Dialectic on the Problem Solving Approach: Illustrating Hermeneutics as the Ground Theory for Lesson Study in Mathematics Education

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Abstract Lesson study is the major issue in mathematics education for developing and sharing good practice and theorize a theory for teaching and curriculum development. Hermeneutic efforts are the necessary activities for sharing objectives of the lesson study and make them meaningful for further development. This paper illustrates hermeneutic efforts with two examples for understanding the mind set for lesson study. The first example, the internet communication between classrooms in Japan and Australia, demonstrates four types of interpretation activities for hermeneutic effort: Understanding, Getting others' perspectives, Instruction from experience (self-understanding), and the hermeneutic circle. Using these concepts, we will illustrate the second example with dialectic discussion amongst students in the problem solving classroom engaged in a task involving fractions.

Introduction

From early 1980s, mathematics educators have established various theories of understanding using cognitive models. These theories have illustrated such various evidence as described by qualitative research methodologies. Those evidence-based theories well illustrated students' understanding and what really happened in the classroom. Those evidences are meaningful for social scientists who are working in the office and go to the classroom for finding something which no one in the office knows about it. In the academic society of the scientists, knowing means using specific frameworks for description and special setting for using procedures which are recognized as scientific in their society. Everyone, including myself, believes that those activities are the basic methods for academic and scientific research in mathematics education which are necessary for developing general theories for mathematics education as an academic discipline.

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On the other hand, even if Newton did not explain the free fall motion of the falling apple, children would still enjoy catching ball by their own efforts and improve their ability for baseball in their team. What is the necessity to explain such activity by cognitive models which are developed by researchers? If a child began to throw the ball along various paths, whether straight or curved, his friends and observing coach would interpret that he is challenging a new ball like a pitcher. Teachers usually observe children for understanding what they are doing for supporting their effort. Children here are engaged in activities which are related to their objectives, aims, or wishes. Through synchronizing willingness of each other, good teachers usually make efforts to set good tasks with their objectives for sharing with children, attentively observing children's thinking and trying to extend children's competencies. They usually recognize children's misconceptions before researchers have a chance to find them. How can we theorize those teachers' recognitions with their practical wisdom? Their wisdom has been called pedagogical content knowledge. How can we share and theorize it for teachers?

Before Newton, many scientists such as Galileo, had already explained the motion with constant acceleration. Through the decades, scientists read the same textbook such as 'Discourses and Mathematical Demonstrations Relating to Two New Sciences' for interpreting the nature and re-organize their theories. The same subject appears recursively in the history of mathematics and in the textbooks for mathematics education. For showing evidence, historians may interpret that they must meet similar difficulties and try to find the evidence from their historical texts.

It is easy to say that these notions of teachers and students are necessary for them and not the matters for the theory for observation because they are living and working in different societies or paradigms. However, beyond this sectionalism, how can we develop the grand theory for teaching and learning for teachers who develop their children? What does it mean to establish such theory for teaching and learning for teachers and children? How can we make it practical? Lesson study is one of the methods whereby teachers can develop a fundamental theory for teaching.

On the World Association of Lesson Study Conference 2011, Yrjö Engeström mentioned that various theories in education introduced until today did not treat the objective/aims of teaching. Instead of the activity of social scientists, Lesson Study is the activity of sharing objective for teaching among teachers, children, and observers for better practice. Then, in Lesson Study, what kind of methodology would be meaningful for sharing and synchronizing their objective? Here, I propose hermeneutics, introduced by Jahnke (1994) in the PME plenary with the title 'Objectifying the Subjective'. Indeed, to ensure being scientific, mathematics educators have been focusing on the objective evidence in the subjective object.

Hermeneutics is a general theory for interpretation of human view of all natural sciences, art and literature, and one of the ways to humanizing mathematics education through objectifying subjective understanding of others. Because it is a fundamental theory of the methodology of qualitative studies, many mathematics education researchers deeply relate to it or implicitly apply it, but there are not so many researches who openly and explicitly have applied it for mathematics education.

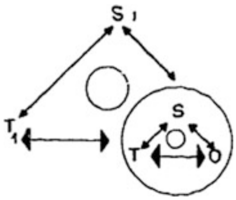
This paper is the extraction from a forthcoming paper which will explain how to design a mathematics class based on the problem solving approach. In this extraction, the author proposes and emphasises the importance of hermeneutic effort as the methodology to theorize the wisdom of lesson study. Firstly, hermeneutics will be defined by two examples which are intended to explain it.

Hermeneutics for the Theory of Understanding Others

Here, hermeneutic effort is defined and applied for analysing mathematical activity and enabling to explain it in a more subjective perspective for humanizing mathematics education.

Hermeneutics in Mathematics Education Research

In mathematics education research, major proposers of hermeneutics were Jahnke (1994) and Brown (1997). Brown clearly described its historical development and focused on current theoretical issues in relation to language for developing research methodology. Jahnke (1994) introduced hermeneutic effort as an activity of the recursive process of interpretation for explaining the role of history for education as follows;



> My principal thesis is that the concept of "hermeneutics" is suitable to describe the pedagogical interaction between synchronous and diachronous culture. …My thesis implies the claim that the historian's perspective represents an important element of an appropriate teaching culture. Seen under the aspect of method, history of mathematics, like any history, is essentially a hermeneutic effort. Theories and their creators are interpreted, and the interpreter is always aware of the hypothetical, even intuitive character of his interpretation. Interpretation itself takes place within a circular process of forming hypotheses and checking them against the text given, in the case of history of science, the objects of his process of interpretation, the scientific subjects (individuals and groups), are again hermeneuticians who interpret fields of objects. Of course, this view of scientific work will be adequate with varying precision in different times and different fields. But if we do not understand it too narrowly, this description can very well be advanced. Scientific interpretation, too, is now subject to the circular process of forming hypotheses, testing and revisiting them. He who is concerned with history, thus, has to do with a complex network of interpreters, problem fields, and interpretations (theories) which I have represented in a little diagram and which I should like to name the "twofold circle". This diagram consists

> of: Primary circle in the right bottom representing the circular relation between a scientist (s), a theory (T) and a field of objects (O) and a large secondary circle representing the historian (S1), a historical interpretation (T1) and the primary circle as his field of objects. My proposition that the teacher should know and understand something about the historian's perspective if he takes history of mathematics into the classroom, refers precisely to the problem that he/she must be aware of this twofold circle and able to move within it. Only this will enable him and his students to acquire a certain freedom against the subject matter to form hypotheses and to be ready to think oneself into other persons who have lived in another time and another culture. For me, this thinking oneself into another person and into a different world seems to be the core of an educational philosophy, providing a basis for historical contents in mathematics teaching (Jahnke 1994, pp. 154–155).

Here, from Jahnke's perspectives, the focus is on hermeneutic efforts which characterize trying to get others' perspectives. At the same time, for clarifying the meaning of hermeneutics within the diversity of the meanings of hermeneutics, here, we introduce the word 'hermeneutic effort' for understanding human activities from the perspective of hermeneutics. Hermeneutic effort is the activity to objectifying the subjective.

Hermeneutic Effort for Clarifying Humanization

Based on various theories of hermeneutics and examples of interpretation (e.g., Isoda et al. 2000; Isoda and Tsuchida 2001; Arcavi and Isoda 2007), Isoda characterized hermeneutic effort as an activity according to four principles: "Understanding," "Getting others' perspectives (the assumption of the positions of others/imaging others' minds)," "Instruction from experience (self-understanding)," and "The hermeneutic circle."

