Constructionism: Theory of Learning or Theory of Design?

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Abstract Constructionism has established itself as an epistemological paradigm, a learning theory and a design framework, harnessing digital technologies as expressive media for students' generation of mathematical meanings individually and collaboratively. It was firstly elaborated in conjunction with the advent of digital media designed to be used for engagement with mathematics. Constructionist theory has since then been continually evolving dynamically and has extended its functionality from a structural set of lens to explanation and guidance for action. As a learning theory, the constructionist paradigm is unique in its attention to the ways in which meanings are generated during individual and collective bricolage with digital artefacts, influenced by negotiated changes students make to these artefacts and giving emphasis to ownership and production. The artefacts themselves constitute expressions of mathematical meanings and at the same time students continually express meanings by modulating them. As a design theory it has lent itself to a range of contexts such as the design of constructionist-minded interventions in schooling, the design of new constructionist media involving different kinds of expertise and the design of artifacts and activity plans by teachers as a means of professional development individually and in collective reflection contexts. It has also been used as a lens to study learning as a process of design. This paper will discuss some of the constructs which have or are emerging from the evolution of the theory and others which were seen as particularly useful in this process. Amongst them are the constructs of meaning generation through situated abstractions, re-structurations, half-baked microworlds, and the design and use of artifacts as boundary objects designed to facilitate crossings across community norms. It will provide examples from research in which I have been involved where the operationalization of these constructs enabled design and analysis of the data. It will further attempt to forge some connections with constructs which emerged from other theoretical frameworks in mathematics education and have not been used extensively in constructionist research, such as didactical

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design and guidance as seen through the lens of Anthropological Theory from the French school and the Theory of Instrumental Genesis.

Keywords Constructionism · Design · Meaning generation · Theory networking · Digital media

Introduction

Constructionism was a term coined by Seymour Papert almost half a century ago with a two-legged agenda based on the context and time of the emergence of constructivist theory as a more sensible approach to study and understand learning than the prevailing behaviorist frame at the time, which had been established a few decades before that. Papert's agenda was on the one hand to challenge Piaget's constructivist theory by drawing our attention to the meanings actually generated by learners rather than to describe their shortcomings in understanding taken-as-ontological meanings at different stages in life. It was also to change the perception that concrete thinking was a 'lesser' kind of cognitive process in relation to abstract thinking by pointing out that proper and rich exposure to the former was pivotal in ever hoping to reach the latter (Ackermann 1985). On the other, it was to claim that meanings are naturally generated in our social, intellectual and physical environment and that digital technology makes it possible for us to enrich this environment so that learners would enjoy more opportunities for the formation of meanings. Papert's work has been an ode to kids mathematical thinking, he has been provocative in arguing that we have not paid enough attention to how children think mathematically and to the nature of meanings they form given the language and tools they use, the activities they engage in and the communicative situations they find themselves in. He also argues that for children, a key to learning is the process of engagement in activity, the ownership of ideas and style of learning and the exposure, i.e. expressing their ideas to others, for reasons of exploration and communication.

So, what happened to constructionist theory since its first articulations? What has the mathematics education community learned so far and how has it been put to use? Has the theory been developing all these years or is it a well recognized but now rather blunt instrument associated with outdated technologies and ideas of how they can be used for learning? We live at a time of growing connectivity and resource availability, at a time where 'watch and practice' technologies and administrational infrastructures are popular and politically publicized and immersion in collective virtual worlds where mathematical representations are given very low priority by media designers. We also live at a time where several theoretical frameworks and constructs in mathematics education are in danger of lying in fragmentation each to be used by a community of researchers close to the context from within which it emerged. So, is constructionism relevant and useful today and in what capacity? Can it be meaningfully connected to other theories and is there some mutual benefit from trying to forge such connections? How can it be useful in the age of jings, blogs, portals and LMS? Is there scope for further development of constructionism theory in an era of ever-changing technologies and a wealth of theoretical frameworks and constructs and how can this be justified? In this paper I attempt to contribute to the argument that constructionism is essentially an epistemology creating continual need for an evolving theory of learning in collectives and individually and at the same time a theory of design of new digital media, new kinds of activities facilitating the generation of meanings and techniques and processes for systemic interventions at various levels such as school cultures, resource systems and educational systems.

Epistemology

Constructionism started out as an epistemology of mathematics as a discipline and of doing mathematics. Mathematics is portrayed as a human construct under continual development rather than an ontological reality to be explained or as the production of unquestionable truths. Papert perceives mathematics as essentially fallible (in the sense of Ernest 1980), i.e. that each mathematical definition, lemma, theorem, proof, has the status of a proposition for others to try to refute. Even in cases where theorems are proven and problems are solved, they lie in wait for other mathematicians to question the process, the context, the point of view of articulating the question, the assumptions. This process is essentially part of doing mathematics and it does not matter in the end whether and to what extent an axiomatic system remains robust or a theorem is proven to have shaky foundation. Engaging in mathematical activity necessarily involves the process of refutation together with logical thought, deduction, generalisation and proof. Lakatos was at the time equally provoking in his book ‘proofs and refutations’ (Lakatos 1976) where he analysed mathematicians’ activity as a process of conjecture as a public expression of thought and subsequent engagement with a cycle of refutation, redrafting and new proofs.

