CONSTRUCTING FUZZY CLUSTERS FOR MARKETING RESEARCH

Jerzy (George) Plocinski
La Trobe University

Using the fuzzy sets theory this study shows how customer subjective preferences for preselected products and their subjective evaluations of attributes can be used to generate market clusters. The findings indicate that if the preferences are quantitatively expressed and treated as membership functions of some fuzzy sets then it is possible to construct purely "objective" (non-dominated) evaluations of the products and the attributes. The evaluations can be combined to form profiles which are then grouped as clusters, depending on what linkage criterion is used (single, complete, average). The considerations are illustrated by a numerical example of a car market represented by four car brands (Ford, Mercedes, Holden, and Toyota) and five types of driver (car buff, young person, wealthy physician, rally enthusiast and amateur racer).

Marketing projects are usually characterised by a large number of complex data which collected are then used to develop an appropriate mathematical market model to analyse and forecast future behaviour of consumers. The complex data are usually multivariate and processed through multivariate analysis . A popular source of market information involves paired comparisons of consumers during their interviews. They are

"judgemental tasks that typically involve exposing an individual to pair of stimuli (e.g. brands, attributes, etc.) one at a time and asking for a judgement about which element in the pair is more preferred, or important, or to select one element of the pair according to some other criterion. In comparison with other data collection formats, paired comparisons are easily administered and provide a relatively simple judgemental task for the collection of comparative measurements. Because of its simplicity paired comparisons have been frequently applied in commercial settings. They have been routinely employed in the following four application areas:

1.Concept screening studies
2.Choice modelling studies
3.Pre-test-market studies
4.Conjoint analysis studies."(Dillon et al. 1993).

There are a number of procedures recommended for studying consumer preferences for definite products. It is worthwile to mention here William Dillon et al.(1993) who proposed an extended Bradly-Terry-Luce (BTL) model incorporating descriptor variables to capture individual differences. In 1993 Joel Huber et al. suggested an unrestricting attribute-elicitation mapping (UAM) to evaluate brands with respect to attributes as a set of configuration represented by matrices. Jan Steenkamp et al. developed a conceptual model on the linkages between the attribute based components of equities in the core product category and a related extension product category. Moreover, the approaches made the assumption that market consumers elicit their preferences according to the rationality criteria.

"The theory of rational choice assumes that preference between options does depend on the presence or absence of other options. This principle called independence of irrelevant alternatives is essentially equivalent to the assumption that the decision maker has a complete preference order of all options, and that -given and offered set - the decision maker always selects the option that is highest in that order" - Amos Tversky 1993.

However all the mentioned methods map consumer preferences as a clear cut act of choice, preformed by an individual respondent and in which states of nature, possible actions, results and preferences are well and crisply defined. Although each of the methods are interesting and important *in ipse*, however, the present paper will concentrate only on the cluster analysis due to its importance for marketing research.

CLASSICAL CLUSTER ANALYSIS

Cluster analysis is a technique used to identify different groups (or clusters) of respondents, such that the respondents in any one cluster are all similar to each other but different from the respondent in the other clusters. The technique applies to a large sample of data that includes a number of variables.
Formally,

Definition One
X^k is the k-th cluster which contains u intervals i.e. it is an u-tuple

$$X^k \triangleq \{X_1^k, X_2^k, \ldots, X_u^k\} \quad \text{such that}$$
$$X_l^k \triangleq \{X_{l_{min}}^k, X_{l_{max}}^k\} \quad \text{and} \quad X_{l_{min}}^k < X_{l_{max}}^k \quad \forall l \quad (l=1,\ldots,u)$$

Definition Two

X^k and X^l are said to be clusters iff $\forall$ k $\neq$ l and $\forall$ i (i = 1,..., u) $\exists!$ i that

$$X^k_i \cap X^l_i = \phi.$$

Definition Three

The profile a is said to belong to a cluster k, $a \subset X^k$,

$$\text{if } crd^i a \subseteq X^k_i \text{ for any i and k.}$$

Clustering techniques are regarded as preclassificatory in the sense that no prior information is used to partition the objects (rows in the data matrix). The data, however, are assumed to be partially heterogeneous, i.e. "clusters" exist. The clustering of different profiles is usually based on some measure of interobject similarity (proximity or resemblance) and carried out pairwise.

