1597	1.0186	1.0	0	1.4979	0.5688	0.2685	309.25
3	3	1.4399	2.2880	1.5	0	1.9095	1.3829	0.3630	098.86

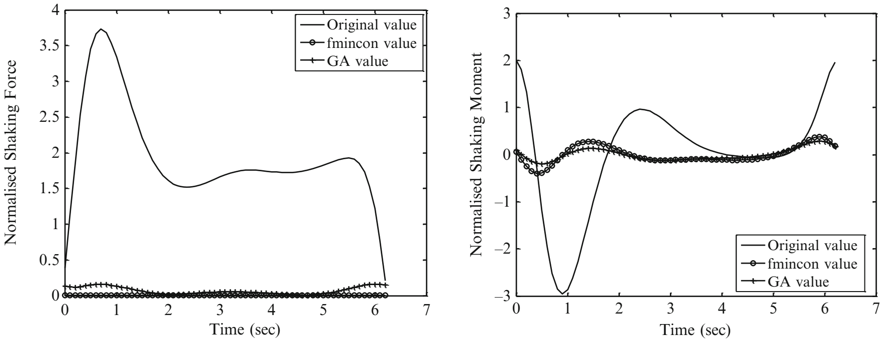
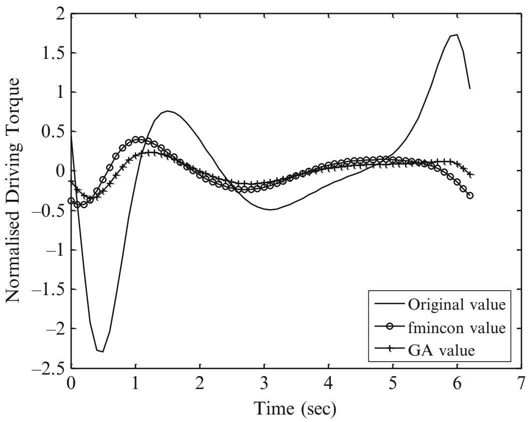


Fig. 11.4 Variations of shaking force and shaking moment for complete cycle

Fig. 11.5 Variations of driving torque for complete cycle



Two different approaches were used to solve this multi-objective optimization problem using GA. In a priori approach, a composite objective function is formed using equal weighting factors to both the objectives, i.e., shaking force and shaking moment as explained in Eq. (11.34). As the shaking force and the shaking moment are of different units, these quantities are made dimensionless with respect to the parameters of the driving link for adding them in a composite objective function for which the results are shown in Table 11.3. The values in the parenthesis denote the percentage increment/decrement with respect to the corresponding RMS values of the standard mechanism.

For the given problem, the genetic algorithm produced better results as compared to the results obtained using conventional optimization technique. With equal weighting to shaking force and shaking moment, about 96 %, 89 %, and 84 % reductions are achieved in shaking force, shaking moment, and driving torque, respectively.

In posterior approach, a set of optimal solutions, known as Pareto front, is found by considering both the objectives separately. Each solution in the Pareto front is

Table 11.3 RMS values of dynamic quantities of standard and optimized mechanisms

Balancing method	RMS values of dimensionless dynamic quantities		
	Shaking force	Shaking moment	Driving torque
Standard mechanism	2.0582	1.1593	0.8613
Conventional algorithm [31] $w_1 = 0.5$; $w_2 = 0.5$	3.78×10^{-6} (−100)	0.1882 (−84)	0.2051 (−76)
Genetic algorithm $w_1 = 0.5$; $w_2 = 0.5$	0.0868 (−96)	0.1233 (−89)	0.1398 (−84)

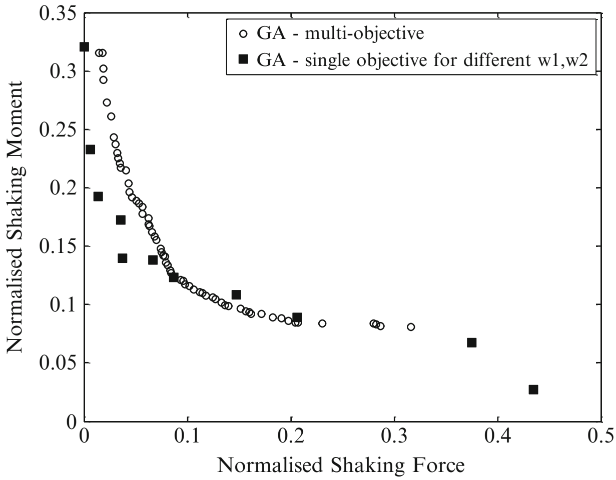


Fig. 11.6 Pareto front for four-bar mechanism problem

an optimum solution as no single solution minimizes both the objectives when compared to other solutions in the set (Fig. 11.6). The results obtained using two approaches are also compared in Fig. 11.6.