> …deductivist style tears the proof generated definitions off their ‘proof-ancestors’ presents them out of the blue in an artificial and authoritarian way. It hides the global counterexamples which led to their discovery. Heuristic style on the contrary highlights these factors. it emphasises the problem-situation: it emphasises the ‘logic’ which gave birth to the new concept (Lakatos 1976, p. 144).

Papert (1972) argues that this kind of mathematical activity cultivates learning not only in established mathematicians but for all, even for young children. He goes further to say that this activity is natural and that traditional schooling somehow denies students the opportunity and encouragement to engage in the logic which gives birth to concepts as Lakatos put it, imposing an artificial picture of mathematics to be the practice of trying to understand the abstract and irrelevant products of mathematical activity rather than the activity itself. Papert discusses mathematical activity which is based on the expression of mathematical meanings through the use of and tinkering with representations in the form of digital artifacts. His argument was that this technology could play an important role in generating learning environments which would be more authentic, rich and dense in opportunities for the type of mathematical activity which was termed as 'heuristic style' by Lakatos. Papert was inspired by the ways in which computer scientists engaged in solving difficult algorithmic problems by using LISP programming as an expressive tool for this kind of heuristic process and thought that there was no reason not to develop a LISP-like tool for children to do mathematics with. Logo was the tool he built together with Feurzig (Feurzig and Papert 1971) maintaining the LISP philosophy for programming as an emergent problem solving process (Sinclair and Moon 1991) but changing the syntax and most importantly the object and the objective of programming to Turtle Geometry. The choice of Geometry was not accidental. At the time, Geometry was perceived by educationalists as a problematic domain for learning mathematics. Although it carried a unique potential to connect children's experience with space to mathematical meaning making, educational approaches had steered it away towards a problematic attempt to teach deduction in an abstract explanatory way. At the time, Freudenthal (1973) had a similar approach to learning mathematics in his discussion about the educational potential of geometry:

> the deductive structure of traditional geometry has just not been a didactical success. people today believe geometry failed because it was not deductive enough. In my opinion the reason was rather that this deductivity was not taught as reinvention, as Socrates did, but that it was imposed on the learner. If geometry as a logical system is to be imposed upon the student it would better be abolished (73, p.402-406). Geometry can only be meaningful if it exploits the relation of geometry to the experienced space. If the educator shirks this duty he throws away an irretrievable chance. Geometry is one of the best opportunities that exists to learn how to mathematize reality (p. 407).

The essence of Turtle geometry as a context for mathematical programming was not only its differential kernel (the state of the turtle entity is defined by its difference to its immediately previous state) but also connectivity with children's experienced space though the embodied metaphor of the turtle and its state changes, something which Papert called 'body syntonicity' (Healy and Kynigos 2010). So the idea was to embody this digital medium with a coherent mathematical kernel, a mathematical language connecting formalism to graphical representations and a naturalistic metaphor to enable even young children to engage in heuristic activity. The latter issue reflected an epistemology towards learning itself which was constructivist in nature.

Papert called Turtle Geometry a mathematical microworld, a term which became central to constructionism and was originally defined as:

> self-contained worlds where students can "learn to transfer habits of exploration from their personal lives to the formal domain of scientific construction" (Papert 1980, p. 177).

In a recent review, Healy and Kynigos (2010) reflect on the ways in which the microworld idea was developed, used and modified within the framework of constructionist theory. They argue that although initial focus was on microworld as a digital object it quickly became apparent that it made much more sense to discuss the term in association with the kinds of activity emerging from the use of microworlds and the scope of each microworld with respect to the conceptual field it was designed to embody. Healy and Kynigos point out that these issues are apparent in a more recent definition of microworlds as computational environments embedding a coherent set of scientific concepts and relations designed so that with an appropriate set of tasks and pedagogy, students can engage in exploration and construction activity rich in the generation of meaning (Sarama and Clements 2002). In a reflective paper discussing dynamic geometry systems and Turtle Geometry Balacheff and Sutherland (1993) use the term 'epistemological validity' to denote that the design of expressive media for mathematical learning inevitably incorporates an epistemological approach to mathematics and to the learning of mathematics.

So, even though research on student activity based on the use of microworlds focused on contributing to learning theory, to the design of digital media and educational interventions and to the processes of learning and teaching as design activities, what has been pertinent in constructionism is epistemology. Constructionist epistemology has shown to be quite challenging at many levels including institutional levels, using the work in the sense used in the Anthropological Theory of the Didactic (Artigue et al. 2011). It has caused both enthusiasm and reaction in many cases and has certainly given inspiration for the term used by Laborde in another context 'perturbation' of norms and mindsets (Laborde 2001). What is interesting is that after so many years it continues to inspire and perturb at the same time and find itself relevant in many societal and technological issues emergent today. Take for instance the instructionist 'watch and practice' paradigm which has recently been enabled technologically and has gained such kudos by the media, the educational administrators and policy makers. The argumentation behind it is that since the instructional part of mathematical education is now covered without the need to spend classroom face to face time, there is space for better supporting experiential learning of mathematics be it constructionist or inquiry learning or problem solving. How sound is this argument and how can it be approached from an epistemological basis?