A relatively large number of proximity methods employ as a clustering criterion a distance in some type of metric space e.g. the Euclidean distance between two points in space in n dimensions:

$$d_{ij} = \sqrt{\sum_{m=1}^{n} (a^i_m - a^j_m)^2}$$

where a^i_m, a^j_m are the m-th standardised coordinates of points a^i and a^j in the space. Due to correlation of variables in most data matrices the above distance is calculated between pairs of points to their scores on component axes.

A technique is needed to categorize respondents int particular clusters. There are a number of rules that might be used for this purpose, however, the most commonly known are the single linkage, the complete linkage and the average linkage rules which result in similar, but not identical clusterings.

All these grouping procedures are based on crisp sets i.e. such that they assume a dichotomous (that is, of the yes-or-no type rather than of the more-or-less type) classification of elements to a given set. They also assume that the set of actions by respondents is precisely defined as the set of possible states (or the state). The utility function is based on a deterministic form.

It seems that the set of well-known and widely used grouping procedures could be extended by operations based on fuzzy set theory and in particular on fuzzy preference relations.

MODELLING PREFERENCES

In many cases the decision-maker encounters situations in which some alternatives are more preferred to others and the he or she needs to work out a way to deal with them. The easiest way is, however, to use preference relations which play an important role in the decision-making process.

A week preference relation R on the set of alternatives A is defined as a binary relation satisfying properties [Ovchinnikov]:

1) $x R y \vee y R x$
2) $(x R y \vee y R z) \rightarrow x R z \qquad \forall x, y, z \in A$

The relation R is complete and transitive.

A given preference relation is associated with the two binary relations:

1) the indifference relation I defined as:

$x I y$ iff $x R y \wedge y R x \qquad \forall x, y \in A$

2) the strict preference relation P:

$x P y$ iff $x R y \wedge \sim y R x \; \forall x, y \in A$.

In some instances, however, the above defined classical binary relations are not adequate models for intuitive concepts of preference and indifference. Especially when an individual wishes to express his/her preference of x to y as a certain degree or as a probability. Then a concept of a valued binary relation is very useful. A valued binary relation R on A is a function $R: A \times A \rightarrow [0, 1]$ and its examples are:fuzzy binary relations and probability relations.

For the purposes of the present paper a fuzzy binary relation R is a fuzzy set defined on the product set $A \times A$, $R = \{[(x,y), \mu_R(x,y)\} \mid (x,y) \subseteq A \times A\}$, of which membership function is denoted by μ_R In order that the fuzzy binary relation reflects a preference it should satisfy some conditions such as an ordinary weak order usually satisfies e.g. reflexity and transitivity.

The conditions are often formulated in the following way:

reciprocity: $\mu_R(x,y) + \mu_R(y,x) = 1 \quad \forall x,y$

max-min transitivity: $\mu_R(x,z) \geq \min[\mu_R(x,y),\mu_R(y,z)] \quad \forall x,y,z$

additive transitivity $\mu_R(x,y) + \mu_R(y,z) - .5 = \mu_R(x,z) \forall x,y,z$

(ir)reflexity: $\mu_R(x,x) =(0)1 \quad \forall x$

symmetry: $\mu_R(x,y) = \mu_R(y,x) \quad \forall x,y$
antysymmetry: $\min[\mu_R(x,y),\mu_R(y,x)] = 0 \quad \forall x,y$

For a given fuzzy preference relation R Orlovsky introduced a strict preference relation R^S:

$$\mu_{\tilde{R}^s}(x,y) = \max[\mu_R(x,y) - \mu_R(y,x),0]$$

which is then used to determine the "non-domination" choice function $\tilde{ND}^{\tilde{R}}(x)$:

$$\tilde{ND}^{\tilde{R}}(x) = \inf_{y\in X}[1 - \mu_{\tilde{R}^s}(y,x)] = 1 - \sup_{y\in X} \mu_{\tilde{R}^s}(y,x)$$

The value of the $\tilde{ND}^{\tilde{R}}(x)$ determines the degree to which the alternative x is dominated by no other alternative in A. In the context of fuzzy sets the non-domination choice function is a fuzzy set of nondominated alternatives (x,y,...). Therefore is seems reasonable to pick up only those alternatives of the set which are relatively near to their extreme values e.g.