"Understanding" is one's interpretation regarding a text (or other objects). "Getting others' perspectives (the assumption of the positions of others)" means that the appropriate interpretations of a text (or other objects) is only possible through a subjective approach whereby we assume the writer's (or speaker's) position, feelings and sympathetically attempt to put ourselves into the position of another (writer or speaker). "Instruction from experience (self-understanding)" means that when one interprets assuming the position of another, one's own subjective opinion (at times, one's preconceived opinion) is reflected, in other words, one obtains an instruction about one's self with comparison of others' perspectives. "The hermeneutic circle" refers to the cycle of hypothetical interpretation and confirmation from further readings or interaction with other object such as textual interpretation whereby understanding the particulars. It broadly contributes to the whole, and overall understanding contributes to understanding the particulars, but broadly refers to the fact that a recursive or multi-layered advance in interpretation leads to a more objective interpretation: if we have some understanding, we apply it to new situations and if it is applicable, it will become more objectively correct.

In particular, "getting others' perspectives (the assumption of the positions of others)" and "instruction from experience (self-understanding)" are acts subjectively carried out through the empathy of the interpreter toward the object of understanding; accordingly the objectivity of interpretation can be stipulated in the subjectively shared act of empathy. Through this, one can see man attempting to recognize mankind as an existence able to think from another's perspective—an existence equipped with a nature that empathizes with others and comprehend human acts by such human subjectivity.

Those four principles for characterization of the hermeneutic effort were identified by the author from various hermeneutic theories and reflecting on personal interpretations of mathematics history and a hundred classroom experiments, which enabled students to perform hermeneutic efforts in mathematics on historical subject matter.[1] Here, four principles are illustrated at first and then, confirmed with some references.

An Example for Illustrating the Four Principles of Hermeneutic Effort

For illustration of the four principle of hermeneutic effort, here we read and interpret the classroom communication between Japan and Australian high school through the Internet (Isoda et al. 2006). The theme was to determine the attributes of the sums of consecutive numbers (see Figs. 1 and 2).

As Isoda et al. (2006), on the meaning of Jahnke (1994), there are two different dimensions of activities which can be noted—the students participating communication are seen as synchronous and the observing researchers interpreting their communication dialog and data are seen as isochronous. And interpreting those kinds of activities, he enhanced the role of hermeneutics. Here, we interpret this extract for illustrating the four principles of hermeneutic effort: Understanding, Getting others perspective (the assumption of the position of others), Instruction from experience (self-understanding), and The hermeneutic circle.

Each students involved in this communication conducted hermeneutic effort. First, in (B), an answer came from the Australian side that expressed three consecutive numbers as x, y and z, which differed from the Japanese customary ways of expressing algebraic expression. In (D), the Japanese side limited themselves to self-introduction, and in (E), the Australian side politely urged and showed concern for the Japanese side's failure to send an answer. In (F), the Japanese side gauged how the Australian side would respond to answer (G), which consisted of algebraic generality when sent. Simultaneously questions were submitted in (I) and (J). In (M), the Australian side explained using the Japanese side's ways of expression.

[1]A huge number of the experimental study of lesson study were done on this theory by Isoda, his lesson study team and his graduate students. Most of the books in Japanese by Masami Isoda on Worldcat are the product of it. Using original historical resources, their produced materials and classroom activities can be seen on http://math-info.criced.tsukuba.ac.jp/Forall/project/history/.

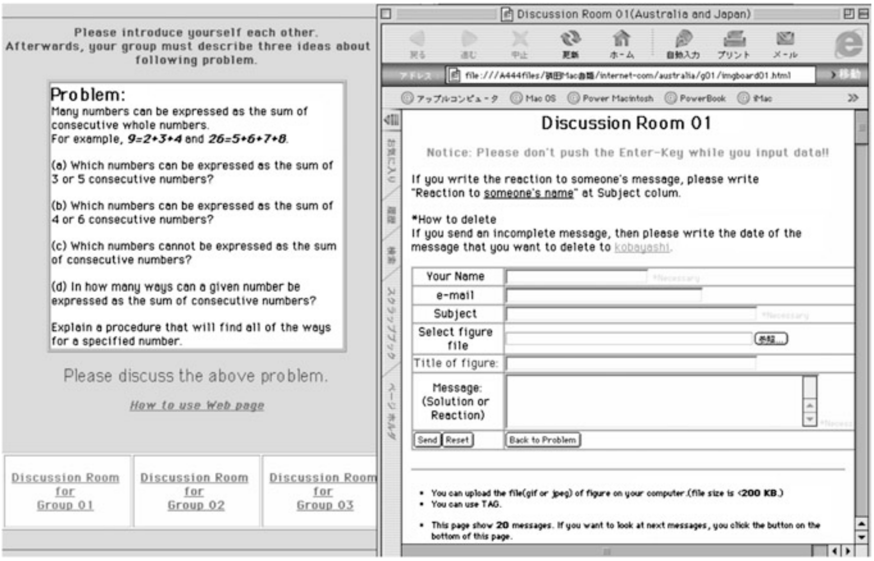


Fig. 1 A problem and discussion room

Fig. 2 The discussion between Japan and Australian

☐ Reacton and our idea to solve (a) ,(b) name :Y K [1999/10/30,22:20:51]

(F) #As you know, we are not so good at writing English. So please let us know if you don't understand.

(OMISSION)

We read your message. The answer is same as ours. But we solve it in a differnt way. I think this way is easier than yours. You used three letters. But to use only X is easier. I'll show you our way.

---- part (a) ------

(G) #In this problem, we have to think 3 consecutive numbers and 5 consecutive numbers separately.

< 3 consecutive numbers >

Let the first number be X. As three numbers are consecutive, the next number must be (X+1).
In the same way, the last number must be (X+2). So the sum of these 3 numbers is...
X+(X+1)+(X+2)=3X+3=3(X+1)
X will be natural number. (It can be taken for only integer which includes negative numbers.)

(OMISSION)

(H) Gentlemen and Gentlemen! (You are only boys) I hope you will understand the meaning of this expression.
Actually, when X=2 3(X+1)=9, when *(OMISSION)* Anyway the answer is <u>multiple of 3 bigger than or equal to 6</u>

< 5 consecetive numbers >

(OMISSION)

(I) #Question from us (1)
About "A" consecetive numbers. When "A" is an odd number, you can express the sum as multiple of "A".
When "A" is an even number, you can't express the sum as multiple of "A". Can you tell us why?

---- part (b) ------

#We considered part (b) in the same way. I'll show you waiting for your pointing out our mistakes.

< 4 consecetive numbers >

(OMISSION)

(J) The expression 2(2X+3) means that when X increases 1, the answer increases 2.
The answer is <u>multiple of 2 bigger than or equal to 12</u>

< 6 consecetive numbers >

(OMISSION)

#Question from us (2)

(K) We considered this problem over an basic condition. It is that the "numbers" means natural numbers.
But as I discribed before, "numbers" can be taken for integer which includes negative numbers.
If "numbers" means integer, how does the answer change?

---- Message -----

(L) Are you happy? Be happy! *(OMISSION)*

☐ Internet Project name :P P [1999/11/03,07:30:13]

(M) If negative numbers were included then the answer would be the same, but include all the answers as a negative as well as the positive.

part (b)

(N) The lowest number is 10 , this is because the numbers can be represented as (x, x+1, x+2, and x+3.) This is for the addition of 4 consectutive numbers.This works out as 4(x+1.5) As with your solution for part (a), It goes up in multiples of the amount of adding consecutive numbers, in this case, 4. This meanns the values are 10. 14. 18. 22. 26, etc...

For 6 numbers...

(OMISSION)

☐ Let's think about part (c)! name :Y K [1999/11/05,21:05:34]

(O) We read your letter. Your answer of <u>#Question from us (2)</u> was perfect!
If negative numbers are included, there is no minimum value. I think we discussed enough about (a) and (b).
But, have you discussed on <u>#Question from us (1)</u> in your group?
I will tell you the answer of it in the next letter. Please think about it again before the next letter comes.
Anyway, we want to go to part(c).

