

# Chapter 25

## Controller Design by Time-Domain Objective Functions Using Cuckoo Search

Huey-Yang Horng

**Abstract** In this research, a new optimization algorithm, called the cuckoo search algorithm, is introduced for design controller. A large proportion of industrial systems are represented by linear time-invariant transfer functions. The proportional-integral-derivative (PID) controller is one of the most widely used functions. The lead-lag controller is a more practical alternative. Traditionally, time-domain or frequency-domain methods have been used to design a lead-lag controller to design specifications. This chapter focused on the design of controller both PID and lead-lag controller, by optimization of the time-domain objective function. The proposed objective function includes time-domain specifications, including the rise time, peak time, maximum overshoot, setting time, and steady-state error. In the chapter, Cuckoo Search algorithm is chosen to finding the optimal solutions. Cuckoo Search is metaheuristic optimization method recently developed. That is a type of population-based algorithm inspired by the behavior of some Cuckoo species in combination with the Lévy flight behavior. Given that the plant is modeled according to a linear time-invariant transfer function, the proposed method designs the controller capable of approaching the specifications.

**Keywords** PID controller • Lead-lag controller • Cuckoo Search • PSO

### 25.1 Introduction

Most industrial plant systems can be represented by a linear time-invariant transfer function. Proportional-integral-derivative (PID) controllers are probably the most commonly used controllers in industrial applications. Numerous methods have been proposed for tuning the PID controller parameters [1–3].

Lead-lag controllers provide a more practical alternative. The design of the lead-lag controller has been studied [4–6]. Ou and Lin proposed a method based on Generic Algorithm (GA) and Particle Swarm Optimization (PSO) to design the PID

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controller, and then compared the results [7]. Horng used cuckoo search to design lead-lag controller [8]. Cuckoo search algorithm is one of the latest metaheuristic techniques, developed by Yang and Deb [9–11].

In this chapter, cuckoo search algorithm that uses a time-domain objective function to design the controller is proposed. If the plant could be modeled as a linear time-invariant transfer function, the proposed method can design a controller that approaches or meets the time-domain specifications. The objective function includes eight specifications, they are delay time, rise time, first peak time, maximum peak time, percent maximum overshoot, percent maximum undershoot, setting time, and steady-state error. This is improved by the formation of reference [8].

## 25.2 Time-Domain Objective Functions

Typical unit-step response of a control system illustrating the time-domain specifications are percentage maximum overshoot  $OS\%$ , delay time  $T_d$ , rise time,  $T_r$ , setting time  $T_s$ , and steady-state error  $E_{ss}$  [1]. Let  $y(t)$  be the unit-step response and  $y_{ss}$  the steady-state error. For a more general system, the following specifications will be considered:

1. First Peak time,  $T_p$ . The time to reach the first peak.
2. Maximum Peak time,  $T_m$ . The time to reach the maximum peak.
3. Percentage Maximum overshoot,  $US\%$  is defined as

$$y_{us} = \min(y(t)), \quad t \geq T_p, \quad US\% = \begin{cases} \frac{(y_{ss} - y_{us})}{y_{ss}}, & \text{if } y_{us} < y_{ss}, \\ 0, & \text{if } y_{us} \geq y_{ss}. \end{cases} \quad (25.1)$$

First define deviation ratio (DR)

$$\begin{aligned} DR(TDS) &= f(\mathbf{x}|TDS : lb, ub) \\ &= \begin{cases} 0, & \text{if } 0 \leq lb \leq f(\mathbf{x}|TDS) \leq ub \\ \frac{f(\mathbf{x}|TDS) - ub}{ub}, & \text{if } f(\mathbf{x}|TDS) > ub \\ \frac{lb - f(\mathbf{x}|TDS)}{lb}, & \text{if } f(\mathbf{x}|TDS) < lb \end{cases} \end{aligned} \quad (25.2)$$

where  $TDS$  is the time-domain specification, i.e., rise time, first peak time, maximum peak time, etc. The proposed objective function is

$$\text{TDOF} = \frac{(w_1 \text{DR}(T_d) + w_2 \text{DR}(T_r) + w_3 \text{DR}(T_p) + w_4 \text{DR}(T_m) + w_5 \text{DR}(\text{OS } \%) + w_6 \text{DR}(\text{US } \%) + w_7 \text{DR}(T_s) + w_8 \text{DR}(E_{ss}))}{\text{TW}} \quad (25.3)$$

where  $\text{TW} = \sum_{i=1}^8 w_i$ . In (25.3),  $w_i$  represents weights reflecting the relative importance of the corresponding term. DR denotes the deviation ratio of desired interval, from lower bound (lb) to upper bound (ub). For second-order system, the first peak time is the maximum peak time. But, for the general system, they are not the same.