Learning Theory

The visionary nature of applying this kind of epistemology to mathematics education for young children coupled with the new technology just made available motivated researchers to study both the process of mathematical meaning generation and the nature and use of meanings. The early eighties was good timing for researchers to investigate mathematical learning processes since a concurrent problem solving movement emerged at least in the US and was applied widely in schools. Process oriented studies identified types of student activity such as (U.D.G.S.) Using, Discriminating, Generalising and Synthesising (Hoyles and Noss 1987) and characterised students' abstractions as situated (Noss and Hoyles 1996), i.e. as emerging from the specific situations including the problem, the notation (such as for instance the notation for variable values), the computer feedback, the types of argumentation around the figural models built with the turtle.

The emergent meanings were inevitably seen as tightly connected with the activity of working with Turtle Geometry. The first characteristic of such activity was the style envisioned by Papert and pedagogically encouraged by researchers in the role of teachers which he characterised as 'bricolage', borrowing the term from the Bourbaki school of thought in France at the time. The idea was that students used the computer to construct models of figures by means of programming the turtle. These models, both their figural and formal descriptions were seen as externalized expressions of ideas and thoughts. The act of constructing a model constituted making a sequence of ideas public for discussion and change. The models or artefacts thus had the status of being malleable, of being questionable or improvable propositions for an on-going discussion around subsequent changes to the artefacts. They had the same role as Lakatos' mathematical propositions expressed by mathematicians with formal notation in order for a refutation and proof dialectic process to start. Only here, the representations albeit mathematical were connected and designed for youngsters to use and give meaning to, the expressive tool used provided feedback, and interactivity and the constructions were extensible (Papert 1980). Mathematical meanings we generated at different levels, at the level of using the notation to express an idea, for instance a variable to express generalized number, at the level of a concept underlying a model and its behaviour or at the level of the model itself, i.e. the model of a variable pentagon and the patterns which can be constructed with this as a building brick.

But of course this kind of learning does not happen just by providing students with the media. Research showed that just like giving the opportunity for constructivist learning by no means sufficed for students to go on and become advanced mathematical thinkers, poorly designed pedagogically unsupported constructionist activity with Turtle Geometry quickly resulted in a-mathematical constructions and plateaus with respect to mathematical development. Furthermore, it showed that the meanings themselves were idiosyncratic and bound to the situations from which they emerged, students found it difficult to synthesise them with more general applications of the same concepts or with traditional mathematical notations. Kaput et al. (2002) describe this mathematics as learnable but also as a different kind of mathematics which is shaped by the users of the medium which in turn shape the medium itself. Edwards talks about functional and structural views of microworlds (Edwards 1988) and more recently Rabardel's ergonomy theory of Instrumental genesis has been elaborated for the use of digital media for mathematics education to highlight the ideas of instrumentation and instrumentalisation (Verillon and Rabardel 1995; Guin and Trouche 1999; Artigue 2002). Both constructs are elaborations of the idea that expressive media are shaped by the people who used them who in the process form evolving schemes regarding the meanings underlying the use of the media.

There has of course been extensive research on the meanings generated by students around specific mathematical concepts which could be mapped onto any standard curriculum. Some such studies have been on symmetry (Healy and Hoyles 1999), geometrical systems (Kynigos 1992, 1993), properties of geometrical figures and functions (Hoyles and Noss 1987), generalised number (Sutherland 1987), fractions and proportional thinking (Psycharis and Kynigos 2009), trigonometric functions (Keisoglou and Kynigos 2006; Kynigos and Gavrilis 2006), navigational mathematics Yiannoutsou and Kynigos 2004), angle and spatial geometry (Kynigos 1997; Clements and Batista 1992; Latsi and Kynigos 2011). There has also been research addressing constructionist learning through the design of digital artifacts (Kafai et al. 1998; Kafai 2006). This research was mainly done in the context of students designing games and playing them, focusing on the mathematical rules of the games. In this paper we will not extensively cover this research, more can be found in Kafai and Resnick (1996). This research has moved on to consider design in teacher professional development contexts where teachers design artifacts and scenarios for student activity creating thus a context for pedagogical reflection (Kynigos 2007a).

Constructionist activity has been addressed from early on as a social activity where typically students discuss and argue over working with a microworld (Hoyles and Sutherland 1989; Hoyles et al. 1992). Meaning generation is afforded by a conjunction of noticing computer feedback, using the representations and the semantics of the microworld and generating a language to express arguments in the context of this kind of activity. It has taken some time for the wider education research community to notice that constructionism was not perceived as an individualistic learning theory and at the same time for constructionists to develop more explicit language and frameworks for the social discursive aspect of this kind of learning (Artigue 2009a, b; Kynigos and Theodossopoulou 2001).

Apart from the early research which perceived constructionist learning as essentially discursive (Hoyles et al. 2002; Simpson et al. 2007), special attention was given to the design of digital support for collective constructionist learning. The first was that of distributed constructionism, the second involved literal artifact exchanges across the net and the third study which is introduced to the mathematics education community and was on-going at the time this paper was written addressed the idea of socio-metacognition, i.e. students learning to learn in collectives engaged in constructionist activity.