(OMISSION)

(P) This chart means 1+2+3+4+5+6

0
0 0
0 0 0
0 0 0 0
0 0 0 0 0
0 0 0 0 0 0

It is similar to a right angled isosceles triangle.
<u>Instead of counting all the points, calculate its area.</u>

☐ Internet Project name :P P [1999/11/08,

Discussion for part (c)

(Q) We like your idea, but we have another idea.
For this problem we will just focus on positive numbers, as you can get any numbers using negatives, eg (−3)+(−2)+(−1)+(0)+(1)+(2)+(3)+(4)=8

(OMISSION)

Fig. 2 (continued)

The Japan side confirmed using the same means as the Australian side. In (N), the Japanese side expressed consecutive numbers using a diagram and explained the total, but in (O), while still being supportive, the Australian side advocated thinking of a different expression that cannot express negative numbers.

In the above communications, the students had an empathetic stance at each stage by the time the sending and receiving content from both sides were synchronized. Each side described their own *understanding*. The other side's message content indicated their understanding, the other side's mathematical ability was appraised using their mathematical expression, and an attempt was made to respond by adopting the perspective of the other side. Then, the other side was asked questions for the purpose of obtaining deep *self-understanding*. Through *the recursive activity of mutual interpretation*, communication was synchronized. For example, Australian students answer (B) was not appropriate for Japanese and thus why they considered explanation (P) which was easier to understand. It is evidence that Japanese students *were trying to get Australian students' perspectives*. Each reply itself described the content they learned and developed from others. Further instruction from their experience was clearly recorded in the comments sheets in both sides. In fact, the students themselves applied their views of mathematics through message interpretation, and as a result, their individual views of mathematics were adjusted and updated. For example, an impression from the Japanese side included, "I usually did mathematics alone, but discussing a problem as a group, in this way, has its own appeal and I think it's a good thing." This comment shows that the student's usual way of studying mathematics became clear through the mirror of this study activity. The instruction included 'leaning how to learn' for developing social norm among their communication. In another impression on the Japanese side, "It was fun that we could talk with students in far-off Australia. We could neither see them nor hear them, but the three of them certainly exist on the other side of the ocean and were thinking about the same questions as we were. Just imagining that makes me happy," said a student. The students were delighted by this synchronized communication carried out with others on the far side of the ocean. By means of mathematics communication, the student himself reappraised his own view of mathematics through their hermeneutic effort of thinking and sharing with others. At the same time, this comment clearly illustrated the *human activity where students were imaging the existence of students on other side*. Because of *trying to understand the other side* (*getting other perspectives*), Australian students kindly waited reply at (E) and, on (O) and (Q), they positively evaluated previous messages at fist and then, they replied their own ideas. The whole processes of developing synchronized communication and developing human relationship. This is a process of *hermeneutic cycle*; For example, Australian students used different characters for consecutive numbers on (B), thus, Japanese students imagined and hypothesised that Australian students may not be using symbolic-algebraic representation well. Japanese students asked numbers on (K) and Australian students simply answered (M). Japanese confirmed the hypothesis, and for easy understanding, Japanese used Pythagorean representation on (P).

What is clear is that the students and all of us who are interpreting students' activity engage in the activity of hermeneutics efforts. We feel empathy with the interpretation that assumes the position of others, as seen in the students' communication acts, with the instruction through self-understanding as evident in their impressions, and the ability to discuss this.

Additionally, this example for illustrating hermeneutic efforts illustrates the synchronized letter style communication which included sympathetic and competitive attitudes (Ishizuka et al. 2002). For example, "We like your idea but we have another idea" on (P) shows sympathetic and competitive attitude. It is not limited in this example but historically well-known on historical mathematical text such as the Method by Archimedes and the letter on probability from Pascal to Fermat. In the next chapter, we will illustrate that those kinds of communication are observed in classroom communication, and analyse it from the viewpoint of the four principles, too.

Four Principles of Hermeneutics Effort in Its Theoretical Background

The above four principles were identified by the author from taking into consideration various hermeneutic theories and reflecting on personal interpretation of mathematics history and classroom activity. As Brown (1997) described, the current meaning of hermeneutics was described by Gadamer. On the other hand, here, we are not considering current meanings of hermeneutics but hermeneutic efforts like the historian engagement which was introduced by Jahnke (1994) into mathematics education. Especially, historians, such as Schubring (2005, pp. 1–7), described hermeneutics as for their methodology and their hermeneutics is more traditional and it is far from Gadamer's Hermeneutics. Here I will identify traditional ideas from the historical development of hermeneutics for explaining the four principles as for clarifying the hermeneutic efforts.

Gadamer (1993), who led the development of hermeneutics from the 1960s and in recent years, said that the development of hermeneutics began with "seeking the will of God in the Bible," advanced with Schleiermacher, D.F. and Dilthey, W., and then, the opinions of Heidegger and Gadamer.

Schleiermacher (1905) generalized hermeneutics from Protestant biblical hermeneutics to methodological theory on literature, history and other textual interpretations: "Two contrasting maxims of understanding. (1) I understand until I encounter a contradiction or nonsense. (2) I do not understand anything that I cannot perceive and comprehend as being necessary." This is an allusion to the confirmation of understanding by necessity and non-contradiction as seen by the subjective, and to the *hermeneutic circle* whereby hypothetical understanding is preserved until the acknowledgement of contradiction, with interpretation continually occurring. "The main point of interpretation is that the person must be able to make the transition from his own mind to the mind of the author." This is an allusion to *getting other's perspective (the assumption of the other's (author's) position)*. The act of seeking to interpret empathetically by assuming the mind of the author and aligning one's own mind with that is described. "Grammatical interpretation is objective interpretation, and technical interpretation (hermeneutics) is subjective interpretation." The act of aligning the mind for getting other's perspective is totally subjective. The position of Schleiermacher based on this kind of subjective interpretation hints at the inclusion of self-understanding whereby interpretation becomes a mirror that reflects the subjective understanding itself; however, I could not find that Schleiermacher himself mentioned about this.

Dilthey (1900) considered hermeneutics to be a methodological theory of mental science that gives objectivity to interpretation based on the subjective: "Comprehension always remains merely relative, and can never be complete." In other words, the hermeneutic circle applies. "The central point of the techniques (hermeneutics) that apply to comprehension is in the interpretation of human existence implicit in the text," "The artful comprehension of the permanently emended in expression of the existence (life) is called 'interpretation'. In terms of the interpretation, the only expression capable of such objective apprehension is linguistic expression." The salient feature of Dilthey's understanding is the point of seeing living human testimony (activity) in the text. The perspective that recognizes objectivity in interpreting subject's empathetic reading of this human nature is a characteristic of hermeneutics. "By comparing myself with others, I am able for the first time to experience individuality in myself." *Dilthey described getting others (the assumption of the position of others) and instruction from experience (self-understanding) through empathy and the superposition of the mind by* "*transferring one's self into the macrocosm of the given expression of existence*."

Gadamer disagrees with the hermeneutics descended from Schleiermacher and Dilthey. Gadamer (1960) said, "The essential character of the historical spirit is not in the restoration of the past, but rather in the mediation of present existence through thinking." For Gadamer, the emphasis is on self-understanding as a manifestation of the interpreter's present existence conducted in the medium of the work of restoring the living testimony of the past: "With all comprehension (in addition to Heidegger's understanding and interpretation), the third performance opportunity arises in order to 'comprehend one's self'." This is an expression of understanding (comprehension) from Gadamer's perspective.