11.3.2 Planar Slider-Crank Mechanism

The optimization method presented in this chapter can be effectively used to balance the mechanisms having revolute and prismatic joints while most of the methods available in the literature are for the mechanisms with revolute joints only. A slider-crank mechanism is balanced here by optimally distributing the link masses [36] while a cam mechanism with counterweight was used to balance the same mechanism in [37].

Table 11.4 Dimensionless parameters of standard and balanced slider-crank mechanism

Link	Length a_i	Standard mechanism				Balanced mechanism			
		Mass m_i	Moment of inertia $I^c_{i,zz}$	d_i	θ_i	Mass m_i	Moment of inertia $I^c_{i,zz}$	d_i	θ_i
1	1.0000	1.0	0.1759	0.5000	0	1.5226	2.5204	1.6171	171.94
2	1.4623	1.5	0.8210	0.7329	0	1.5015	0.4222	0.1842	357.65
3	–	2.0	–	–	0	2.0011	–	0.3750	269.12

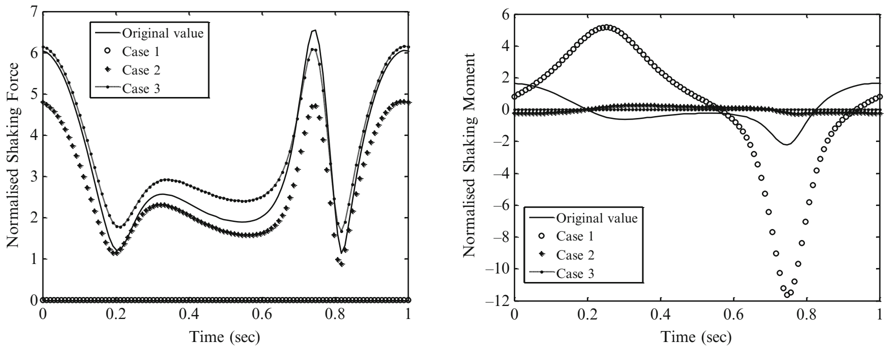


Fig. 11.7 Variations of shaking force and shaking moment for different cases

Table 11.5 RMS values of dynamic quantities of normalized standard and optimized mechanisms

Balancing method	Shaking force	Shaking moment
Standard mechanism	3.6877	1.0047
Conventional algorithm	2.9132 (−21)	0.1883 (−81)
Genetic algorithm	2.0051 (−46)	0.0105 (−99)

The problem is considered here to balance it using optimization procedure described in the chapter. The parameters of standard and balanced slider-crank mechanisms are shown in Table 11.4 whereas Fig. 11.7 shows the variation of shaking force and shaking moment over the complete cycle.

The results corresponding to different combinations of the weighting factors using conventional optimization algorithm are shown in Fig. 11.7. The case 1 is complete shaking force balancing in which the RMS value of shaking moment increases to four times that of the unbalanced mechanism. Similarly, in case 3, shaking force increases while shaking moment reduces substantially. Reductions in both the quantities occur in case 2, in which equal weights are assigned to them.

Then the same problem is solved using GA with equal weighting factors for both the quantities. The comparison of the original RMS values with the optimum RMS values of the shaking force and shaking moment obtained using conventional and genetic algorithm is presented in Table 11.5 and Fig. 11.8. The optimized link parameters are then found by using the equimomental conditions corresponding to GA solution and shown in Table 11.4.