Distributed constructionism was the first attempt to focus on collaboration and discussion around and about constructions (Resnick 1996). It was based on the idea of combining constructionism with distributed cognition, i.e. that is addressing learning ‘not as a property of a person but as the process of interaction with others and the environment’ (p. 281). Computer networks mediated three main types of constructionist activities enabled by the available networking infrastructure at the time. The first type involved discussing constructions through email, newsgroups and bulletin boards, the second type of activity involved sharing constructions which made use of a web based version of Logo which allowed file storing in LogoWeb, a network created to support this activity due to technological restrictions that did not allow the use of world wide web, which offered easy access to the files among users. The third type of activity involved collaboration on constructions where students were expected to collaborate in real time in one project. The aim of the project as described by Resnick (1996), was the creation of an ocean ecosystem where each student was expected to program the behavior of an artificial fish and place it to function in the ecosystem. Student collaboration was facilitated by chat tools and focused on the programming and functioning of the ecosystem through discussions emerging from the observed interactions of the programmed fish.

The concept of sharing discussing and collaborating constructions through the internet was revisited in 2002 in a multi-organizational European project titled 'WebLabs' which was based on an explicit attempt to develop technologies supporting the social aspect of constructionism. Web Labs built a system that allowed early teenage students to construct models of their emerging mathematical ideas, to share the models and to pay attention to the process-based descriptions of the models (Simpson et al. 2007). Collaboration in weblabs was mediated by "webreports" which included working models—not just descriptions of models—along with multi-media descriptions, interpretations and reflections. According to the project' design, the complex behavior of generating meaning by tinkering with the rules with which digital models operate, questioning their behaviors and thinking about the phenomena they simulate requires the students to have shared models, argumentation skills, and practice with reflection on their learning activities.

Another approach to collective constructionism has been taken in an on-going multi-organizational project in the European TEL setting titled 'METAFORA'.[1] The project is about socio-metacognition, i.e. enhancing students' awareness of group learning processes and their roles in learning collectives. The phrase used in the project is 'learning to learn together'—L2L2 (Dragon et al. 2013). The project is based on a wider 'challenge based learning' paradigm where attention is given to learning process in students addressing challenges consisting of fuzzy broad realistic problematic situations. A challenge typically refers to a complex open problem situation relating to a real or realistic phenomenon within some kind of socially relevant situation. In a constructionist challenge the pedagogical intervention strategy and the design of the challenge is such that students will try to understand the rules, properties and relations underlying a model in order to change or re-create a model of their own, generating meanings from mathematics as they work. In METAFORA students are encouraged to focus on how they learn with and from each other as they work in groups to solve challenging problems in science and math. L2L2 is perceived as a complex enterprise not easily decomposed into a set of underlying skills. However some key skills were identified as necessary to any process in which students are learning together, and on a higher level, learning

[1]The Metafora project is co-funded by the European Union under the Information and Communication Technologies (ICT) theme of the 7th Framework Programme for R&D (FP7), Contract No. 257872.

how to learn together. The students must gain skills at working collectively to be more successful group learners. These competencies included Distributed leadership, Mutual engagement, Help seeking/giving and Reflection on the group learning process. This process of group meaning generation requires skills from across the different aspects of L2L2, organizing, discussing, seeking and offering help from peers when needed. A digital L2L2 built within the METAFORA project integrated tools for group planning and discussion with microworlds for students to collectively engage in constructionist activities. Within this framework, the "social" in constructionism involves not only the process (discussion, sharing and collaboration) but also the output that is, the actual product of construction. Constructions have the special status as public entities which means that they are open for others to inspect how they work, to comment on the underlying mechanism of the construction, to change, customize and re-use parts of this mechanism. These activities which focus on "how things work" and which are central and rather unique in constructionist environments can shape collaborative activity and differentiate it from other learning environments (such as inquiry based learning). So, the question raised here and that is going to be further investigated in "METAFORA" is how the placing of constructions at the centre of collaboration shapes L2L2 activity. Microworlds in this case are perceived as boundary objects which offers an analysis of how meaning is generated through discussion and negotiation about constructions among participants that belong to different communities (see also Hoyles et al. 2004; Kynigos 2007b).

Artifact Design

The design of constructionist media quickly expanded beyond mathematics education. Even from the days of the MIT-initiated LCSI Logo the idea of programming as a means of expression for all, a new kind of literacy, grew quite rapidly making the idea of constructionist mathematics look like a special case of something outside the mathematics education community. The idea of constructionism without programming in the sense of using a formal language also gave birth to digital applications, such as the use of macros, building with ready-made construction kits. Furthermore, programming got fused in icon-driven interfaces and semantics making the use of a formal language not necessary.

Important ideas however emerged for constructionism not necessarily addressing the learning of mathematics per se, such as that of principled design to enhance deep structural access to the functionalities of digital tools (diSessa 2000), the idea of interplay between private and public expression, of transparency (Resnick et al. 2000; Eisenberg 1995) and of black-and-white box designs (Kynigos 2004).