Gadamer enlarged the object of interpretation from text to other area. For example, with regard to conversation he states, "Placing one's self in another's position is on all occasions an element of true conversation." (Gadamer 1960; Warnke 1987) As we already mentioned and illustrated, the subject of hermeneutic effort, whereby one aligns one's mind with another's position and understands one's own subjective thinking, is not limited to the diachronic text. After Gadamer, there also appeared movements to see hermeneutics as a philosophy that applies to natural science and all sciences. This included viewing the relationship between theory and observation as a hermeneutic circle in the natural sciences.

As shown above, the four principles, “understanding,” “getting others’ perspectives (the assumption of the positions of others),” “instruction from experience (self-understanding),” and “the hermeneutic circle” are known on the historical development of hermeneutics even if they are not mentioned by the same terms. Instead of the discussion of difference of every philosophers’ terms, here, we prefer to use traditional hermeneutics which is usually used on the historians’ activity and choose those four principles of hermeneutic efforts for describing human activity on mathematics education and for designing good teaching practices.

Hermeneutic Effort in Dialectic Discussion on Problem Solving Approach in Mathematic

On the part of illustrating the significance of hermeneutic effort, the dialectic communication in the classroom are shown for demonstrating the getting others’ perspectives and knowing ways of argumentation in the classroom which is designed through the teachers for constructing mutual understanding.

Japanese problem-solving approach in mathematics classes (Isoda and Shigeo 2012) is comprised of both individual solving of an unknown problem using students’ previous knowledge (known or learned), as well as a whole classroom work (communication or dialectic discussion) that utilizes individual ideas used in their problem solving. In particular, the classroom work, which is aimed at using each other’s thought (individual problem solving) and reorganizing through sublation that can be shared publicly based on values of mathematics such as simple, understandable, reasonable, general, easier and so on (Isoda et al. 2010). If one focuses on the dialectic communication of information between different answer-groups, one notices that this is a process of interaction between groups. If one views this as a process of each individual cognition, it becomes evident that this is a process of reviewing one’s own thinking as perceived based on information from others. This chapter focuses on the difficulties of students for getting others’ perspectives and analysing the argumentation planned by the teacher among children who are engaging in every dialectic argument for the correctness of their own thoughts, in a manner that makes it easy to elicit both sides.

First, the dialectic communication of fraction in classroom (Isoda 1993) will be described and ways of argumentations will be illustrated.

Case Study: Divisional (Partitive) Fractions Versus Quantitative Fractions

The 45 min class (first lesson) and the 15 min class (second lesson) at fifth grade were taught by Hideaki Suzuki (Sapporo Elementary School attached to Hokkaido

University of Education) regarding the problem of “making (creating) a $\frac{2}{3}$ m piece of tape from a 2 m piece of tape”. This exercise is known for its very low percentage of children and even higher grade students of giving a correct answer (Nohda 1981). It involves discriminating the different understandings of fraction between **divisional fraction** (fraction in partition; n parts from among m equally divided parts of the whole) which is studied at grade 3 on the 1989 curriculum and **quantity fraction** (n parts from among m equally divided parts of a unit quantity (such as ‘1 m’), where m < n is also possible such as ‘$\frac{3}{2}$ m’) which studied at grade 4 based on the 1989 curriculum in Japan (Ministry of Education 1989). Both of them were learned in the previous grades. But in the case of the classroom students, the actual results of a previously implemented test showed that only one out of 38 students gave a correct answer. For those students answering incorrectly, some simply misread the question, or others simply focused on the “from 2 m” part and automatically applied the procedure of divisional fractions (Fig. 3).

The class proceeds in two groups: (1) Matsuura’s Group, those who followed the divisional fraction method of thinking, whereby they came up with an answer that was “two parts of the three equal parts of 2 m (it means $\frac{4}{3}$ m as quantity)”, and (2) Minamiyama’s Group, those who followed the quantitative fraction method of thinking, whereby they came up with an answer that was “$\frac{2}{3}$ m”. Groups were originated from each student’s individual solution. Groups exchanged each of their argument while attempting to convince the other side. Through the argumentation, students changed their idea and move their position. What followed was an overview of how a student Suzuki repeatedly said the same thing in an attempt to persuade from the perspective of the listener, whereas the student Minamiyama failed to persuade from the perspective of the listener. This overview focuses on the intervention by the teacher: Teacher chose speakers among students who raised their hands and intervened for synthesizing parallel discussion. Otherwise, the communication goes parallel without connection between different opinions and will be in discrepancies.

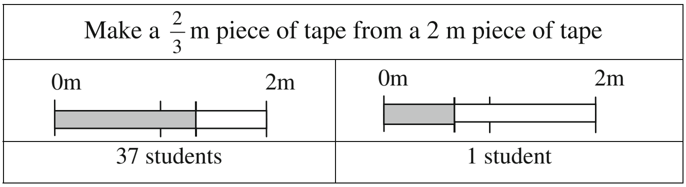


Fig. 3 The Results of pre-test

Class Overview

First class (45 min)

Scene 1 Presentation of the results of individual problem solving

When the teacher described the problem and distributed out pieces of tape (see it as 2 m each), telling students to cut the tape, complains from students such as "this is a pain" or "this is too simple." can be heard. The majority (Matsuura's Group) sees this exercise as a problem of divisional fractions and cuts the tape into two parts of the three equal segments of the 2 m (resulting in $\frac{4}{3}$ m). Only two students (Minamiyama's Group) see this exercise as a problem of quantitative fractions with the desired quantity as $\frac{2}{3}$ m, or $\frac{2}{3}$ m with a base quantity of 1 m (from the perspective of Matsuura's Group, this is one part of three equal segments of 2 m) (Table 1).

(In the following extractions, the named children appeared several times. Other children are distinguished by the names of groups.)

The teacher asks students to present the cut tape on the blackboard. Matsuura presents the answer without saying anything while Minamiyama says the following:

Minamiyama (1): Basically, it's half of this (Matsuura's tape).
Matsuura Group: No, it's two thirds!
Matsuura Group to Minamiyama's Group: Wait, I get it; I'm in Minamiyama's Group, too.
Matsuura Group: The way the teacher wrote this isn't "get $\frac{2}{3}$ of 2 m from 2 m of tape"…
Suzuki (1): Teacher, this can be interpreted either way.

The teacher asks the students to indicate their positions which they agree with their Named Magnets on the blackboard. "Matsuura Group" has 22 students, "Minamiyama Group" has 4 students, "Either Way Is Fine" has 11 students (included Suzuki), and "Undecided" has 2 students. At this point, the student Suzuki is in the "Either Way Is Fine" group (Table 2).

Table 1 The result of first selection

Matsuura group: two parts of the three equal segments of the 2 m	Minamiyama group: $\frac{2}{3}$ m
37 students	2 students

Table 2 The result of second selection

Matsuura group: two parts of the three equal segments of the 2 m	Either way is fine	Undecided	Minamiyama group: $\frac{2}{3}$ m
22 students	11 students	2 students	4 students

Scene 2 Exchanges regarding each side's position (Each others' opinion)

Either Way Is Fine Group: Maybe the problem is coming from the way of question. According to Matsuura Group, the answer is $\frac{2}{3}$ of 2 m.
Minamiyama Group: But it says "from 2 m."
Either Way Is Fine Group: So the reason for Matsuura's answer of $\frac{2}{3}$ is that since 1 or 1 m is taken from the 2 m tape.
Matsuura Group: You just divide it into three and take two of those segments.
Minamiyama Group: But it's $\frac{2}{3}$ from the 2 m tape (Note: the "m" of "$\frac{2}{3}$ m" is missing in his explanation).
Suzuki (2): This is a 2 m piece of tape, so with 2 m, you get $\frac{2}{3}$ m, right? Usually when you have a fraction, the base number is 1. Since it's $\frac{2}{3}$ m here, you have to get the base to 1. It says $\frac{2}{3}$ m, right? Since there's an "m" on it, that means $\frac{2}{3}$ of 1 m. So it's $\frac{2}{3}$ m from 2 m of tape, and Minamiyama first threw out this half (1 m), and I think you use two of the three segments of the remaining tape. If Matsuura's Group did this without the "m" in "$\frac{2}{3}$ m", I think it would be just like Matsuura's answer.
Minamiyama Group: Now there's the "m", so wouldn't Minamiyama be right?

