Multi-objective and Multi-physics Optimization of Fully Coupled Complex Structures

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Abstract. This work presents an improved approach for multi-objective and multi-physics optimization based on the hierarchical optimization approach of the typical MOCO ("Multi-objective Collaborative Optimization") whose objective is to solve multi-objective multi-physics optimization problem. In this document, we propose a new hierarchical optimization approach named Improved Multi-objective Collaborative Optimization (IMOCO) whose goal is to decompose the optimization problems of the complex systems hierarchically in two levels (system and disciplinary level) according to the disciplines. In other words, according to the different physical (mechanical-electrical-acoustical) involved in the mechanical structures design. The presented approach uses a NSGA-II "Non-dominated Sorting Genetic Algorithm II" as an optimizer, and uses a coordinator between the system optimizer and the disciplinary optimizer, which has the role, is to ensure consistency between the various disciplines of the complex system. For the purposes of validation of the proposed method, we chose two examples: (i) numerical problem and (ii) engineering problem. These examples are solved using the proposed IMOCO method and the previous approaches. The obtained results are compared well with those obtained from the previous approaches: (i) non-hierarchically based AAO optimization approach and (ii) hierarchically based MOCO optimization approach, which show the good performance of our proposed IMOCO method.

Keywords: multi-objective optimization, multi-physics optimization, hierarchical optimization, MOCO, IMOCO, NSGA-II, disciplines.

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1 Introduction

The complex engineering systems in the real world often involve several physical (mechanical, electrical, acoustic and thermal, etc.), subsystems and components. Many examples of advanced engineering systems may be find in the industry, such as the design of aircraft systems, automotive and nano, micro-electro-mechanical systems (NEMS, MEMS), which are intrinsically linked them by interdisciplinary interactions. The classical optimization approaches that ignore the interactions between the different disciplines are unable to give the desired optimal solutions. The use of advanced optimization techniques are required for these types of problems in order to achieve practical solutions. Multi-disciplinary Design Optimization (MDO) is a relatively field recent to engineering science which is based on the decomposition of these complex systems based on the different specialties where disciplines whose aim is to respond more effectively to design problems integrating different disciplines. The decomposition of these complex systems can be done in a number of different ways according to different criteria. The most frequent decomposition methods are: (i) decomposition by disciplines, (ii) decomposition according to the structural components. Typical MDO approaches of such large systems is characterized by the interdisciplinary couplings, the multiple objectives, a large design variable space and a significant number of design constraints (Tappeta and al 1997). The MDO approaches can be classified into two groups: Single level optimization approaches e.g. AAO ("All At Once") and multilevel optimization approaches (Aute and Azarm 2006). Due to the multiple criteria of multi-disciplinary optimization problems, recent work has focused on the formulation of the optimization problem of a multi-objective and multidisciplinary manner. For example, (Aute and Azarm 2006) proposed a new genetic algorithm based approach for Multi-objective Collaborative Optimization (MOCO) to handle the multi-objective and multi-disciplinary optimization problems. (Ghanmi and al 2011) developed an algorithm for multi-objective and multi-level optimization to solve the optimization problems of complex structures with low Multi-physics coupling.

This paper is organized as follows: Section 2 gives a brief description of the strategy IMOCO (Improved Multi-Objective Collaborative Optimization). Sections 3.1 and 3.2 show the application of our proposed approach in the cases of a numerical problem and an engineering problem of capacitive Micomachined Ultrasonic Transducers (cMUT). The obtained solutions from the IMOCO approach compare well with those obtained from a non-hierarchically based AAO optimization approach and from a hierarchically based MOCO optimization approach.