There were however issues particularly pertinent to mathematics education. Firstly, the issue of programming as a mathematical activity. Logo was intentionally designed to be as mathematical as possible in syntax, rules and semantics maintaining however in full its property of being a full functional, list processing, recursive and structured programming language (Harvey 1993). However, the very role of a mathematical formalism to be the notation for programming involved differences to traditional formalism in mathematics (Abelson and diSessa 1981). In the old days, this was a relative detail since there was no other way to approach mathematical expression with a computer. However, in the past decade with the advent of mathematical text editors and Computer Algebra Systems a programming language is a programming language and the attempts to call it mathematical formalism look feeble. There is a deeper issue at hand however than wether the connection between mathematical formalism and programming is technically avoidable: it is that now we can reflect on how conducive is formal representation to learning mathematical thinking, especially constructionist mathematics. This is an open question: there are those who perceive formalization as an obstacle for students to access mathematical meaning and thus a design objective for digital media is to find ways to by-pass formalism by means of alternative representations (Laborde et al. 2006; Kaput et al. 1992). There are those who argue that digital media has made it possible for formalisation to become meaningful to students since they can use it to construct models and other mathematical representations such as graphs (Kynigos and Psycharis 2009; Dubinsky 2000; Artigue 2002). The proponents of the latter perceive formalization as one of the representation registers which lie interconnected in digital media and that meaning is generated through expression with connected representations precisely because students gradually dissociate meaning from a single representation.

Another issue is why program in order to do mathematics now that it's not necessary? Dynamic Manipulation Systems, Computer Algebra Systems, Data Handling systems and even simulation authoring systems allow for constructionist learning often with mathematical representations but only with an indirect notion of programming and certainly not by means of a formal programming language.

Furthermore, technical advances in e-book technologies are allowing for the consideration of narratives containing constructionist 'widgets'.

This is now been attempted in the 'M C Squared' project where 'c-books' (c for creativity) invite the reader to also tinker with a variety of widgets and the author of such c-books to re-think the role of the reader and the this particular resource.

Intervention Design

The previous section discussed some issues of constructionist media design pertaining to mathematics education. As more technological choices become available, it is a continual task to re-appraise what is a rich constructionist tool for doing mathematics with respect to the representations, to the functionalities and semantics of the medium, the ways representations can be manipulated and to the relationship of constructionism to programming and the exact definition of programming itself. In the section before that, the notion of design as a learning process was discussed within the framework of students designing games and other digital artefacts.

In this section we look at a different aspect of design, that of designing and implementing an intervention in an institutionalized educational practice. This issue has been termed 'design research' and has been elaborated in the past 10 years or so as a research method (Cobb et al. 2003). Researchers working within the frame of constructionism however, always had a special agenda for intervention and challenge in some established educational practice. Artigue uses the term 'concerns' to describe researchers' agendas which are not fully elaborated or expressed even in academic publications yet shape the method and the produced knowledge to an important extent (Artigue 2009a, b). Constructionist epistemology was from the beginning articulated as something which was real and natural but which the institutionalization mechanisms and processes had somehow corrupted or diverted mindsets, values and practices to more artificial views of knowledge teaching and learning (Papert 1972). The growth of very large communities outside educational institutions such as the Scratch or the NETLOGO communities are an indication of this (Brennan et al. 2011). An important part of constructionist research was carried out to challenge institutionalized education through a design research paradigm. There are different ways of synthesising such research of course. In this section we look at constructionist interventions at the level of teacher professional development, the school as an educational institution and the education system at a wide scale (Childs et al. 2006; Healy and Kynigos 2010).

With respect to teachers, an early study—the 'microworlds' project (Hoyles et al. 1991)—centred on allowing a design role for teachers by supporting them in the design and construction of microworlds. This focus has seen the development of new methodologies, tools and experiences of teachers as designers has been progressing (Funglestad et al. 2010). What does design for constructionist activity have to offer, not only to planning for and assessing learning but also to professionals engaging in such designs (Kynigos 2007a). In the framework of exploring such issues, the idea of "half-baked" microworlds as digital artefacts designed so that learners would directly engage with, question and change their structure was introduced (Kynigos 2007b). These are artifacts explicitly designed so that their users would want to build on them, change them or de-compose parts of them in order to construct an improved artifact for themselves or one designed to be changed by others. They are meant to operate as starting points, as idea generators and as resources for building or decomposing. The essence of such microworlds is not only that they are built to allow changes but also that they are mediated as malleable, questionable and improvable objects. Half-baked microworlds can be designed for teachers to engage in pedagogical or epistemological reflection as they de-construct or re-construct the microworlds in a context of designing tools for students. Half-baked microworlds operate like diSessa's toolsets (diSessa 1997) in that they are not built and presented as ready-made environments to be understood by the teachers and then used by students. Instead, the point is to change and customize them and thus to gain ownership of the techniques and the ideas behind microworld construction. In this way the operate s boundary objects for teachers and researchers to discuss pedagogy and intervention agendas (Star and Griesemer 1989; Kynigos 2007a).

Constructionism has naturally not only been perceived as a theory for the design of tasks and digital artifacts for students. That would mean that to get students to work in a contructionist way would be unproblematic in traditional schools and schooling systems. On the contrary, both constructionist epistemology and practice have been portrayed by Papert himself as constituting a challenge involving the need for re-thinking the educational paradigm of schooling and the epistemological approach of mathematics (Papert 1993). This rationale came about in the historical context of some bold attempts for curricular changes in the United States based on the problem-solving movement which focused on problem-solving methods. Constructionism was seen as the ideal boost to the movement through the use of technology. Inevitably, the attention of the wider community was on the technology that accompanied the constructionist vision at the time which became a victim of the educational innovation pendulum (Agalianos 1997; Noss 1992). Constructionism was seen as a technological ticket to generate mathematical thinkers on a wide scale. When this not surprisingly proved to be a much lengthier and complex issue than the advent of a specific technological support, constructionism was in many ways attached both to a specific ageing technology and to a sense of an unfulfilled promise at the cultural level (Papert 2002). Technologies were very hard to access, machines were slow and non-dependable, the internet had not arrived and people were not widely using digital technologies in their daily activities. Many of these disappointing features were attributed to the original perspectives and theories of how digital media could be used for mathematical learning.