Here, from the viewpoint of observers who knew which is correct, Suzuki (2) is speaking with a good understanding of both sides, and so that should cause Minamiyama's Group to win the discussion. Logically, this should have finished the discussion however the students are not satisfied.

Matsuura Group: Why is it that Matsuura's right if there is no "m" (in "$\frac{2}{3}$ m")?

Some of Matsuura Group do not quite understand what Suzuki is saying. Then, Teacher asked Minamiyama to explain the meaning once more.

Minamiyama (2): I thought that $\frac{3}{3}$ is equal to 1 m.
Teacher (1): One more time.
Minamiyama (3): $\frac{3}{3}$ means 1 m, right?

In spite of the fact that he, Minamiyama, will not be able to persuade Matsuura Group until he clarifies the fact that this is a quantitative fraction, the "m" quantity is consistently missing in Minamiyama's explanations from Minamiyama (1) to (3) such as "$\frac{3}{3}$ is equal to 1 m" but the divisional procedure is represented as well as the explanations of Matsuyama group. Even if the "m" is clearly written in Minamiyama's note, when he explains what he has done to other students, he equates $\frac{3}{3}$ with 1 m rather than stating "$\frac{3}{3}$ m". Minamiyama is applying divisional fractions with 1 m as the unit quantity, and is overlooking the fact that this is a quantitative fraction despite of the repeated prompting from the Teacher (1). This makes it impossible for Minamiyama to deny the "three equal parts of 2 m" idea of Matsuura's Group.

Because Minamiyama himself did not explain well, the teacher diverts the discussion away from what Minamiyama is saying in the following way and starts stirring things up.

Scene 3 Stirring things up (Teacher intervention 1)

Teacher (2): $\frac{3}{3}$ m is 1 m, no doubt about it (Note: he emphasized the "m").
Minamiyama Group: Right!
Matsuura Group: No, absolutely not.

The teacher focuses on whether or not $\frac{3}{3}$ m is 1 m for trying to find sharable ground of discussion.

Suzuki (3): Teacher, it is not related with the problem (Note: the original question), isn't it?

Suzuki (3) takes this to mean that other than the original question, anyone would think that $\frac{3}{3}$ m $= 1$ m, or in other words that the "base is 1 m". This is also a counterargument and then, they start to include the original question in teacher's new question.

Matsuura Group: You take $\frac{2}{3}$ from 2 m, right? So maybe Minamiyama's $\frac{3}{3}$ m is 2 m.

Matsuura group still assumes that their answer is correct and for persuading Minamiyama group, they begin to follow Minamiyama's thinking that $\frac{3}{3}$ m is 2 m if $\frac{2}{3}$ from 2 m but it just reflects on their interpretation. On the other hand, Minamiyama explanation continued to be expressed as a divisional procedure and failed to explain using the basis (meaning) of a quantitative fraction.

Minamiyama (4): (Pointing at the 2 m figure) This half is 1 m, and these two segments are $\frac{2}{3}$.
Matsuura Group: No mentioned $\frac{2}{3}$ of "1 m" in the original problem.
Minamiyama Group: It doesn't say create "2 m" tape. It just says is "from 2 m of tape".
Minamiyama Group: Since the original problem doesn't say to make this only from a 2 m tape, you can make it from 1 m as well.

The teacher reorganizes the conflicting arguments. Matsuura's Group sees the "base as 2 m", and the Either Way Is Fine Group sees "both 2 and 1 m can be the base". In order to summarize their viewpoints, the teacher questions Minamiyama Group as follows.

Teacher (3): Minamiyama, if your answer is $\frac{2}{3}$ m, then we would like to say that the base is 1 m. This is the reason why $\frac{3}{3}$ m is 1 m, and the base is 1 m, according to what you are trying to say, right, Minamiyama?

This questioning clarifies the ground of Minamiyama's inference as opposed to the aforementioned Matsuura Group's ground.

Scene 4 Sharing the argument

Teacher (4): Well, this is a problem, isn't it?
Matsuura Group: Since Minamiyama has left 1 m over, doesn't that mean what really remain is 1 and $\frac{2}{3}$?
Matsuura Group: So Minamiyama does not take 1, but $\frac{1}{6}$.
Teacher (5): No. Minamiyama's answer works when he's only using this (1 m). The remaining 1 m is irrelevant for him.

Many of Matsuura Group still reflect on their interpretation. Matsuura Group which "divides 2 m", interprets Minamiyama's unit quantity $\frac{1}{3}$ m as $\frac{1}{6}$ of 2 m. Accepting the teacher's statement that "the remaining 1 m is irrelevant", Suzuki stated that she is moving (to Minamiyama Group) and started to talk.

Suzuki (4): If the original problem involves making $\frac{2}{3}$ of a 2 m tape, then Matsuura's side is right, I mean, I think Matsuura's argument is easier to understand. Since you're supposed to create $\frac{2}{3}$ m from a 2 m piece of tape then it must be $\frac{2}{3}$ m. So you ignore the 1 m, and this $\frac{3}{3}$ m is also 1 m. Since you are going "from", you've got to deal with both "from" and "m". If there wasn't this "m", and if "from" was "of", then I would agree with Matsuura. (Repeating while reviewing the figure) This $\frac{2}{3}$ m means that the base is 1 m. If there wasn't an "m", then you could use any amount of "m" as the base, but since there is an "m", then 1 m must be the base.
Matsuura Group: If the problem is "create $\frac{2}{3}$ from a 2 m tape", or "create $\frac{2}{3}$ m of a 2 m tape"?
Matsuura Group and Minamiyama Group: The first one, "create $\frac{2}{3}$ from a 2 m tape", is Matsuura Group but what about the second one?
Matsuura Group: 2 m might be the base, but since its $\frac{2}{3}$ m, 1 m might be the base, too.
Teacher (6): So the second one would be strange and contradicting.

Ever since Suzuki (4)'s statement, the semantic interpretation of each group was not the same. On the other hand, the statement of Matsuura Group here has been influenced by the teacher's specification of the base amounts and the group now shares Suzuki's statement. Matsuura Group should now focus on "from" and "of" while considering questions that they come up with themselves in order to review the points they presented themselves. This awareness of contradiction then causes some members of Matsuura Group to begin sharing Minamiyama's idea that the base quantity for the case of $\frac{2}{3}$ m is 1 m, indicating that they are considering joining Minamiyama Group. Teacher (6) mentioned that "create $\frac{2}{3}$ m from a 2 m tape" do not contradict but "create $\frac{2}{3}$ m of a 2 m tape". It's already implicated for the people who have appropriate knowledge that Matsuura group is inappropriate but they do not well understand teacher's saying even if they felt it as strange in Japanese.

In order to articulate this state where ideas have changed, the teacher asks the students to move their named magnets (for the second time). The results are 16

Table 3 The result of third selection

Matsuura group: two parts of the three equal segments of the 2 m	Either way is fine	Undecided	Minamiyama group: $\frac{2}{3}$ m
16 students	No student	2 students	20 students

students in Matsuura Group, 20 students in Minamiyama Group, no student in the Either Way Is Fine Group and 2 students in the Undecided Group (Table 3).

Scene 5 Stirring things up (Teacher intervention 2)

In order to stir things up again, the teacher asked the students to forget the original question and whether or not "$\frac{3}{3}$ m is 1 m" temporarily. Some members of Matsuura group are still the opinion that "it is three equally divided parts of 1 m or 2 m". This opinion indicates that some students are still caught up in the idea of divisional fractions. The teacher asks "can we change tracks?" and continued as follows.