The design of controller becomes in the minimization of the TDOF for all possible parameters. Moreover, some tolerances in the time-domain specifications are allowable as in (25.2).

### 25.3 The Controller

Two kinds of controllers are considered, there are PID and lead-lag (or lag-lead) controller. For PID control, one typically has

$$G_c(s) = K_p + \frac{K_i}{s} + K_d s. \quad (25.4)$$

The transfer function of a lead-lag (or lag-lead) controller is written as:

$$G_c(s) = K \left( \frac{T_1 s + 1}{\alpha T_1 s + 1} \right) \cdot \left( \frac{T_2 s + 1}{\beta T_2 s + 1} \right), \quad (25.5)$$

where  $K > 0$ ,  $\alpha > 1$ ,  $T_1 > 0$ ,  $\beta < 1$ ,  $T_2 > 0$ .

### 25.4 Cuckoo Search Algorithm

In this chapter, a new optimization algorithm, called the cuckoo search algorithm, is introduced for design controller [9–11]. The algorithm uses a combination of a local random walk and the global random walk, controlled by a parameter  $p_a$ . This allows for proper balance between exploration and exploitation of the solution space. The local random walk can be written as

$$x_i^{t+1} = x_i^t + \alpha s \otimes H(p_a - \varepsilon) \otimes (x_j^t - x_k^t) \quad (25.6)$$

where  $x_i^t$  and  $x_k^t$  are two different solutions taken by random permutation. In (25.6),  $H(u)$  is a Heaviside function,  $\varepsilon$  is a random number got from a uniform distribution,

and  $s$  is the step size. On the other hand, the global random walk is figured out using Lévy flights:

$$x_i^{t+1} = x_i^t + \alpha L(s, \lambda) \quad (25.7)$$

where

$$L(s, \lambda) = \frac{\lambda \Gamma(\lambda) \sin(\pi\lambda/2)}{\pi} \cdot \frac{1}{s^{1+\lambda}}, \quad s \geq s_0 \geq 0 \quad (25.8)$$

Here,  $\alpha = 0.01$  is the step size scaling factor.

In the following design procedure, set maximum generation equals to 750, and  $P_a = 0.25$ . Generate initial population of 15 host nests, which cause closed-loop stable (by using Routh-Hurwitz criterion).

## 25.5 Illustrative Examples

*Example 1* A unity feedback system has the forward transfer function:

$$G_p(s) = \frac{150}{s^2 + 15s + 50}. \quad (25.9)$$

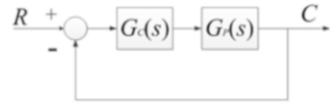
as in Fig. 25.1. The closed-loop system is stable. The system type refers to the order of the pole of  $G_p(s)$  at  $s=0$ . Thus, the closed-loop system having the forward-path transfer function of (25.9) is type 0. The step response of the uncompensated system is shown in Fig. 25.3. Here, the peak time is 0.249 s, the percentage overshoot is 13.3 %, and the steady-state error  $E_{ss}$  is equal to 0.273.

As shown, the uncompensated system does not satisfy the design specifications in Table 25.1. Hence, the controller is used to improve the transient response as in Fig. 25.2. All the weight  $w_i$  are set to 1. First, the PID controller will be designed for the system. When the design procedure is completed, the parameters are  $K_p = 5.3161$ ,  $K_i = 10.2197$ , and  $K_d = 0.2299$ . Apart from delay time, most of the time-domain specifications are very close to demand. Next, the lead-lag controller will be designed. When completed, the parameters are  $K = 59.6384$ ,  $\alpha = 26.0208$ ,  $T_1 = 0.2319$ ,  $\beta = 0.1520$ , and  $T_2 = 0.0911$ . All the desired specifications are fulfilled. If the deviation ratio of delay time would be reduced, the weight of delay time will increase to 6, and keep the others the same value. After the adjustments of weights, and redesign the controller again. The parameters found are,  $K_p = 3.0046$ ,  $K_i = 7.3653$ , and  $K_d = 0.1339$ . The deviation ratio of all specifications are listed in Table 25.2. Since the specifications of both led-lag controllers are full-filed, the adjustments produce influence of no importance as shown in Fig. 25.4.