$$\max_{x_i\in A} \tilde{ND}^{\tilde{R}}(x_i) = 1 - \min_i \max_j \mu_{\tilde{R}^s}(y_j,x_i)$$

FUZZY CLUSTERING

There are many different clustering methods which have been programmed and are available in computer programs (e.g. BMD P Series, SAS, SPSS) but their full specification might become a very heavy task. However, all of them are based and operate on proximity measures viewed as distances in a sort of metric space. The distances are simply crisp sets.The present paper suggests the use of fuzzy sets as the basis for a procedure which will then form, evaluate and qualify respondent profiles to various clusters.
The problem may be formulated as follows:
let some market be characterised by a set of alternatives $A = \{a_1, a_2, ..., a_p\}$, the set of customers $B = \{b_1, b_2, ..., b_q\}$ and the set of criteria $C = \{c_1, c_2, ..., c_r\}$. Let the set C be represented by outcomes which are contained by set A. The customers assess each alternative with respect to the degree with which the alternative dominates the market. Next the alternatives are combined to form profiles which in turn determine clusters. The evaluation of the alternatives is performed pairwise , thus showing how much each alternative is preferred by customers to some other alternative within a selected group of alternatives. The preference is expressed as a number contained in the interval of [0,1], which is a membership function.
Let $\tilde{R}^k_n$ be the fuzzy preference relation which determines the evaluation of the alternatives made bv the k-th respondent (b_k, k = 1, ..., q) with respect to the criterion c_n

$$\text{e.g} \qquad \tilde{R}^k_n = \{(x_i,x_j)/\mu_{\tilde{R}^k_n}(x_i,x_j)\,|\,x_i,x_j \in A \wedge \mu_{\tilde{R}^k_n}(x_i,x_j) \in [0,1]\}$$

where $\mu_{\tilde{R}_k}(x_i,x_j)$ is a membership function which shows the respondent k's preference of x_i over x_j. Let us assume that the preference relation is reflexive, then by Zadeh's definition $\mu_{\tilde{R}_k}(x_i,x_i) = 1$. Moreover, assume that individual respondent's preferences are aggregated i.e. they may be expressed in the simplest way by the relation:

$$\tilde{R}_n(x_i,x_j) = \frac{1}{q}\sum_{k=1}^{q} \tilde{R}^k_n(x_i,x_j)$$

or using Tanino's relation [1984]:

$$\tilde{R}_n(x_i,x_j) = \begin{cases} \dfrac{1}{2}\dfrac{\sum_{k=1}^{q}[\tilde{R}^k_n(x_i,x_j) - .5]}{\sum_{k=1}^{q}|\tilde{R}^k_n(x_i,x_j) - .5|} & \text{if } x_i \neq x_j \\ .5 & \text{if } x_i = x_j \end{cases}$$

Let the alternative set A be split into mutually exclusive and exhaustive subsets A_i and A_j i.e. $A_i \cap A_j = \phi$ and $\cup A_i = A$ where x^r_i denotes and evaluation of the i-th attribute with respect to the r-th criterion. Each alternative x^r_i is pairwise assessed with another alternative x^r_j by the respondent's membership function $\mu_{\tilde{R}^r}(x^r_i,x^r_j) \in [0,1]$ which denotes the degree to which the alternative x^r_i is preferred to x^r_j with respect to criterion r. Then the so-determined fuzzy preference relation $\tilde{R}^r$ is used to calculate the "nondomination choice" function [Orlovski]:
that:

$$\tilde{ND}^{\tilde{R}^r}(x^r_i) = 1 - \max_j \mu_{\tilde{R}^r}(x^r_j,x^r_i) \qquad \forall i$$

which will show how much the alternative x_i^r is regarded nondominated by x_j^r, or simply, the degree to which the respondents subjectively see x_i better than x_j with respect to r.The choice function will determine the fuzzy set of nondominated alternatives such

$$\tilde{X}^{ND\,r}(x_i^r) = \sum_{i=1}^{i_r} \frac{1 - \min_i \max_j \mu_{\tilde{R}^r}(x_j^r, x_i^r)}{x_i^r}$$

The above-outlined procedure applies to all of the subsets A_i $(i = 1,...,u)$,to obtain fuzzy sets of nondominated alternatives $\tilde{X}_i^{ND}$ $(i = 1,2,...,u)$. As a consequence u fuzzy preference relations will portray the respondent's evaluations by various criteria used.