Scene 6 Stirring things up (Teacher intervention 3)

Teacher (7): If we have 0.5 m, then do we indicate what the length is?
Matsuura Group and Minamiyama Group: Yes, it's the same as 50 cm.
Teacher (8): Can we express this as a fraction? (Detailed discussion omitted) So is it the same as $\frac{1}{2}$ m, or is it different?
Minamiyama Group: It's the same.
Matsuura Group: Wow! (Note: this is taken to mean that they are realizing their contradiction.)
Matsuura Group: It's different.
Suzuki (5): If 0.5 m is the same as $\frac{1}{2}$, then what is $\frac{1}{2}$?
Teacher (9): And if I asked you to express $\frac{1}{2}$ m as a decimal of m, what would that be? (Note: he added 'm'.)
Matsuura Group: 0.5. (Note: 'm' is still missing.)
Matsuura Group: It might be $\frac{1}{2}$ of 2 m.

Some members of Matsuura Group now think that "if $\frac{1}{2}$ m is an invariant then maybe $\frac{3}{3}$ m is 1 m". However even now, some members of Matsuura Group still recognize the fact that $0.5\,\text{m} = \frac{1}{2}$ m, but do assert $\frac{1}{2}$ m $\neq$ 0.5 m because '$\frac{1}{2}$ m is $\frac{1}{2}$ of 2 m' is also true. These members insisted that their own explanation on the original problem is correct and therefore account their thinking on divisional fractions with a division target of 2 m as the base. Minamiyama continued his explanation.

Minamiyama (5): $0.5\,\text{m} = \frac{1}{2}$ m and $\frac{1}{2}$ m $= 0.5\,\text{m}$ are the same thing, all you're doing is reversing the order. So I think you can say that $\frac{3}{3}$ m is 1 m. But if the base changes, I'm not sure if you can still say that $\frac{1}{2}$ m $= 0.5\,\text{m}$.

It is evident now that Minamiyama himself recognized the way of thinking of Matsuura Group. It looks that both groups had now reached a state where they recognized the thinking of the other group. At the same time, Minamiyama himself

Table 4 The result of forth selection

Matsuura group: two parts of the three equal segments of the 2 m	Either way is fine	Undecided	Minamiyama group: $\frac{2}{3}$ m
14 students	No student	1 students	23 students

is dealing with the problem of how to find “what unassailable ground of discussion can be shared” with Matsuura Group which is still fixated on divisional fractions. Time runs out at this point and the teacher returns to the argument at hand about “if ‘m’ is affixed on $\frac{2}{3}$ m, whether or not 1 m is the base”, asked the students to move their magnets for the third time. At this point, Matsuura Group has 14 students, Minamiyama Group has 23 students, the Either Way Is Fine Group has no student and the Undecided Group has 1 student (Table 4).

Second class (15 min)

Scene 7 The next day class

The lesson began with the review on the explanation of what had been discussed in the previous lesson for students who were absent yesterday. The question of “from” or “of” is examined once again, with the aim of articulating the difference between interpretations that determine whether one is a member of either Matsuura Group or Minamiyama Group. However the discussion between the groups is not as heated as it was yesterday. The teacher noted the mood of the classroom and started the guiding instruction for concluding.

Teacher (10): The class seems to be in Minamiyama’s direction. Matsuura Group, do you have anything to add to the discussion?
Matsuura Group: It says “from” a 2 m tape, right? If it said “from a 1 m tape”, or if it didn’t say “from” (“of 1 m”), then Minamiyama would be right, but it does say “from”, so 2 m is the base.
Minamiyama Group: 2 m is larger than 1 m, right? So we can just forget 1 m of the 2 m for the moment, and take $\frac{2}{3}$ m from 1 m, for instance.
Minamiyama Group: Just ignore where it says “from”.
Teacher (11): So that you are saying, just “create a $\frac{2}{3}$ m tape” is the same as the original question.

The teacher asked the student Suzuki to explain the answer by focusing on the original problem from the children regarding “how the exercise changes depending on whether m is affixed or not.”

Suzuki (6): For instance, you have a blackboard and you have $\frac{2}{3}$ of a blackboard. We say this is $\frac{2}{3}$. For instance, if you have a blackboard eraser, you could say $\frac{2}{3}$ of this blackboard eraser. Understand?
Teacher (12): I know what you’re trying to say. I really do understand.

Suzuki (7): You can go with anything whatever. But it says $\frac{2}{3}$ m. Since it has an "m" on it, that "m" must be the base. We studied that it was determined by the distance from the Equator to the North Pole divided by some tens of millions, right? Before they standardized it that way, "1 m" was not always equal, right? If you use 2 m as the base, you back then against the standardization. Anyway, since m has 1 m as the base. This is the difference between when you have a given base and when you don't.

The teacher then asked other students to explain how they understood what Suzuki had explained in their own words and summarized the discussion as follows:

Teacher (13): Suzuki wants to say that since there is a unit affixed, the base is already completely settled. So that's why she feels she has to join the Minamiyama Group.
Teacher (14): We haven't heard from the Matsuura Group at all lately. Can we end this discussion now, then? Since "m" is affixed, the base is "1 m", but since we are dealing with $\frac{2}{3}$, we can change the base accordingly. It's as simple as that, isn't it? Is this fine with everyone? (The class ends at this point.)

The teacher ends the class by using Suzuki's statement as the basis of bringing the discussion to a conclusion. Although there are no longer any counterarguments from Matsuura Group, some of the members remain unconvinced. The teacher continued by reformulating the lesson using fractions of various sizes of quantity such as more than 1 m as a subject matter in order to convince those who still remain in doubt.

This overview includes excerpts focusing on the following phenomena, which are recognized as part of the argument process.

(a) At the beginning, Matsuura Group failed to share Minamiyama's way of thinking and interpreting it wrongly. This prevented connection between the two groups.
(b) Minamiyama's explanation without using the unit "m" in his words is unsuitable for persuading Matsuura Group.
(c) Suzuki's statements were consistent from the beginning but still failed to fully persuade Matsuura Group.
(d) The teacher intervention for trying to conclude the contradiction with respect to quantity was effective in persuading the class.
(e) As both sides interacted, they shifted from a state of misunderstanding each other to a state of shared understanding.
(f) Controversially, there were some students in Matsuura Group who developed the hard core on insisting that their believed conclusion was true which enabled them to insist that $0.5\,\mathrm{m} = \frac{1}{2}\,\mathrm{m}$ but $\frac{1}{2}\,\mathrm{m} \neq 0.5\,\mathrm{m}$.
(g) Some students were convinced, but others were not completely convinced by the efforts of the persuasion.

The flow of the class overview is shown in Fig. 4 on the next page.