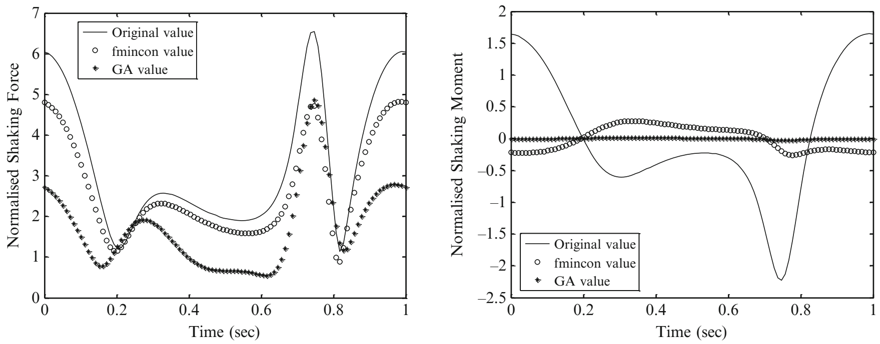


Fig. 11.8 Variations in shaking force and shaking moment for complete cycle

Note that the reductions of 21 % and 81 % in the RMS values of shaking force and shaking moment, respectively, are achieved in the conventional method. The application of the genetic algorithm results in reduction of 46 % and 99 % in the shaking force and shaking moment, respectively. The moment of inertia of slider about CG doesn't affect the values of shaking force and shaking moment and hence it is not provided in Table 11.4.

11.3.3 Planar Six-Bar Mechanism

The optimization methodology can also be used to minimize the shaking force and shaking moment in multiloop planar mechanisms. A Stephenson six-bar mechanism [12] shown in Fig. 11.9 is optimally balanced using counterweighting method [38]. First, the force balancing is achieved by optimizing the point-mass parameters of the counterweights. Next, the balancing problem is formulated as a multi-objective optimization problem which minimizes the shaking force and shaking moment simultaneously. The parameters of original unbalanced Stephenson six-bar mechanism are given in Table 11.6 whereas Fig. 11.10 shows the variation of shaking force and shaking moment over the complete cycle.

The kinematic simulation was carried out using the MotionView and Motion-Solve of Altair HyperWorks 11.0 software [39]. For the problem considered, the standard and optimized values of the shaking force and shaking moment for different combinations of weighting factors using conventional optimization algorithm are presented in Table 11.7 and shown in Fig. 11.10.

For only the shaking force balancing case, results show 63.87 % reduction in the shaking force whereas 190.93 % increment in shaking moment occurred. For only the shaking moment balancing case, reduction of 39 % was found in the shaking moment while the shaking force is increased by 127 %. These two cases support the fact that the reduction in one dynamic quantity increases the other. Thus a trade-off is necessary to reduce both the shaking force and shaking moment. To reduce both

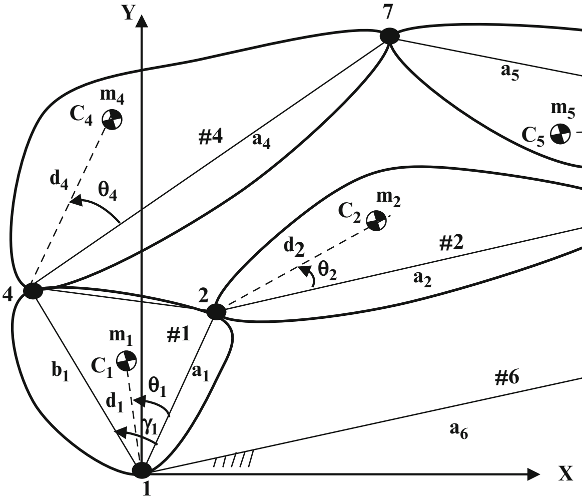


Fig. 11.9 Stephenson six-bar mechanism

Table 11.6 Parameters of unbalanced Stephenson six-bar mechanism

Link i	1	2	3	4	5	6
a_i (m)	0.0559	0.1206	0.0032	0.1397	0.0444	0.1238
b_i (m)	0.0584	–	0.0030	–	–	–
γ_i (deg)	6	–	16	–	–	–
θ_i (deg)	3	0	5	19	0	11
d_i (m)	0.0286	0.0630	0.0031	0.0836	0.0197	–
m_i (kg)	0.0608	0.0825	0.0757	0.1732	0.0395	–