2 Improved Multi-objective Collaborative Optimization

The presented multi-disciplinary optimization approach called IMOCO, which uses NSGA-II, is suitable for multi-physics problematic whose objective is to divide the optimization problem of widely multi-physics structures according to different disciplines involved in the design. For simplicity, a multi-physical system is considered in the present document that involves two disciplines (mechanical and electrical) in order to illustrate the optimization procedure of the IMOCO method as shown by the Figure 1. As illustrated in Fig.1, the system level optimizer has objectives functions f_G and constraints g_G. The vector X_G includes the share variables x_{sh}^G between the two disciplines previously mentioned and the system auxiliary variables (t_{me}^G, t_{em}^G), with the super script 'G' referring to a system level. The mechanical optimizer has a set of disciplinary objectives functions f_m, disciplinary constraints g_m and a vector of disciplinary design variable X_m includes the local design variables x_m, the local shared variables x_{sh}^m, and the local auxiliary variable t_{em}^m. The write y_{me} represents the coupling variable, which is calculated by the disciplinary state equation (represented by Y_{me}) depending of X_m. Likewise, the electrical optimizer has a set of disciplinary objectives functions f_e, disciplinary constraints g_e and a vector of design variable X_e, which includes the local design variables x_e, the local shared variables x_{sh}^e, and the local auxiliary variable t_{me}^e. The write y_{em} represents the coupling variable: the output of the electrical optimizer used as an input variable in the mechanical optimizer. In addition, each discipline requires shared variables x_{sh}^G and inputs from other disciplines indicated by (y_{me}, y_{em}). The coordination problem $C(X)$ is introduced between the two optimizers (mechanical and electrical). The aim of the coordinator (coordination problem) is to minimize the difference between the shared parameters (x_{sh}^m, x_{sh}^e) transferred from the disciplinary level and the coordination shared variables x_{sh}. On the other hand, the coordination problem aims to minimize the difference between the auxiliary variables (t_{me}, t_{em}) and the auxiliary parameters (t_{me}^m, t_{em}^e) transferred from the disciplinary level. In addition, it reduces the difference between the auxiliary variables (t_{me}, t_{em}) and the coupling variables (y_{me}, y_{em}) transferred from the disciplinary level.

After the optimization of coordination problem and the optimal solutions with the smallest value of the penalty function we chose the optimal variables (x_m^*, x_e^*) at disciplinary level which are then transferred towards the system optimizer. Note that, the optimal variables (x_m^*, x_e^*) are transferred from the disciplinary optimizer and temporarily stored in the coordination problem. In the literature there are many methods aimed at addressing the coupling variables. In our proposed approach IMOCO, we use the auxiliary variables ((Aute and Azarm 2006) (Tappeta and al.1997)) to decouple all disciplines and add a constraint of consistency, which aims to ensure compatibility interdisciplinary. New form defines the constraint of consistency of the mechanical discipline such as $\left\|1-\left(x_{sh}^{m}/x_{sh}^{G}\right)\right\|_2+\left\|1-\left(y_{me}/t_{me}^{G}\right)\right\|_2+\left\|1-(t_{em}^{m}/t_{em}^{G})\right\|_2$ and to integrate in the objective functions of the optimization problem of the mechanical discipline. In addition, the constraint of consistency of the electrical discipline is defined by $\left\|1-\left(x_{sh}^{e}/x_{sh}^{G}\right)\right\|_2+\left\|1-\left(y_{em}/t_{em}^{G}\right)\right\|_2+\left\|1-(t_{me}^{e}/t_{me}^{G})\right\|_2$ and to integrate in the objective functions of the optimization problem of the electrical discipline