Constructionist interventions at the level of school originated as far back as the early eighties either as a school wide implementation of an innovation (Noss 1985) or at the level of a longitudinal mathematics curriculum for a particular group of students (Hoyles and Sutherland 1989). They also addressed wider issues of implementing an innovation as a means to challenge epistemology and school culture (Blikstein and Cavallo 2002; Kynigos 2002). Both at the school and at the institutional level however, these interventions did not generate any kind of self-growing culture change. On the contrary, the system diluted the essence of constructionist learning in a number of ways insightfully analyzed by Hoyles (1993).

Up till now, however, constructionist interventions in educational institutions have been perceived by the community at large as designed by researchers for radical innovation to be implemented in small scale situations. The era when this approach was valid and seductive to the research community seems to be ending. There is now demand for large scale initiatives and accreditation of new efforts before they have had the chance to become infused in educational curricula. The ideas behind the constructionist culture are proving hard to grasp and accept not only by school systems but also by other stakeholders in education such as new computer science and telecommunication communities. A mathematics designed with a constructionist agenda in mind can only become part of school mathematics if the associated practices are given legitimacy by the various stakeholders involved. This poses new challenges to the community of finding methods and avenues for communicating these ideas to a wider audience in a language which is widely understood (Papert 1996, 2002).

This kind of language can only emerge from experience of communicating and negotiating ideas and reform processes with such communities, it is not something which can be defined at a theoretical level. The process will require being explicit about what happens when collaborative design and implementation is taking place. An example of such an effort in a broader context of digital media in mathematics education including microworlds is that of a European project titled 'ReMath, Representing Mathematics with Digital Media' where six research teams engaged in cross-experimentation in order to develop a more specific language to communicate theories, contexts and use of representations (Bottino and Kynigos 2009).

Connectivity and Networking

Within the constructionist community, there is recent discussion on whether constructionism is better addressed as an epistemology or a learning and design theory. At the crux of the discussion is not only what is more prevalent in the articulation of constructionism and in the ways it has been put to use in designing and studying educational processes but also, if it is to be addressed as a learning theory what kind of theory has it turned out to be. There are several possible reasons why reflections on the essence and the status of contructionism as a theory for mathematical learning have been intensified in the past few years. Some may be seen as a result of challenges put to the community by other developing theories in mathematics education and also by pendulum-like trends in educational systems, such as the different versions of back-to-basics- reforms. Others, as a result of perceiving constructionism as a theory prescribing a method through which deeper mathematical understanding will be achieved, attaching, that is, an element of predictability and controllability to the theory (see for instance the Pea—Papert debate in the eighties, Papert 1987).

A different kind of challenge comes from the changing trends in what is considered as added value in the uses of digital media in education and in society at large. A first wave was the dynamic manipulation and mathematical text editing technologies which questioned programming as an effective mathematical meaning-making activity. Programming and formal code were seen as a kind of unnecessary noise to doing mathematics. A second wave was the advent of social media, portals, LMS and the recently widely advertised 'watch-and-practice' video portals considered as an infrastructure relieving teachers of the need for frontal lecturing. This means that attention is currently given to the use of technology for mathematics education which supports traditional curriculum delivery so that human time and focus can be given to discussing questions and supporting the generation of meaning. So is constructionism going to be considered as an unnecessary noise to content delivery?

Reflections on the place and role of contructionism in amongst mathematics education theories however have also emerged as a result of a wider initiative to consider the landscape of theories in the field, to better identify their nature, status and functionalities and to develop strategies for integrations amongst them so that there is a better understanding and communicability of the progress of mathematics education as a field to stakeholders outside academia and educational reformers. These initiatives began without reference to theories in the use of digital media dating from the CERME conference in 2005 and have already output important literature on the ideas of integration and networking between theories (for example, Prediger et al. 2008; Niss 2007).