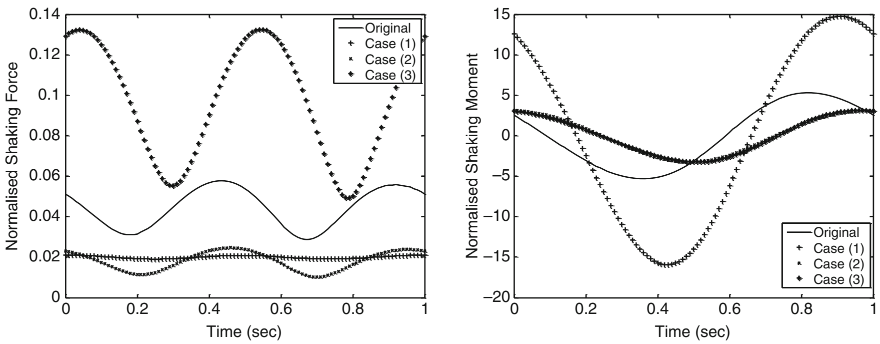


Fig. 11.10 Variations in shaking force and shaking moment for complete cycle

Table 11.7 RMS values of shaking force and shaking moment in Stephenson six-bar mechanism

	Shaking force	Shaking moment
Original value	0.0450	3.7332
Only shaking force Case (1) $w_1 = 1.0$; $w_2 = 0.0$	0.0164 (−63.87 %)	10.8610 (+190.93 %)
Both shaking force and shaking moment Case (2) $w_1 = 0.5$; $w_2 = 0.5$	0.0192 (−57.7 %)	2.2651 (−39.32 %)
Only shaking moment Case (3) $w_1 = 0.0$; $w_2 = 1.0$	0.1031 (+127.09 %)	2.2516 (−39.68 %)

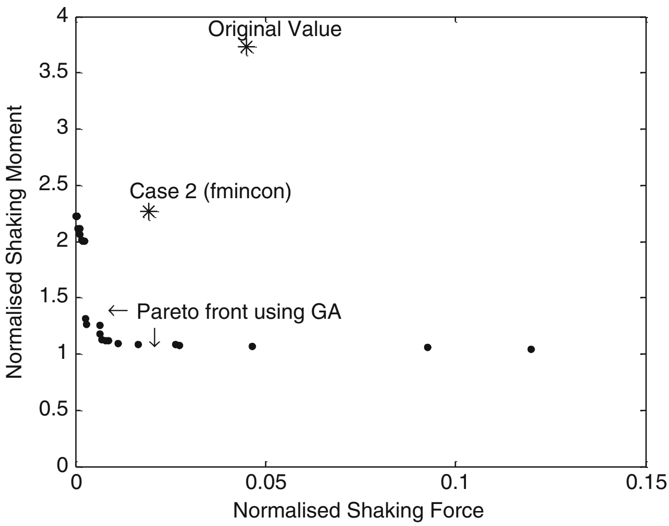


Fig. 11.11 Pareto front for six-bar mechanism problem

the shaking force and shaking moment simultaneously in the given mechanism, both the quantities are assigned equal weighting factor value, i.e., 0.5 in the objective function (Case 2). The result shows 57.7 % and 39 % reduction in the shaking force and shaking moment, respectively.

The optimum design variables obtained for case 2 are then taken as the initial population and the genetic algorithm was used to find the solution of this optimization problem. The genetic algorithm produces multiple optimal solutions (Pareto front) as shown in Fig. 11.11.

This plot shows the trade-off between the two objective functions, i.e., the shaking force and shaking moment. Thus it is advantageous to use GA for finding multiple optimal solutions without running the traditional algorithm many times. Figure 11.11 shows that the GA results are better than the results obtained using traditional optimization algorithm. The values of the shaking force and shaking moment corresponding to the best solution among available Pareto optimal solutions are given in Table 11.8.

Table 11.8 Results from GA algorithm

	Shaking force	Shaking moment
Original value	0.0450	3.7332
Optimized value	0.0069 (−84 %)	1.1260 (−69.83 %)

Table 11.9 Optimum counterweight parameters using GA algorithm

Counterweights	CW 1	CW 2	CW 3	CW 4	CW 5
Mass (kg)	0.3046	0.1124	0.0570	0.0116	0.0144
d (m)	0.0750	0.2318	0.0132	0.3336	0.2209
θ (deg)	200.55	250.13	136	144.68	214.21

The counterweight parameters for optimum design variables are calculated using the equimomental conditions and are presented in Table 11.9.