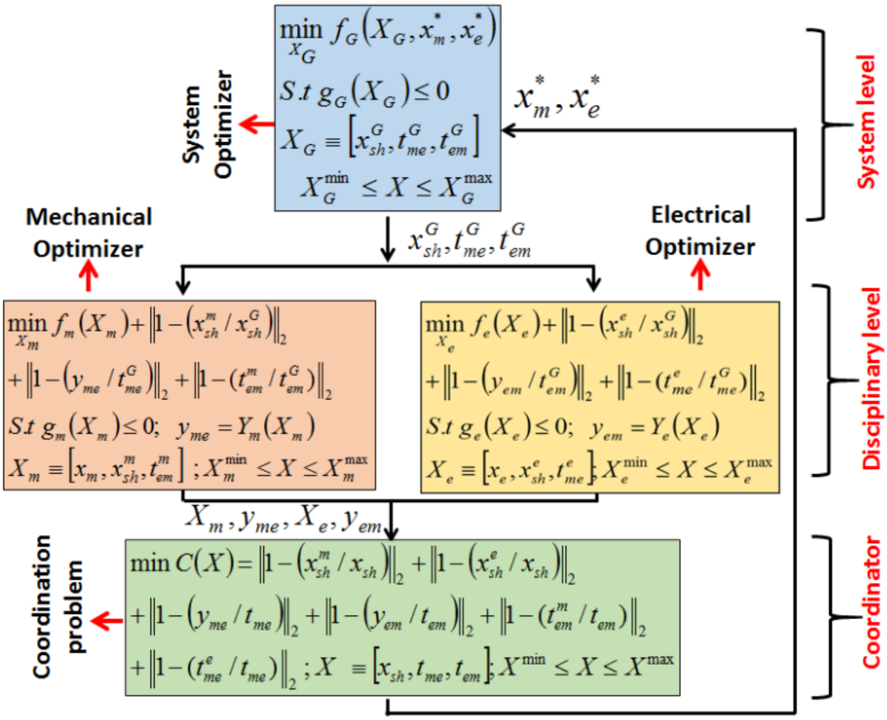

Fig. 1 Procedure of the IMOCO

3 Numerical Simulations

3.1 Numerical Problem

This first bi-objective numerical problem taken from a test case is used by (Aute and Azarm 2006). The two-objective optimization formulation for this problem, in the non-hierarchical AAO method is given in Equation (1). The problem with two cost functions f_1 and f_2 as to minimize admits six design variables and six inequality constraints. To decompose the problem, we use the proposed method, which is illustrated in Fig.1. The non-hierarchical optimization problem is decomposed into two-optimization level : (i) optimization problem at the system level is given by Equation (2) and (ii) two optimization problems at the disciplinary level is given by Equation (3-4). The formulation of typical MOCO approach is omitted here can be found in (Aute and Azarm 2006).

$$\begin{aligned}
&\text{minimize } f_1(x) = -\left[25\left(x_1-2\right)^2+\left(x_2-2\right)^2+\left(x_3-1\right)^2+\left(x_4-4\right)^2+\left(x_5-1\right)^2\right]\\
&\text{minimize } f_2(x) = x_1^2+x_2^2+x_3^2+x_4^2+x_5^2+x_6^2\\
&\textit{subject.to } g_1 = 2-x_1-x_2 \le 0;\quad g_2 = x_1+x_2-6 \le 0\\
&\qquad g_3 = x_2-x_1-2 \le 0;\quad g_4 = x_1-3x_2-2 \le 0\\
&\qquad g_5 = \left(x_3-3\right)^2+x_4-4 \le 0;\ g_6 = 4-\left(x_5-3\right)^2+x_6 \le 0\\
&\qquad\qquad 0 \le x_1, x_2, x_6 \le 10;\ 1 \le x_3, x_5 \le 5;\ 0 \le x_4 \le 6
\end{aligned} \tag{1}$$

❖ Decomposition according to the proposed approach IMOCO: System level Optimization

$$\begin{aligned}
&\text{minimize} f_1(X_G, \overset{*}{X}_{D1}) = f_{11}+f_{21}\\
&\text{minimize } f_2(X_G, \overset{*}{X}_{D1}, \overset{*}{X}_{D2}) = f_{12}+f_{22}\\
&\textit{subject.to } g_1, g_2, g_3, g_4 \ge 0\\
&x_{sh} = \left[x_1, x_2\right],\ X_G \equiv \left[x_{sh}, t_{21}^G\right]
\end{aligned} \tag{2}$$