Significant work on bringing constructionism and other theories developed or shaped to study the uses of digital media for mathematical learning into this game was done through the work of six European research teams for a period of 6 years [2004–2009, the TELMA European research Team in the Kaleidoscope Network of Excellence and the European Information Society Technologies programme (FP6) titled 'Representing Mathematics with Digital Media' (ReMath)]. Theories such as the Anthropological Theory of the Didactique, The Theory of Didactical Situations, Social Semiotics, Semiotic Mediation, Activity Theory, Instrumental Genesis were considered together with Constructionism to be part of the same phenomenon happening more widely in mathematics education, i.e. a fragmentation and polysemy slowing down and diluting the production of knowledge in the field. The teams worked under the initiatives of Michele Artigue (2009a, b) to elaborate a process of networking amongst these frameworks initially at the level of conceptualizing and proposing a networking process and subsequently at the level of operationalizing the process to actively articulate connectivities between frameworks through joint research. The initial framing of the networking process involved an articulation of these theories through the lens of their didactical functionality and the language of concerns. Special attention was give to the aspect of representations of mathematical concepts through digital media (Artigue 2009a, b) and the formative influences of the context of the educational system and the processes of design and development of both media and research interventions (Kynigos and Psycharis 2009). Pilot methods for implementing research enabling this kind of networking were originated in the TELMA ERT involving the process of cross experimentation, i.e. a research team designing and carrying out research based on the use of a digital medium designed and developed by another and vice versa. In the ReMath project, networking involved the whole cycle of designing and developing six original state of the art digital media for learning mathematics, the design of interventions and classroom experiments and the implementations of these analyzing students' meanings in realistic classroom situations. Several networking tools were developed for cross experimentation which operated as boundary objects to identify and articulate connectivities between frames. A key element of the project was the cross-case analysis of these studies, i.e. an integrated meta-analysis or two research studies carried out by two different teams in respectively different contexts involving the use of the same digital artifact. This section discusses a set of three examples of the formation of an elaborated language connecting theoretical frames through the ReMath work and in particular as a result of the cross-case analysis method. The examples are draw from the ways in which one particular theory, constructionism, which was developed specially as a tool to think about learning mathematics with digital technologies, was connected to three different theories in respective cross-case analyses. These were two-way connectivities articulated between constructionism and (a) instrumental theory (Kynigos and Psycharis 2013), (b) social semiotics (Morgan and Kynigos 2014) and (c) the anthropological theory of the didactique (Artigue and Mariotti, in preparation). What is particular about the enterprise of connecting constructionism with other theories is that as perhaps the oldest theory on this particular issue, it has had enough time to become fragmented largely due to its interpretation as a static theory and in parallel, enough time has passed for it to evolve and develop from a theory focusing on the individual to addressing social and distributed cognition, many types of technologies and representations, new ventures such as for instance the design of activities and interventions and most importantly interventions challenging institutions. This developmental nature has not really been recognized or noticed much outside the constructionist community and yet connectivities with at least some other theories could provide mutual benefit and reveal complementarities useful to elaborate in the future.

Take for instance the theory of instrumental genesis. With respect to connectivity, it was originally seen as a tool to explain the instrumentation of CAS-based techniques as discussed earlier within an anthropological framework. There have also been some perceptions of IG providing a more elaborated tool to describe the process of mediation within the framework of Activity Theory (e.g. Lagrange and Vandenbrouk, in preparation). IG has given a lot of attention to instrumentation as a notion to describe what happens when digital artifacts are put to use by denoting the formation of a conceptual schema which users develop about the functionality of the artifact in question, the underlying concepts, the kinds of things is can be used for, the meaning of its representations etc. The process of instrumentation has been seen as incorporating changes made to the medium itself and this aspect has been termed intrumentalization. Instrumentalization was coined to show that the artifact itself is shaped by each individual through its use and that there is a reciprocal relationship between these two processes, i.e. that instrumentation is affected by instrumentalization and vice versa. Little attention however has been given to instrumentalization itself. Activity theory was not articulated at a time when the medium was susceptible to functional and operational changes as is the case with digital media and therefore gave no detail into the process by which schemes of artifact use were formed through the mediation of artifacts. Instrumental theory identifies instrumentalization and situates this process within the context of mediation and schemes of use but does not elaborate on its definition. What is meant by changes to the artifact? What constitutes a change? What constitutes a change which is relevant to instrumentation and are there changes which are less relevant or irrelevant? Is instrumentalization a process which inevitably happens during instrumentation or does it depend on the design and the nature of the activity and on the nature of the artifact. Are there artifacts which invite instrumentalization more than others? What are the issues involving the design for instrumentalization (Kynigos and Psycharis 2013). These ideas are coherent with the notions articulated about a decade earlier by Noss and Hoyles (1996) that a medium shapes the mathematical meanings generated through its use and at the same time is itself shaped by use reciprocally. What is interesting however is that the design element of constructionist theory offers a more elaborate articulation of the process of designing media so that they afford useful and rich kinds of instrumentalization. A relevant notion here is that of 'half-baked microworlds' developed by Kynigos (see e.g. 2007a, 2009), i.e. digital artifacts intentionally designed and given to students as malleable and improvable asking of them to engage in discovering faults and shortcomings and changing them. This process is at the heart of fallibility and bricolage activity and discusses instrumentalization processes through a language of concerns pertaining to design and meaning generation.

With respect to social semiotics the focus is on the use of external representations and their connectivity and interdependence in digital media. Each medium carries one or more interconnected representational registers which are used together with traditional representations such as language, tangible manipulatives, gestures and written language. Digital media have the particular property that representational registers are connected and therefore expressing on manipulating one representation has immediate reciprocation on the other. They also have the unique property that representations can be dynamically manipulated and the manipulation becomes part of the representation itself. The social semiotics perspective takes a pragmatic view of the use of these registers alongside with others outside the medium such as language and pencil-paper notations (Halliday 1978). What constructionism seems to bear on such a perspective is an educational weight, i.e. on addressing a representation as a facilitator or an obstacle to the generation of meanings, of designing representations for the former and of giving primary importance to one representation over others in cases where there are more than one in connection. For example, mathematical formalism is placed in a driving role of creating graphical representations with the didactical intention to find ways in which formalism may become meaningful to students. In that sense, it can be placed in digital media with educational goals even though it may be possible from a technical or ergonomic point of view to avoid it by an icon driven interface for instance. Constructionism also gives importance to the idea of artifacts or models as representations and thus the activity of making changes to representations by making changes to the artifacts. It encompasses the idea of levels of representations. Functions represent the properties and behaviors of models of newtonian objects and at the same time the objects themselves and their behaviors are representations of science and mathematical concepts.