11.4 Summary

In this chapter, balancing problem of planar mechanisms is formulated as an optimization problem. The main focus of the chapter is to reduce the shaking force and shaking moment. The design variables and the constraints on them are identified by introducing the equimomental system of point masses. Using the equimomental three point masses, equations of motion are reformulated to determine the shaking force, shaking moment, and other dynamic quantities. Three planar mechanisms, namely four-bar, slider-crank, and Stephenson six-bar mechanism, are optimally balanced using the methodology given in this chapter.

Acknowledgment The following material is used "With kind permission of Springer Science + Business Media." Section 4.1 (Chapter 4) and Sects. 5.1, 5.2 (Chapter 5) including Figs. 5.1, 5.3, 5.4 and Table 5.1; pp: 87–92, 100–101, 104–105, 110–117 from book Dynamics and Balancing of Multibody Systems, Lecture notes in applied and computational mechanics, Vol. 37 by Himanshu Chaudhary, Subir Kumar Saha, published by springer-Verlag Germany, 2009 ISBN 978-3-540-78178-3.

References

1. Routh, E.J.: Treatise on the Dynamics of a System of Rigid Bodies. Elementary Part I, p. 28. Dover Publication Inc., New York (1905)
2. Wenglarz, R.A., Forarasy, A.A., Maunder, L.: Simplified dynamic models. Engineering **208**, 194–195 (1969)
3. Huang, N.C.: Equimomental system of rigidly connected equal particles. J. Guid. Control. Dyn. **16**(6), 1194–1196 (1983)
4. Sherwood, A.A., Hockey, B.A.: The optimization of mass distribution in mechanisms using dynamically similar systems. J. Mech. **4**, 243–260 (1969)

5. Lee, T.W., Cheng, C.: Optimum balancing of combined shaking force, shaking moment, and torque fluctuations in high speed linkages. Trans. ASME J. Mech. Transm. Automat. Des. **106**(2), 242–251 (1984)
6. Hockey, B.A.: The minimization of the fluctuation of input-shaft torque in plane mechanisms. Mech. Mach. Theory **7**, 335–346 (1972)
7. Carson, W.L., Stephenes, J.M.: Feasible parameter design spaces for force and root-mean-square moment balancing an in-line 4R 4-bar synthesized for kinematic criteria. Mech. Mach. Theory **13**, 649–658 (1978)
8. Attia, H.A.: A matrix formulation for the dynamic analysis of spatial mechanisms using point coordinates and velocity transformation. Acta Mech. **165**, 207–222 (2003)
9. Gill, G.S., Freudenstein, F.: Minimization of inertia-induced forces in spherical four-bar mechanisms. Part 1: The general spherical four-bar linkage. Trans. ASME J. Mech. Transm. Automat. Des. **105**, 471–477 (1983)
10. Rahman, S.: Reduction of inertia-induced forces in a generalized spatial mechanism. Ph.D. Thesis, Dept. of Mech. Eng., The New Jersey Institute of Technology. http://archives.njit.edu/vol01/etd/1990s/1996/njit-etd1996-017/njit-etd1996-017.pdf
11. Quang, P.R., Zhang, W.J.: Force balancing of robotic mechanisms based on adjustment of kinematic parameters. Trans. ASME J. Mech. Des. **127**(3), 433–440 (2005)
12. Berkof, R.S., Lowen, G.G.: A new method for completely force balancing simple linkages. Trans. ASME J. Eng. Ind. **91**(1), 21–26 (1969)
13. Fattah, A., Agarwal, S.K.: On the design of reactionless 3-DOF planar parallel mechanisms. Mech. Mach. Theory **41**, 70–82 (2006)
14. Shchepetil'nikov, V.A.: The determination of the mass centres of mechanisms in connection with the problem of mechanism balancing. J. Mech. **3**, 367–389 (1968)
15. Kochev, I.S.: A new general method for full force balancing of planar linkages. Mech. Mach. Theory **23**(6), 475–480 (1988)
16. Lowen, G.G., Tepper, F.R., Berkof, R.S.: The quantitative influence of complete force balancing on the forces and moments of certain families of four-bar linkages. Mech. Mach. Theory **9**, 299–323 (1974)
17. Elliott, J.L., Tesar, D.: The theory of torque, shaking force, and shaking moment balancing of four link mechanisms. Trans. ASME J. Eng. Ind. **99**(3), 715–722 (1977)
18. Foucault, S., Gosselin, C.M.: Synthesis, design, and prototyping of a planar three degree-of-freedom reactionless parallel mechanism. Trans. of ASME Journal of Mechanical Design **126**, 992–999 (2004)
19. Kamenskii, V.A.: On the questions of the balancing of plane linkages. J. Mech. **3**, 303–322 (1968)
20. Tricamo, S.J., Lowen, G.G.: A novel method for prescribing the maximum shaking force of a four-bar linkage with flexibility in counterweight design. Trans. ASME J. Mech. Transm. Automat. Des. **105**, 511–519 (1983)
21. Berkof, R.S.: Complete force and moment balancing of inline four-bar linkage. Mech. Mach. Theory **8**, 397–410 (1973)
22. Arakelian, V.H., Smith, M.R.: Complete shaking force and shaking moment balancing of linkages. Mech. Mach. Theory **34**, 1141–1153 (1999)
23. Arakelian, V.H., Smith, M.R.: Design of planar 3-DOF 3-RRR reactionless parallel manipulators. Mechatronics **18**, 601–606 (2008)
24. Lowen, G.G., Tepper, F.R., Berkof, R.S.: Balancing of linkages—an updates. Mech. Mach. Theory **18**(3), 213–220 (1983)
25. Kochev, I.S.: General theory of complete shaking moment balancing of planar linkages: a critical review. Mech. Mach. Theory **35**, 1501–1514 (2000)
26. Arakelian, V.H., Smith, M.R.: Shaking force and shaking moment balancing of mechanisms: a historical review with new examples. J Mech. Des. **127**, 334–339 (2005)
27. Berkof, R.S., Lowen, G.G.: Theory of shaking moment optimization of forced-balanced four-bar linkages. Trans. ASME J. Eng. Ind. **93B**(1), 53–60 (1971)