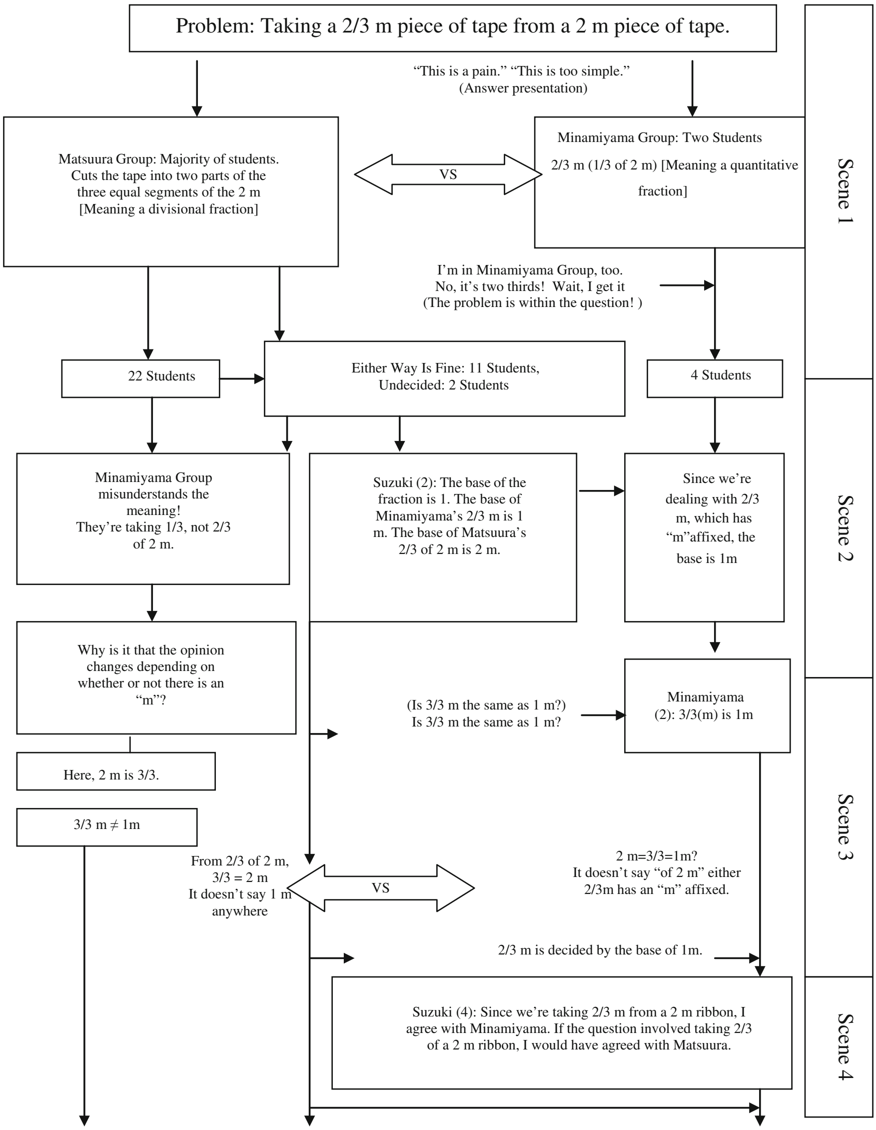


Fig. 4 The process of argumentation

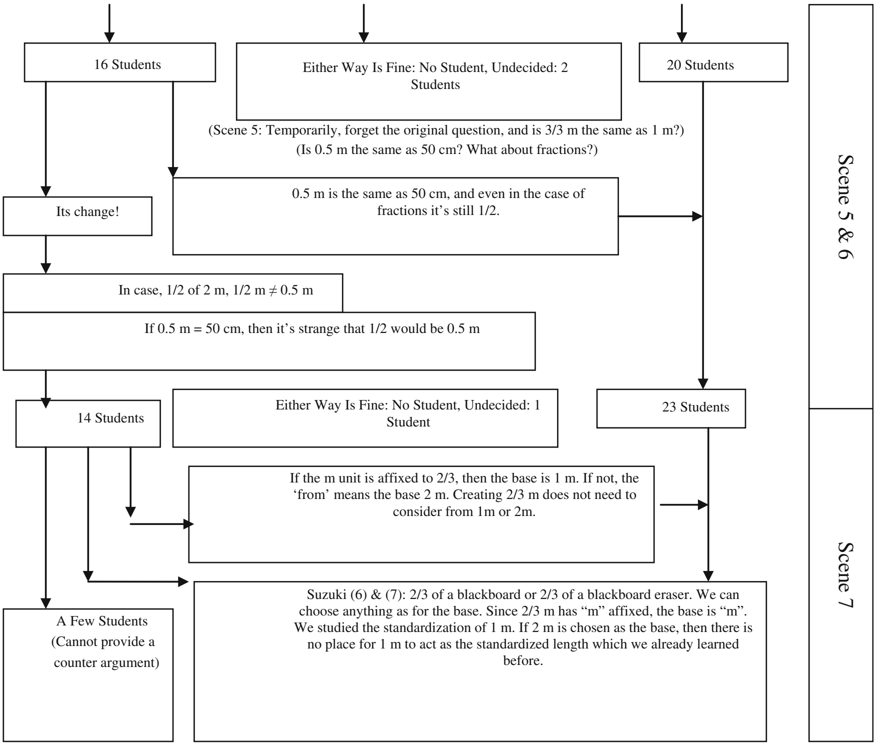


Fig. 4 (continued)

Analysing Argumentations Between Groups

Japanese problems solving approach usually goes in a whole classroom work based on individual solution and the position of each individual opinion (mathematical ideas) is recognized on categorized solution (positioned discussion). A Teacher usually planned the process of arguments based on the category and conducts argumentations in a whole classroom with categorized solutions after solving individually. The difference of individual ideas will be communicated in relation to categorized solutions which are presented by students. On this context, here, we analyze the process of argumentation with categorized groups which are shown on Fig. 4a, b.

From the view point of dialectic, the process of argumentations on Fig. 4a, b are summarized by the following structure on Fig. 5.

The process of argumentations were controlled by the teacher. He intervened in the following ways for completing dialectic;

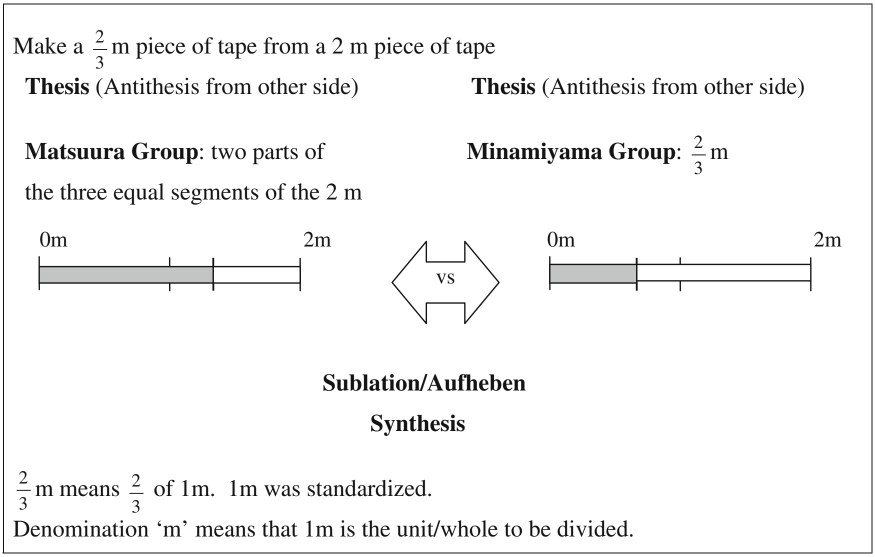


Fig. 5 The dialectic at the class

Firstly, he gave opportunity to present every argument/assertion from both sides (Scene 1),
Secondly, he allowed Suzuki who understands both sides' arguments to explain,
Thirdly, he gave a counter example $\frac{3}{3}$ m in relation to the original question (Scene 3),
Fourthly, he gave generalized counter example $\frac{1}{2}$ m (Scene 6), and
Fifthly, he supported Suzuki who explained the necessity to fix '1 m' as a unit for conclusion.
Finally, because the teacher supported Suzuki and there was no new counter argument, Matsuura Group did not give their comment further more. Teacher set remedial instruction about the quantitative fraction more than 1 m such as $\frac{3}{2}$ m: Students already learned those concepts in grade 4.