Discipline level optimization 1

$$\begin{aligned}
&\text{minimize} f_{1.1}\left(X_1\right) = -\left[12.5\left(x_1-2\right)^2+0.5\left(x_2-2\right)^2+\left(x_3-1\right)^2+\left(x_4-4\right)^2\right]\\
&+\left\|1-(x_{sh}^1 / x_{sh}^G)\right\|_2+\left\|1-(t_{21}^1 / t_{21}^G)\right\|_2\\
&\text{minimize} f_{1.2}\left(X_1\right) = 0.5\left(x_1^2+x_2^2+x_3^2+x_4^2+x_5^2\right)+\left\|1-(x_{sh}^1 / x_{sh}^G)\right\|_2+\left\|1-(t_{21}^1 / t_{21}^G)\right\|_2\\
&\textit{subject.to } g_5 \ge 0\ ;\ x_{sh}^1 \equiv \left[x_1, x_2\right];\ X_{D1} \equiv \left[x_3, x_4\right];\ t_{21}^1 \equiv x_5\ X_1 \equiv \left[X_{D1}, x_{sh}^1, t_{21}^1\right]
\end{aligned} \tag{3}$$

Discipline level optimization 2

$$\begin{aligned}
&\text{minimize} f_{2.1}\left(X_2\right) = -\left[12.5\left(x_1-2\right)^2+0.5\left(x_2-2\right)^2+\left(x_5-1\right)^2\right] \\
&+\left\|1-(x_{sh}^2 / x_{sh}^G)\right\|_2+\left\|1-(y_{21} / t_{21}^G)\right\|_2 \\
&\text{minimize} f_{2.2}\left(X_2\right) = 0.5\left(x_1^2+x_2^2\right)+x_6^2+\left\|1-(x_{sh}^2 / x_{sh}^G)\right\|_2+\left\|1-(y_{21} / t_{21}^G)\right\|_2 \\
&\textit{subject.to } g_6 \geq 0 \\
&x_{sh}^2 \equiv \left[x_1, x_2\right];\ X_{D2} \equiv \left[x_6\right];\ y_{21} \equiv x_5;\ X_2 \equiv \left[X_{D2}, x_{sh}^2, y_{21}\right]
\end{aligned} \tag{4}$$

After the optimization of coordination problem in Equation (5) and the optimal solutions with the smallest value of the penalty function we chose the optimal variable at disciplinary level (x_{D1}^*, x_{D2}^*) which are then transferred towards the system optimizer (Equation (2)).

Coordination problem

$$\begin{aligned}
&\text{minimize } C\left(X\right) = \left\|1-(x_{sh}^1 / x_{sh})\right\|_2+\left\|1-(x_{sh}^2 / x_{sh})\right\|_2+\left\|1-(y_{21} / t_{21})\right\|_2 \\
&+\left\|1-(t_{21}^1 / t_{21})\right\|_2\ ;\ X \equiv \left[x_{sh}, t_{21}\right];\ X^{\min} \leq X \leq X^{\max}
\end{aligned} \tag{5}$$

The constrained Pareto optimal front for numerical example generated by AAO, MOCO and IMOCO approaches using NSGA-II (Deb 2001) are plotted in the objective space as in Fig.2. The Pareto solutions obtained by MOCO and IMOCO approaches are compared well to those from AAO approach, which is considered as a reference. Figure 2 shows that the Pareto optimal solutions of the IMOCO method are in good concordance than those from the MOCO. This is also was confirmed by the Mahalanobis distance (D_M^2) used by (Ghanmi and al 2011) between AAO and MOCO on the one hand, and AAO and IMOCO on the other hand. To better compare the performance (precision and convergence time) of the IMOCO and the MOCO methods we used (i) the OS metrics (Overall Spread metric) (Wu and Azarm 2001) aims to measure the precision and (ii) the CPU time to measure the speed of convergence. The results in Table 1 show the advantage of our proposed IMOCO method.

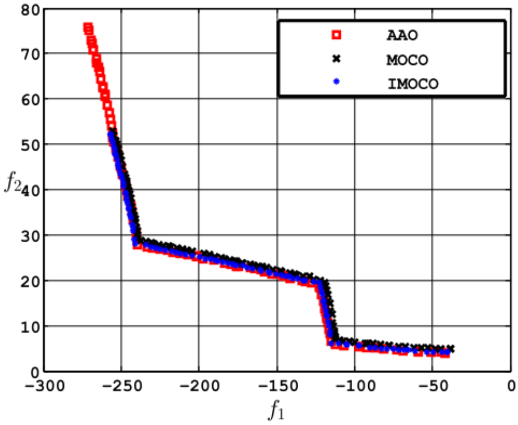

Fig. 2 Feasible objective space for numerical example. Generated by AAO, MOCO and IMOCO

Table 1 Performance of the proposed IMOCO method.