The fuzzy nondominated cluster is defined through a fuzzy set:

$$\tilde{Y}_v = \{(X_{1,v}^{ND}, X_{2,v}^{nd}, ..., X_{u,v}^{ND})/\mu_v | X_{1,v}^{ND}, X_{2,v}^{ND}, ..., X_{u,v}^{ND} \in A, \mu_v \in [0,1]\}$$

i.e.

$$\text{supp}\,[\tilde{X}_1^{ND}] \times \text{supp}\,[\tilde{X}_2^{ND}] \times ... \times \text{supp}\,[\tilde{X}_u^{ND}] \to [0,1]$$

the membership function $\mu_v(X_{1,v}^{ND}, X_{2,v}^{ND}, ..., X_{u,v}^{ND})$, may be determined :

-by the single-linkage (minimum preference degree provided) rule,

$$\mu_v = \min\,[\mu_{\tilde{R}^1}, \mu_{\tilde{R}^2}, ..., \mu_{\tilde{R}^u}] \quad \text{, or}$$

-the complete-linkage rule

$$\mu_v = \max\,[\mu_{\tilde{R}^1}, \mu_{\tilde{R}^2}, ..., \mu_{\tilde{R}^u}] \quad \text{, either}$$

-the average linkage rule

$$\mu_v = E[E[E[E[...E[E(\mu_{\tilde{R}^k}, \mu_{\tilde{R}^l}), \mu_{\tilde{R}^o}]]]...]_{k \neq l}$$

where v is a number of clusters to determine.

The support of the fuzzy set $\tilde{Y}_v$ are v crisp sets - the clusters and any profile P will be said to belong to a specified cluster:

$$P \subset \text{supp}\,[\tilde{Y}_v] \quad \text{if} \quad \mu_P \leq \mu_v$$

where μ_p is a profile membership function.

For a given set of profiles $P = [P_1,...,P_k]$ there is a sequence of values of the membership function μ_P which may be arranged in the ascending order:

$$\mu_{P_{k_1}} \leq \mu_{P_{k_2}}, ..., \leq \mu_{P_{k_k}}$$

The sequence is then used to determine v semi-closed intervals, mutually exclusive and exhaustive such that any profile P_l $l = 1, ...,k$ belongs to

$$\mu_{P_l} \in (\mu_{v,l}^{min}, \mu_{v,l}^{max}>$$

where $\mu_{v,l}^{min}, \mu_{v,l}^{max}$ are endpoints of the i-th interval which is one of v clusters generated by all the profiles available. The cluster intervals may be determined in many ways, however the partitioning (i.e. into a set number of parts u) seems most reasonable since it is optional and simple.

COMPUTATIONAL EXAMPLE

Let us assume we are interested in the segmentation of car market. Let the market be characterised by respondents grouped in profiles which reflect e.g. interest in car brands, types of drivers, technical and economical performance and comfort attributes. Furthermore let some of these attributes be expressed in nominal scales. For simplicity of calculations we consider that it is a pair P=<CB,DR> where CB -means a car brand such as e.g. Ford, Mercedes, Holden, Toyota, briefly F,M,H,T and DR denotes a type of driver e.g. car buff, young person, wealthy physician, rally enthusiast and, finally, amateur racer, shortly CB, YP, WP, RE, AR.

The respondents, randomly picked up, reveal their preferences for the selected car brands and driver types in the form of the fuzzy preference relations $\tilde{R}_{CB}$ and $\tilde{R}_{DR}$ whose values of membership functions are shown as entries of the matrices shown below:

$$\tilde{R}_{CB} = \begin{array}{c} \begin{array}{cccc} F & M & H & T \end{array} \\ \begin{bmatrix} 1.00 & 0.70 & 0.77 & 0.80 \\ 0.62 & 1.00 & 0.65 & 0.88 \\ 0.92 & 0.40 & 1.00 & 0.52 \\ 0.83 & 0.50 & 0.72 & 1.00 \end{bmatrix} \end{array} \begin{array}{c} F \\ M \\ H \\ T \end{array} \qquad \tilde{R}_{DR} = \begin{array}{c} \begin{array}{ccccc} CB & YP & WP & RE & AR \end{array} \\ \begin{bmatrix} 1.00 & 0.75 & 0.62 & 0.85 & 0.53 \\ 0.95 & 1.00 & 0.79 & 0.57 & 0.59 \\ 0.92 & 0.31 & 1.00 & 0.50 & 0.87 \\ 0.74 & 0.98 & 0.77 & 1.00 & 0.54 \\ 0.88 & 0.87 & 0.80 & 0.59 & 1.00 \end{bmatrix} \end{array} \begin{array}{c} CB \\ YP \\ WP \\ RE \\ AR \end{array}$$