Finally, a comparison between the Anthropological Theory of the Didactique (Chevallard 1992) and Constructionism may allow for socio-constructivism to play the role of a common basis. A key issue where these two theories are complementary however, is the role and status of control of the didactical process. This may well be attributed to epistemology or simple to the notion of concern. Constructionism takes on board the notion that meanings are in anyway generated to some extent outside the control of a teacher or the sequencing of an activity. In designing educational activities therefore didactical intervention can at most aim to help create an environment rich or dense in opportunities and challenges for meaning generation. There is an element of randomness and uncontrollability in that process which needs to be appreciated if there is learning to be done. Otherwise, intense attempts to control the learners activities may result in disengagement and trying to guess what's in the teacher's head rather than ownership of knowledge. This does not mean that design is 'looser' with respect to activity sequencing, the designed tools to be used or the interactions between teacher and student collectives. It means however that the kinds of interactions are more strategic from the teacher's side, more participatory in a joint enterprise and more allowing for the unexpected. The teacher elicits meanings in formation and mathematics in use and helps students elaborate emergent ideas and generalizations. Also they allow and recognize fallibility, i.e. the status of suggestions, student created artifacts, student solutions etc. to be in evolution or in flux rather than that of an expression of thought awaiting a final verdict. In this wake the construct of half-baked microworlds was developed to describe artifacts especially designed to invite changes and improvements and given to the students in that capacity, rendering them engineers (Kynigos 2007a). ATD on the other hand elaborates controlled scenarios and designs where didactical interventions are pre-designed, expectations of activities and understandings are precise and stepwise and teaching sequences are defined in terms of responses to specific pre-defined questions and tasks.

> From the identification of fundamental situations expressing the epistemological characteristics of a mathematical concept or theme to the determination of the didactical variables which condition the efficiency of solving strategies or condition students' adidactical interaction with the milieu, the design of situations reflect an ambition of control and optimization. The importance attached to a priori analysis and to its anticipative dimension also attests this ambition, deeply rooted in the role of *phenomenotechnique,* with the meaning given to this term by Bachelard, devoted to didactical engineering (Artigue, in preparation and 1989).

These are three kinds of connectivity elaborations between constructionism and other theoretical frameworks in mathematics education. The process of networking is perceived as essential for the de-contextualization of the theories and a better sense of the richness of theory building in the field. Constructionism as a theory which studies meaning generation through activities of collective and individual bricolage with expressive artefacts (mostly but not exclusively digital) where meaning is drawn through the use of representations, engagement with discussion and reflections on how to make changes to them and on their behaviors as they change.

Discussion

In this paper I argue that Constructionism can be addressed as an epistemology of learning associated with a theory of learning and design. There is also an implicit suggestion that it may be useful to think of Constructionism not only from a scientific perspective but also from a strategic perspective with respect to intervening and pushing for change in institutionalized educational practices. Can constructionism transcend time and be considered as a theory in continual flux and relevance as society and expressive media change? can it be taken seriously in the networking process of theories for mathematics education? what niche does it cover in today's landscape and how can it be used today—for explanation, for guidance for action, as a structural set of lens? Today's society is torn between (a) the digital natives, complex and changing society, the need for flexibility, agency, identification of problems lying in fuzzy realities, integrated domains with respect to traditional schooling and (b) school mathematics, the need for engagement with mathematical processes such as generalization, proof, rigour, analytic-synthetic ability and also the need to understand and use traditional concepts such as number sense, algebra, analysis, geometry, space, navigation, statistics, probability, digital models. Constructionism is relevant since digital society is full of objects to be tinkered with and tools for collective mathematical activity and communication. Traditional mathematics can be given meaning since representations can be used to create models, can be manipulated, connected and visualized. Mathematics concepts and representations in use. Didactic explanatory approaches can become an infrastructure rather than the object of education. This may leave new institutionalized space for constructionist practices to develop. So in the paper I argue that constructionist epistemology is transcendentally relevant and useful to drive strategy and educational knowledge in a society where expression, bricolage, collectivity change with media.

Selected Projects and Products

Microworlds Pro, Imagine, NETLOGO, TNG—StarLogo, Scratch, E-slate Turtleworlds, ToonTalk, FMS-Berkeley Logo, Elica Logo, Scheme, BYOB-Scratch, MachineLab Turtleworlds.

Weblabs, http://www.lkl.ac.uk/kscope/weblabs/theory.htm.

TELMA, ReMath, http://telma.noe-kaleidoscope.org, http://remath.cti.gr.

METAFORA http://www.metafora-project.org/.

Mathematical Creativity Squared, http://mc2-project.eu.

The 'Constructionism' conferences, 2010, 2012, 2014, http://constructionism2014.ifs.tuwien.ac.at/.

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