Approach	CPU (%)	D^2_M	OS
AAO	100		
MOCO	82	0.2633	0.3456
IMOCO	74	0.2748	0.3461

3.2 Engineering Problem : cMUT

The Capacitive Micomachined Ultrasonic Transducers (cMUT) is a complex system that involve the coupling between three physical: electrical, mechanical (mechanical structures) and acoustical. The cMUT device manufactured according to the technologies of micro-electro-mechanical systems (MEMS) and that generates or detects acoustic waves. As it is sketched in Fig.3 (a), the geometry of the mobile part of a cMUT cell is composed of a Silicon Nitride membrane of e_{mbr} thickness, partially covered with aluminum electrode of e_{elec} thickness, and a vacuum cavity h_{Gap} .

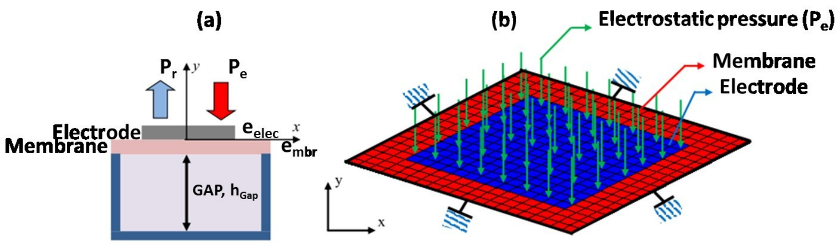

Fig. 3 (a) Representation of cMUT cell and definition of the geometric parameters, (b) a finite element model of cMUT cell.

The cMUT cell is biased with a constant voltage value (V_{dc}) thus creating an electrostatic pressure (P_e) which goes cause a displacement of the membrane. This movement himself generate a radiated pressure in fluid (P_r) in front face of the membrane. The boundary conditions considered in the numerical simulation model are illustrated in detail in fig.3 (b). The details of material and the geometric properties of the cMUT cell can be found in (Meynier and al 2010).

The multi-objective optimization problem of the cMUT cell is given by Equation (6) to solve following the non-hierarchical approach AAO using NSGA-II admits two cost functions, eight design variables and four inequality constraints:

(1) Minimize the mechanical resonance frequency of a cMUT ($f_{rm}(MHZ)$;

(2) Maximize the electromechanical coupling coefficient (K_T^2).

The multi-objective optimization problem to solve is then:

$$\begin{cases} \text{minimize}\left(f_{rm}(x), -K_T^2(x)\right) \\ \textit{subject.to} \quad g_1 = e_{mbr} + e_{elec} - 700\,e^{-9} \geq 0 \\ \quad g_2 = u_{dc} - \frac{2}{3}h_{eqv} \leq 0 \;\; ; h_{eqv} = h_{Gap} + \dfrac{e_{mbr}}{\varepsilon_{mb}} \\ \quad g_3 = \mathrm{V}_{dc} - 105 \leq 0 ; g_4 = \boldsymbol{V} - 105 \leq 0 \\ \quad x \in (E_{mbr}, E_{elec}, e_{mbr}, e_{elec}, \rho_{mbr}, \rho_{elec}, \mathrm{V}_{dc}, \boldsymbol{V}) \end{cases} \tag{6}$$

Where, u_{dc} and h_{eqv} represent the static deflection of the membrane and the effective height between the two electrodes respectively. The electromechanical coupling coefficient of CMUT is calculated in (Yaralioglu and al 2003) as follows:

$$K_T^2 = 1 - \frac{C^S}{C^T} \tag{7}$$

The fixed capacitance C^S of a single membrane can be calculated as follows:

$$C^S = \frac{Q}{\mathrm{V}_{dc}} = \frac{\varepsilon_0 S}{2\left(h_{eqv} + u_{dc}\right)} \;\; ; \; avec \;\; Q = \frac{\varepsilon_0 \mathrm{V}_{dc}}{2\left(h_{eqv} + u_{dc}\right)} \tag{8}$$