28. Hains, R.S.: Minimum RMS shaking moment or driving torque of a force-balanced linkage using feasible counterweights. Mech. Mach. Theory **16**, 185–190 (1981)
29. Arakelian, V., Dahan, M.: Partial shaking moment balancing of fully force balanced linkages. Mech. Mach. Theory **36**, 1241–1252 (2001)
30. Wiederrich, J.L., Roth, B.: Momentum balancing of four-bar linkages. Trans. ASME J. Eng. Ind. **98**(4), 1289–1295 (1976)
31. Chaudhary, H., Saha, S.K.: Balancing of four-bar linkages using maximum recursive dynamic algorithm. Mech. Mach. Theory **42**(2), 216–232 (2007)
32. Conte, F.L., George, G.R., Mayne, R.W., Sadler, J.P.: Optimum mechanism design combining kinematic and dynamic-force considerations. Trans. ASME J. Eng. Ind. **95**(2), 662–670 (1975)
33. MATLAB, R2008b: Optimization Toolbox, Version 7.7.0.471
34. Farmani, M.R.: Multiobjective optimization for force and moment balance of a four-bar mechanism using evolutionary algorithms. J. Mech. Sci. Technol. **25**(12), 2971–2977 (2011)
35. Chaudhary, K., Chaudhary, H.: Dynamic balancing of planar mechanisms using genetic algorithm. J. Mech. Sci. Technol. **28**(10), 4213–4220 (2014)
36. Chaudhary, K., Chaudhary, H.: Optimum balancing of slider-crank mechanism using equimomental system of point-masses. Procedia Technol. **14**, 35–42 (2014)
37. Arakelian, V., Briot, S.: Simultaneous inertia force/moment balancing and torque compensation of slider-crank mechanisms. Mech. Res. Commun. **37**, 265–269 (2010)
38. Chaudhary, K., Chaudhary, H.: Minimization of shaking force and shaking moment in multiloop planar mechanisms. In: Proc. of 1st International and 16th National Conference on Machines and Mechanisms (iNaCoMM2013), IIT Roorkee, 18–20 Dec 2013, pp. 346–352
39. Altair HyperWorks, Version 11