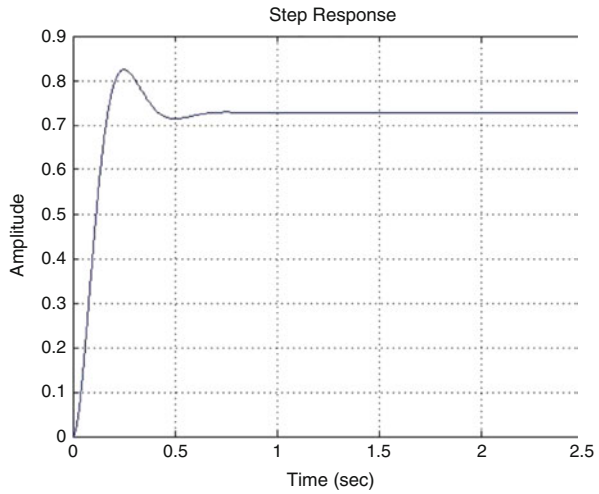
**Fig. 25.1** Unity feedback system without controller



**Fig. 25.2** Unity feedback system with controller



**Fig. 25.3** Uncompensated system in Example 1



**Table 25.1** Example 1: Controller design with  $w_i = 1$ , for  $i = 1, 2, \dots, 8$

Spec.	Desired interval	$w_i$	PID 1	PID 1 DR	Lead-lag 1	Lead-lag 1 DR
$T_p$	[0.0988, 0.1008]	1	0.1014	0.0063	0.1006	0
$T_m$	[0.0988, 0.1008]	1	0.1014	0.0063	0.1006	0
$T_r$	[0.0477, 0.0487]	1	0.0466	0.0223	0.0487	0
$T_d$	[0.0307, 0.0314]	1	0.0185	0.3970	0.0311	0
OS %	[0.0, 0.03]	1	0.0300	0	0.0245	0
US %	[0.0, 0.02]	1	0	0.0200	0.0021	0
$T_s$	[0.1014, 0.1035]	1	0.1031	0	0.1023	0
$E_{ss}$	[0.0, 0.22]	1	0.0326	0	0.0045	0

*Example 2* A unity feedback system has the following forward transfer function:

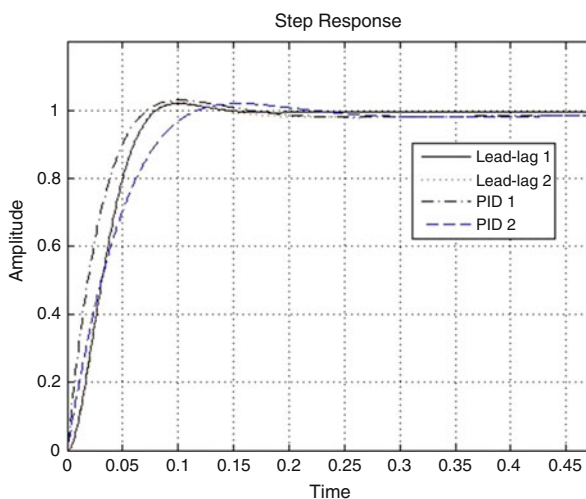
$$G_p(s) = \frac{700}{(s + 2)(s + 4)(s + 6)}$$

The closed-loop system is unstable. The step response of the uncompensated system is shown in Fig. 25.5. The design specifications are listed in Table 25.3.

**Table 25.2** Example 1: Controller design with  $w_4 = 6$ , and  $w_i = 1$ , for  $i \neq 4$

Spec.	Desired interval	$w_i$	PID 2	PID 2 DR	Lead-lag 2	Lead-lag2 DR
$T_p$	[0.0988, 0.1008]	1	0.1542	0.5300	0.1006	0
$T_m$	[0.0988, 0.1008]	1	0.1542	0.5300	0.1006	0
$T_r$	[0.0477, 0.0487]	1	0.0749	0.5382	0.0485	0
$T_d$	[0.0307, 0.0314]	6	0.0307	0.0000	0.0308	0
$OS\%$	[0.0, 0.03]	1	0.0300	0	0.0300	0
$US\%$	[0.0, 0.02]	1	0.0200	0	0.0148	0
$T_s$	[0.1014, 0.1035]	1	0.1055	0.0197	0.1023	0
$E_{ss}$	[0.0, 0.22]	1	0.0453	0	0.0061	0