The free capacitance C^T is defined as the slope of the voltage-voltage curve:

$$C^T = \left.\frac{dQ(V)}{dV}\right|_{u_{dc}, V_{DC}} = V \left.\frac{d\left(C^S\right)}{dV}\right|_{u_{dc}, V_{DC}} = \frac{C_{\boldsymbol{V}}{}^S\left(\boldsymbol{V} - \mathrm{V}_{dc}\right) - C^S \times \boldsymbol{V}_{dc}}{\left(\boldsymbol{V} - \boldsymbol{V}_{dc}\right)} \tag{9}$$

$$with\, \boldsymbol{V} = \mathrm{V}_{dc} + \Delta \mathrm{V}_{dc}$$

In the present work, the cMUT cell operates in a vacuum, therefore two physical are considered in our optimization problem. The objective function for this problem in Equation (6) can be decomposed into two levels of optimization (system and disciplinary level) Using the proposed IMOCO approach.

Equation (10) represents the system level optimization problem. Although each discipline has its own design variables, four variables are shared between the optimization problem at the system level and at the disciplinary level and are the Young's modulus (E_{mbr}), the thickness of the membrane (e_{mbr}), the Young's modulus (E_{elec}), and the thickness of the electrode (e_{elec}).

System level optimization problem

$$\begin{cases} \text{minimize}\left(f_{rm}(X_G, \overset{*}{x}_m), -K_T^2(X_G, \overset{*}{x}_e)\right) \\ subject.to\ g_1, g_3 \geq 0 \\ x_{sh} = \left[E_{mbr}, E_{elec}, e_{mbr}, e_{elec}\right],\ X_G \equiv \left[x_{sh}, t_{em}^G\right] \end{cases} \tag{10}$$

In the system level optimization problem, $X_G \equiv [\,x_{sh}, t_{em}^G\,]$ includes the four shared design variables [$E_{mbr}, E_{elec}, e_{mbr}, e_{elec}$]; and the auxiliary variable t_{em}^G corresponds to the coupling variable y_{em} .

The system level optimization problem aims the minimization of the mechanical resonance frequency (f_{rm}) and the maximization of the electromechanical coupling coefficient (K_T^2). The super script (*) indicates that the design parameters are optimized at the disciplinary level.

Mechanical Optimizer | **Electrical Optimizer**

$$\begin{array}{l|l} \text{minimize}\ f_{rm}\left(X_m\right) + & \text{maximize}\ C^T\left(X_e\right) \\ \left\|1-(x_{sh}^m / x_{sh}^G)\right\|_2 + \left\|1-(t_{em}^m / t_{em}^G)\right\|_2 & +\left\|1-(x_{sh}^e / x_{sh}^G)\right\|_2 + \left\|1-(y_{em} / t_{em}^G)\right\|_2 \\ subject.to\ g_2 \geq 0 & subject.to\ g_4 \geq 0 \\ x_{sh}^m \equiv \left[E_{mbr}, E_{elec}, e_{mbr}, e_{elec}\right] & x_{sh}^e \equiv \left[E_{mbr}, E_{elec}, e_{mbr}, e_{elec}\right] \\ x_m \equiv \left[\rho_{mbr}, \rho_{elec}\right];\ t_{em}^m \equiv \left[\mathrm{V}_{dc}\right] & x_e \equiv \left[\boldsymbol{V}\right];\ y_{em} \equiv \left[\mathrm{V}_{dc}\right] \\ X_m \equiv \left[x_m, x_{sh}^m, t_{em}^m\right] & X_e \equiv \left[x_e, x_{sh}^e, y_{em}\right] \end{array} \tag{11}$$