Note that the set of alternatives A in this example is A={F,M,H,T,CB,YP,WP,RE,AR} and there are two subsets of A: A_1 = {F,M,H,T} and A_2 = {CB,YP,WP,RE,AR} which correspond to the set of criteria, C, C = {C_1, C_2}. According to Orlovski, the corresponding strict preference relations for the above fuzzy sets are:

$$\tilde{R}^S_{CB} = \begin{bmatrix} 0.00 & 0.08 & 0.00 & 0.00 \\ 0,00 & 0.00 & 0.25 & 0.38 \\ 0.15 & 0.00 & 0.20 & 0.00 \\ 0.10 & 0.00 & 0.20 & 0.00 \end{bmatrix} \qquad \tilde{R}^S_{DR} = \begin{bmatrix} 0.00 & 0.00 & 0.00 & 0.11 & 0.00 \\ 0.20 & 0.00 & 0.48 & 0.00 & 0.00 \\ 0.30 & 0.00 & 0.00 & 0.00 & 0.07 \\ 0.00 & 0.41 & 0.27 & 0.00 & 0.00 \\ 0.35 & 0.28 & 0.00 & 0.05 & 0.00 \end{bmatrix}$$

and hence the fuzzy sets of nondominated elements are

$$\tilde{X}^{ND}_{CB} = \frac{0.85}{F} + \frac{0.92}{M} + \frac{0.75}{H} + \frac{0.62}{T}$$

$$\tilde{X}^{ND}_{DR} = \frac{0.65}{CB} + \frac{0.59}{YP} + \frac{0.82}{WP} + \frac{0.89}{RE} + \frac{0.93}{AR}$$

then the supports of there sets are used to generate the set of profiles, P,

$|\mathrm{Supp}\,[\tilde{X}^{ND}_{CB}]| \times |\mathrm{Supp}\,[\tilde{X}^{ND}_{DR}]| = 20$ profiles, which are next grouped into u=2,3,4 clusters derived in accord with the single, complete and average linkage rules. The final results are presented in the table below. These results point to a high variety in the contents of the clusters determined on the basis of the assumed linkage criteria. The arrangements are strictly and exclusively due to data obtained from the respondents and map their preference relations. By analogy we are able to develop much more complex
profiles by increasing a number of attributes evaluated by the respondents, e.g., in our case they could concern such additional attributes as maximum speed, driving comfort,

TABLE

Table of Clusters

	Single Linkage	Complete Linkage	Average Linkage
Two Clusters	(F,CB); (M,CB); (H,CB); (T,CB); (F,YP); (M,YP); (H,YP); (T,YP); (H,WP); (T,WP); (H,RE); (T,RE); (H,AR); (T,AR);	(H,CB); (T,CB); (H,YP); (T,YP); (H,WP); (T,WP);	(F,CB); (H,CB); (T,CB); (F,YP); (M,YP); (H,YP); (T,YP); (T,WP); (T,RE);
	(F,WP); (M,WP); (F,RE); (M,RE); (F,AR); (M,AR);	(F,CB); (M,CB); (F,YP); (M,YP); (F,WP); (M,WP); (F,RE);(M,RE); (H,RE);(T,RE); (F,AR);(M,AR); (H,AR); (T,RE);	(M,CB); (F,WP); (M,WP); (H,WP); (F,RE); (M,RE); (H,RE); (F,AR); (M,AR); (H,AR); (T,RE);
Three Clusters	(F,CB); (M,CB); (H,CB); (T,CB); (F,YP); (M,YP); (H,YP); (T,YP); (T,WP); (T,RE); (T,RE);	(T,CB); (T,YP);	(H,CB); (T,CB); (H,YP); (T,YP);
	(H,WP); (H,RE); (H,AR);	(H,CB); (H,YP);	(F,CB); (M,CB); (F,YP); (M,YP); (H,WP); (T,RE); (T,WP); (T,RE);
	(F,WP); (F,RE); (M,RE); (M,WP); (F,AR); (M,AR);	(F,YP); (M,YP); (F,CB); (M,CB); (F,WP); (M,WP); (H,WP); (T,WP); (F,RE); (M,RE); (H,RE); (T,RE); (F,AR); (M,AR); (H,AR); (T,AR);	(F,WP); (M,WP); (F,RE); (M,RE); (H,RE); (F,AR); (M,AR); (H,AR);
Four Clusters	(F,CB); (M,CB); (H,CB); (T,CB); (F,YP); (M,YP); (H,YP); (T,YP); (T,WP); (T,RE); (T,RE);	(T,CB); (T,YP);	(T,CB); (H,YP); (T,YP);
	(H,WP); (H,RE); (H,AR);	(H,CB); (H,YP);	(F,CB); (F,YP); (M,YP); (H,CB); (T,WP); (T,RE);
	(F,WP); (M,WP);	(F,CB); (F,YP); (F,WP); (H,WP); (T,WP);	(M,CB); (F,WP); (H,WP); (H,RE); (H,AR); (T,RE);
	(F,RE); (M,RE); (F,AR); (M,AR);	(F,RE);(F,AR); (M,CB);(M,YP); (M,WP);(M,RE); (M,AR);(H,RE); (H,AR);(T,RE); (T,AR)	(M,WP); (F,RE); (M,RE); (F,AR); (M,AR);