**Fig. 25.4** Compensated system in Example 1



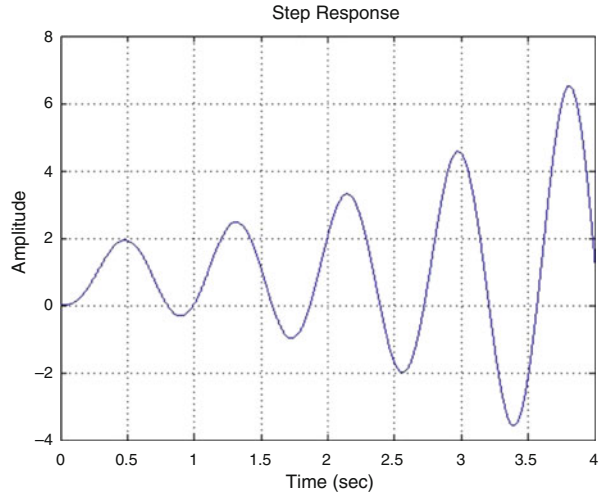
When the design procedure is finished, the PID parameters are  $K_p = 0.2431$ ,  $K_i = 0.3030$ , and  $K_d = 0.2299$ . Furthermore, the lead-lag parameters are  $K = 7.1739$ ,  $\alpha = 67.1652$ ,  $T_1 = 0.4054$ ,  $\beta = 0.0010$ , and  $T_2 = 0.4112$ . The deviation ratio of delay time has been improved (Fig. 25.6).

The weight of delay time  $T_d$  increase by a factor of 6, while keeping the others the same. Redesign the controller again; the PID parameters found are  $K_p = 0.2431$ ,  $K_i = 0.2840$ , and  $K_d = 0.0523$  (Table 25.4).

## 25.6 Conclusions

A large part of an industrial plant system may be represented by the linear time-invariant transfer function. A simple procedure is used to design the controller to meet or approach the specification with cuckoo search algorithm has been proposed

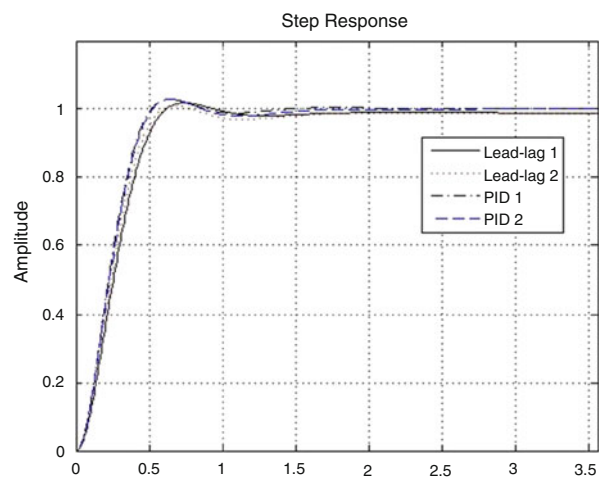
**Fig. 25.5** Uncompensated system in Example 3



**Table 25.3** Example 2: Controller design with  $w_i = 1$ , for  $i = 1, 2, \dots, 8$

Spec.	Desired interval	$w_i$	PID 2	PID 2 DR	Lead-lag 2	Lead-lag 2 DR
$T_p$	[0.7411, 0.7561]	1	0.6434	0.1319	0.7423	0
$T_m$	[0.7411, 0.7561]	1	0.6434	0.1319	0.7423	0
$T_r$	[0.3581, 0.3653]	1	0.3251	0.0921	0.3711	0.0159
$T_d$	[0.2305, 0.2352]	1	0.2229	0.0329	0.2514	0.0687
$OS\%$	[0.0, 0.03]	1	0.0300	0	0.0300	0
$US\%$	[0.0, 0.02]	1	0.0114	0	0.0085	0
$T_s$	[0.7606, 0.7760]	1	0.6557	0.1379	0.0107	0
$E_{ss}$	[0.0, 0.22]	1	0.2263	0.0286	0.0058	0

**Fig. 25.6** Compensated system in Example 3



**Table 25.4** Example 2: controller design with  $w_3 = 6$ , and  $w_i = 1$ , for  $i \neq 3$ 

Spec.	Desired interval	$w_i$	PID 2	PID 2 DR	Lead-lag 2	Lead-lag 2 DR
$T_p$	[0.7411, 0.7561]	1	0.6495	0.1235	0.6805	0.0818
$T_m$	[0.7411, 0.7561]	1	0.6495	0.1235	0.6805	0.0818
$T_r$	[0.3581, 0.3653]	1	0.3306	0.0767	0.3490	0.0254
$T_d$	[0.2305, 0.2352]	6	0.2277	0.0119	0.2361	0.0040
$OS\%$	[0.0, 0.03]	1	0.0300	0.0304	0.0141	0
$US\%$	[0.0, 0.02]	1	0	0	0.0200	0
$T_s$	[0.7606, 0.7760]	1	0.1016	0	0.6619	0.1297
$E_{ss}$	[0.0, 0.22]	1	0.0089	0	0.2414	0.0974

in this chapter. The proposed time-domain objective function is expressed in terms of peak time, maximum overshoot, maximum undertow, and setting time of the unit-step response. Computer simulations support the usefulness of the method.

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