At the disciplinary level in Equation (11), we find two disciplinary optimizers: (i) mechanical optimizer and (ii) the electrical optimizer. The mechanical optimizer has his own design variables; the two variables also include the density of the membrane (ρ_{mbr}), the density of the electrode (ρ_{elec}), and the local copy of the auxiliary variable for t_{em}^{G} i.e., t_{em}^{e} which allows minimizing the mechanical resonance frequency (f_{rm}) of the mechanical discipline. Equally, the electrical optimizer includes local design variable is the bias voltage (**V**), and the interdisciplinary coupling variable y_{em} , whose value is evaluated in electrical optimizer and is used in Mechanical optimizer. This optimizer aims to maximize the free capacitance C^{T} .

After the optimization of coordination problem in Equation (12) and the optimal solutions with the smallest value of the penalty function we chose the optimal variables (x_{m}^{*}, x_{e}^{*}) at disciplinary level which are then transferred towards the system optimizer (Equation (10)).

Coordination problem

$$\begin{aligned} &\text{minimize } C(X) = \left\| 1-(x_{sh}^{m} / x_{sh}) \right\|_2 + \left\| 1-(x_{sh}^{e} / x_{sh}) \right\|_2 + \left\| 1-(y_{em} / t_{em}) \right\|_2 \\ &+\left\| 1-(t_{em}^{m} / t_{em}) \right\|_2 \;\; ; \; X \equiv \left[x_{sh}, t_{em} \right] ; \; X^{\min} \le X \le X^{\max} \end{aligned} \tag{12}$$

The Feasible optimal solutions obtained by IMOCO method are compared with those generated by MOCO as shown in Fig.4. In terms of proximity to the reference (AAO) cloud of the optimal solutions, the spread in the objective space and the CPU time, the IMOCO solutions are better than the MOCO solutions. We can observed in the Table 2 that the CPU time, the values for Mahalanobis distance (D^{2}_{M}) and Overall spread (OS) from the IMOCO are better than MOCO.

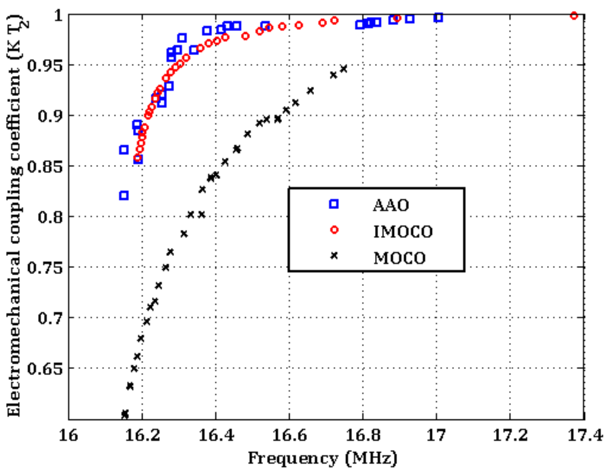

Fig. 4 Feasible objective space for cMUT cell obtained using AAO, MOCO and IMOCO

Table 2 Performance of the proposed IMOCO method

Approach	CPU (%)	D^2_M	OS
AAO	100		
MOCO	75	5.7348	0.4221
IMOCO	72	0.0632	0.3455

4 Conclusion

In this work, we have presented the IMOCO method using NSGA-II for the design of fully coupled complex systems. The IMOCO enhances the convergence by adding a coordination problem between the system level and the discipline level optimizer. This improvement allows the IMOCO to achieve convergence more efficiently than the previous MOCO method. Based on the results from the numerical example and the engineering example we can be concluded that the optimal solutions obtained by IMOCO are better than the MOCO solutions. First, in terms of proximity to the reference (AAO) cloud of Pareto optimum solutions and the spread in the feasible objective space. Secondly, in terms of the CPU time to measure the speed of convergence. Consequently, we conclude that the proposed IMOCO method is able to solve a fully coupled Multi-objective Multi-physics Optimization Problem more efficiently than the MOCO approach. Finally, in our future research we propose to apply our method on complex systems involve several disciplines (mechanical-electrical-acoustical…) fully coupled.

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