mileage, body colours, et cetera. Each of them would be assesed by fuzzy preference relations which in turn would underlie fuzzy sets of non-dominated elements. Those sets could then provide a basis for generating multidimensional profiles of the form:

$$P = <A_1, A_2, \ , A_n>$$

determined by a definite set of their membership functions derived by set operations (max,min) from their original membership functions.

Thus we have obtained a method for allocating respondents int their respective clusters. And obviously all the clusters constitute a universe whose numbers (profiles) have membership functions covering up the full interval [0,1].

CONCLUSIONS

The suggested fuzzy clustering technique allows us to construct clusters based on membership functions characterising their profiles. It makes use of the concept of a fuzzy preference relation which proved its quality in group decision making with fuzzy evaluations. It is then employed to compute the "nondomination choice" function that makes possible to have evaluations of single attributes on the basis of the individual's preferences. The technique applies relatively simple mathematical operations such as the minimum and maximum operations and therefore it is easy to be algorithmisized to obtain results almost instantly.

It seems a very promising tool that could significantly increase the existent repertoire of crisp distance-based clustering techniques.

REFERENCES

Boyd H. Westfall R., Stasch S., 1982, *Marketing Research, Text and cases, Richard D. Irwin Inc.*

Delequie P., (1993), *"Inconsistent Trade-offs Between Attributes: New Evidence in Preference Assessment Biases", Management Science, Vol.39, No 11.*

Dillon V., Kumar A., Smith de Borrero M., (1993), *"Capturing Individual Differences in Paired Comparisons: An extended BTL Model Incorporating Descriptor Variables", Journal of Marketing Research, February.*

Green P., Tull D., Albaum G. (1988), *Research For Marketing Decions, Prentice Hall,*

Huber J., Wittink D., Fiedler J., Miller R., (1993), *"The Effectiveness of Alternative Preference Elicitation Procedures in Predicting Choice", Journal of Marketing Research, February.*

Ovchinnikov S., (1990), *"Modelling Valued Preference Relations", in J.Kacprzyk and M.Fedrizzi (eds):Multiperson Decision Making Models Using Fuzzy Sets and Possibility Theory, Kluver Academic Publishers*

Steenkamp J., van Trijp H., Berge J., (1994), *"Perceptual Mapping Based on Idiosyncratic Sets of Attributes", Journal of Marketing Research, February.*

Tversky A., Simonson E., (1993), *"Context Dependent Preferences", Managment Science, Vol. 39 no.10*

Zahariev S., (1990), *"Group Decision Making with Fuzzy and Non-Fuzzy Evaluations", in J.Kacprzyk and M.Fedrizzi (eds):Multiperson Decision Making Models Using Fuzzy Sets and Possibility theory, Kluver Academic Publishers,*

Zimmermann H., (1987), *Fuzzy Sets, Decision Making and Expert Systems, Kluver Academic Publishers.*