Figure 4a, b showed that students shifted their position of argument depending on the changes of ideas through persuasion by others. From the changes of their position, we can identify following types of students in Matsuura Group from the viewpoint of conceptual and procedural knowledge (Hibert 1986; Isoda 1992, 1996):

Type A Students who changed immediately at Scene 1
Although they knew both the procedure of division and the meaning of the quantity on fractions, they did not used well or understand the meaning of the quantity fraction at the beginning. They applied the dividing procedure of divisional fraction at the original question and some of them immediately remembered the quantity

fraction from Minamiyama's answer and recognized the difference between 'from' and 'of' by themselves. Even if they moved 'either way is fine' but did not moved to Minamiyama Group at Scene 1, it means that they well understand both the procedure of dividing and the meaning of the quantity on fractions. They well understood that the confusion originated from the difference between 'of' and 'from' and that is why they could understand both position.

Type B Students who changed to Minamiyama Group until Scene 6
They knew the divisional procedure and did not well understand the quantity on fractions at the beginning. They applied the dividing procedure to the original question. They did not change their position when they met Minamiyama's answer. It means that their understanding of the quantity on fractions was not enough to distinguish the difference of 'from' and 'of' in this moment. From the contradiction of $\frac{3}{3}$ m = 2 m and additional contradiction of $\frac{2}{2}$ m = 2 m which deduced from their divisional procedure at original question, they gradually recognized the difference of 'from' and 'of' and until Scene 6, they understood the meaning of the quantity on fractions. Then, they moved to Minamiyama Group. We can say that their understandings of the meaning of the quantity on fractions and the quantity itself were different depending on when they had changed.

Type C Students who did not changed until Scene 6.
They knew the dividing procedure. They produced their arguments for explaining how their conclusion was correct and did not want to think of others' ideas which were based on the quantity on fractions, and finally did not understand the relationship among quantity. In their case, their diving procedure produced their belief which is strong enough to disregard the contradiction and change the meaning of the quantity. For their persuasion, they have to use their dividing procedure on original question and they must support it. Their confrontational dialectic position for trying to explain how their original idea was correct, enhanced their procedure as the hard core, which should be kept.

From the view point of the meaning and procedure, Type A students used well dividing procedure and understood the quantity on fraction at the beginning. Type B students used well dividing procedure but did not understand the quantity on fraction at the beginning and finally learned the quantity on fraction from the contradictions until Scene 6. Type C students used dividing procedure but they did not need to understand the contradiction, sympathetically, and failed to conceive the meaning of the quantity on fraction. Some of them rejected to conceive the quantity on fraction for asserting their conclusion to be true. In their case, their procedure is functioning like a meaning as the base to explain why their conclusion is true. They had to choose their necessary assumption for deducing conclusion to be true and never share the assumption which can be generalize. Those categorization from the viewpoint of the procedural knowledge and the conceptual knowledge which were observed in the process of argumentation is summarized in following Table 5.

Table 5 Students understanding of fraction from the conceptual and procedural knowledge

In the process, they applied	Type A	Type B	Type C
The appropriate procedure of division	Kept	Kept	Kept
The appropriate meaning of the quantity	Kept	non → having	non → non

Getting Others' Perspectives; Ways of Persuasion and Moments of Conviction

The four principles of hermeneutic effort are "Understanding," "Getting others' perspectives (the assumption of the position of others)," "Instruction from experience (self-understanding)," and "The hermeneutic circle." Here, I illustrate the difficulty of getting other's perspective for knowing the ways of persuasion and the moments of conviction.

We usually say the "logic of persuasion" states that "it is not possible to easily persuade people unless one does so with an understanding of their perspective first". This is a kind of ancients' dialectic which begins the reason: "if what you say is true…." Suzuki knew the idea of both groups and repeated the same explanation. But depending on students, understandings were different because they do not share the same reasoning.

In Fig. 3, even though the reasoning and understanding of each student are not the same we could categorize Matsuura Group into Type A, Type B and Type C. Suzuki's explanations did not change through the discussion but depending on their understanding their decisions were different. Type B students could not agree with Suzuki from the beginning because they do not have a sharable ground. As Hegel, G. described, the antithesis works positively and negatively. In the case of Type B, counter examples given by teachers supported them to develop (or remember) the meaning of the quantity. In the case of Type C, counter examples influenced them to develop the hard core for reasoning based on their conclusion. Both developments are different instructions from experience in this lesson. It means that counter examples given by teachers functioned positively for developing ground of discussion for Type B students but works negatively for Type C students.

However, in case, students can share the ground of discussion, they can share the ideas. In this lesson, the teacher prepared several strategies for developing the ground of discussion for the conviction. First, he gave students the opportunity to exchange the different answers. Second, he gave the opportunity to exchange their reasoning. Third, he gave counter examples. Fourth, he fixed the ground of reasoning by supporting Suzuki's explanation, especially, he posed different counter examples: $\frac{3}{3}$ m, 0.5 m and $\frac{3}{2}$ m in the next lesson. $\frac{3}{3}$ m is limited within fractions, 0.5 m is related with decimal notations and $\frac{3}{2}$ m is the extension of divisional fraction to the quantity. Each counter example has their different roles for developing the concept of quantitative fractions. He posed them in the sequence from specific to general: students could not represent $\frac{3}{3}$ by decimal notations. Divisional fraction divide whole and is not larger than 1.

Through these teaching strategies, students engaged in the hermeneutic cycle and were able to develop others perspective as the ground for sharable discussion, and developed appropriate understanding.

Significance of Hermeneutic Effort on Lesson Study for Humanizing Mathematics Education

Lesson study will succeed just in case the objective for teaching and learning among teachers, students and researchers in classroom are well shared. On the open class and the post-class discussion on lesson study, the necessary point at the post-class discussion is knowing the objective of the teacher's teaching activities and students' learning activities. Even if participants of open class give their alternative comments against the teacher's teaching activities, they have to make clear the difference of objectives for sharing their ideas reasonably in the lesson study community. To show the difference of objectives, they also respect the originality of teacher's activities. The students' behaviours are usually referred for showing evidence to make clear explanation of the effect of teaching practice on its hermeneutic cycle.

Hermeneutic efforts are usually done for making clear the human activity. Main component of hermeneutic efforts is based on the activity for getting others perspective. Even if it is purely subjective activities, it can produce sharable interpretation and deeper understanding of others.

In mathematics education, Piaget's epistemology and Constructivism had been functioning as the major theory for learning beyond contradiction and reorganizing organisms to be more viable. Vygotskiian epistemology and Social Constructivism had been functioning as the major theory for learning from intersubjective to subjective. The former perspective enhances solipsism in the environment and explains one's understanding, but not easy to explain the difference of environments and relationship between each individual. The later perspective enhances materialism, and existence of intersubjective knowledge such as instrument and norm, and explains learning as instrumentalization but not easy to explain difference of every mind. As Glassersfeld (1995) mentioned, there is no contradiction between Constructivism and Social Constructivism. However the weakness of both epistemologies has existed in their adaptations. Both epistemologies are independently adapted for describing the phenomena of learning metaphorically. On this context, each epistemology functioned as the model to explain learning phenomena in different manner and never connected. On the adaptations, researchers usually did not treat the objectives or aims of subjects such as students. For example, Freudenthal (1973) criticized Piaget's and others description of experiment as the inappropriate interpretations of students' activity beyond their willingness to think.

Traditional perspectives on Hermeneutics, which mentioned in this paper by the four principles on hermeneutic efforts provides our convictions of understanding others and appropriateness of interpretation. Hermeneutic efforts provide solipsism the possibility of understanding others and existence of others. Hermeneutic efforts also provide materialism the possibility of subjective interpretation of the object, such as humanities of mathematics. For example, students can say that I am thinking likely Pythagoras for using Pythagorean theorem even if we are not sure that he found the theorem or not.

As Yrjö Engeström problematized, the theories of education failed to provide the platform for treatment of objective or aims of teaching practice. Hermeneutic efforts provide major theory for lesson study through getting other's perspectives.

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