Integration of Technology into Mathematics Teaching: Past, Present and Future

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Abstract This paper deals with my endeavor as a researcher and lecturer within the world of educational computing to integrate technology into mathematics teaching. I started with the book titled "New Horizons in Educational Computing". In this book Saymor Papert enthusiastically says that computers as powerful learning tools will change tomorrow's classrooms. It is difficult to use this potential of computers for changing teacher's role and practice within an educational setting based on telling and showing. It was not easy for me to shift from traditional notions of teacher to constructivist teacher using Logo, Cabri and GeoGebra as primary tools for doing and exploring mathematics in classrooms.

Introduction

This paper represents a more than 20 year effort made relentlessly since I have been started doing postgraduate studies at the UNB in Canada. As a learner I started with Logo in 80s and continues up till today with Cabri and GeoGebra. When I was an undergraduate student in 70s I just heard the name of the computer, but I have never seen it. I touched the computer for the first time in my life, year 1988. I came up with a book titled "Mathematical Applications of Electronic Spreadsheets" by Deane Arganbright, It was the first book of mine about educational computer. Activities and problems in this book were all what I already knew in school mathematics. They were not really interesting for me in terms of learning and teaching mathematics from a constructivist paradigm. Nothing was new for me in this book in terms of constructing and exploring new mathematical ideas.

My second book on educational education was "Computers in the Mathematics Curriculum" published by The Mathematics Association and edited by David Tall. This book included many open ended activities for constructing mathematical ides.

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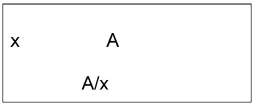
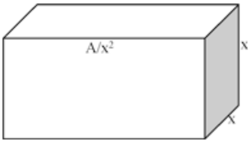
One of them was an example of iteration for finding the square root of a number A. When A is a square number like 49, we can use square to represent the number geometrically and one side of the square (7) will be square root of the number A. When A is not a square number like 55, we can use a rectangle to represent the number geometrically and calculate the square root by using the iteration method. In the classroom this method may be introduced by starting with a rectangle of area A.



The problem is to find the length of one side of the square which has the same area as the rectangle. If one side of the rectangle is of length x, the other side will be of length $\frac{A}{x}$. It is clear that the length of the square of area A will lie between x and $\frac{A}{x}$. In this case, the best approximation is likely to be given by replacing x by $\frac{1}{2}\left(x + \frac{A}{x}\right)$ and continue with this iteration until the difference between x^2 and A is less than a prescribed amount, say 0.0001.

	A	B	C	D	E
1	A =	55	x^2	ROOT	DIFFERENCE
2	X=	28			
3	28	14,98214	419,5	DEVAM	-364,5
4	14,982143	9,32659	139,7323	DEVAM	-84,7323023
5	9,3265899	7,611854	70,99264	DEVAM	-15,9926396
6	7,6118539	7,418713	56,47016	DEVAM	-1,4701598
7	7,4187131	7,416199	55,018652	DEVAM	-0,01865169
8	7,4161989	7,416198	55,000003	DEVAM	-3,1605E-06
9	7,4161985	7,416198	55	DEVAM	-9,2371E-14
10	7,4161985	7,416198	55	7,41619849	0
11	7,4161985	7,416198	55	7,41619849	0
12	7,4161985	7,416198	55	7,41619849	0
13	7,4161985	7,416198	55	7,41619849	0
14	7,4161985	7,416198	55	7,41619849	0

In a similar way, an iteration for cube roots can be obtained by starting with a cuboid of square cross section whose volume is A and finding the length of side of the equivalent cube.



Later, I came up with the book titled "New Horizons in Educational Computing". 1984 This was the real turning point for me in my endeavor of the educational computing. In this book, Saymor Papert enthusiastically says that "*computers as powerful learning tools will change tomorrow's classrooms*".

At the beginning, I actually had difficulty to see this potential of computers in changing teacher's role and classroom practice. I tried to compromise my teaching approach based on telling and showing with the approach based on Papert's constructivist ideas about using Logo. It was not easy for me to shift from traditional notions of teacher to constructivist teacher using Logo as primary tools for doing and exploring mathematics in classrooms.

When I was a postgraduate student at the Institute of Education in University of London I found opportunity to work in Microworld Project with Celia Hoyles and Richard Noss. Their perspectives and approaches to educational technology helped me gradually to see what Seymor Papert points out about the potential of computers in changing teacher's role and classroom practice. I saw Logo as a paradigm for thinking about the use of mathematical software. My experiences at the Institute of Education had led me to believe that Logo is a powerful medium for confronting teachers with their preconceptions about teaching and learning mathematics. Many of the mathematical ideas which are used within the Logo environment (e.g., turtle geometry and recursion) were knew for me and excited me to learn more about Logo.

Programming as a Problem Solving

After finishing my doctoral program I returned to Karadeniz Technical University as a lecturer with many books on Logo such as "Approaching Precalculus Mathematics Discretely" edited by Philip Lewis, "Learning Mathematics and Logo" edited by Celia Hoyles and Richard Noss, "Turtle Geometry" edited by Harold Abelson and Andrea diSessa. With my expecting to explore mathematical concepts in a Logo-based environment I used these books in mathematics courses at undergraduate level and to investigate a model of developing the concepts of calculus and algebra using Logo. We (I and my students in these courses) learned some mathematics by experimenting with the ideas and developed our own structures.

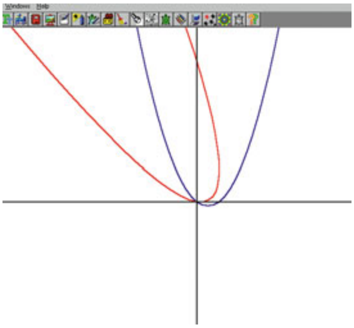
Through my first experience as learner and as teacher within a Logo based environment, I have realized that programming as a mathematical activity. Although I have sought to use Logo as a mathematical language and a good deal of the problem-solving activity is mathematical, it is also clear that many of the problems are problems of programming.

Problem solving	Programming
Understanding the problem	Understanding the problem
Planning for solution	Coding the program
Carrying out the plan	Running the program
Evaluation	Debugging the program

Representing function graphically and rotate the graphs of functions in Logo environment, it is possible both representing the function graphically and rotate the graphs. I worked with my students on the following task:

First, we define f(x) = x(x − 2) function in Logo. If we want to rotate the graph $\theta°$ counter-clockwise about the origin. We need to use the matrix.

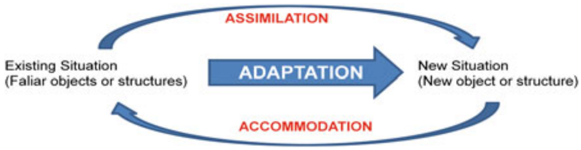
$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$



```
For 30°
to f :x
op :x*(:x-2)
end
to dönx :x :y :k
op ((:x*cos :k)+(:y*(-sin :k)))
end
to döny :x :y :k
op ((:x*sin :k)+(:y*cos :k))
end
to dönme :fonk :x :k
setpenwidth 2
if :x=15 [pu home pd penerase setpos se -75 (koş :fonk -15) pu home
Setheading 0 setpc 1 pd eksen stop]
make "y koş :fonk :x pd setpc 5 setpos se (5*:x) :y
make "t koş :fonk (:x+1) setpos se 5*(:x+1) :t
make "m dönx (5*:x) :y :k
make "n döny (5*:x) :y :k pu setpc 10 setpos se :m :n
make "r dönx (5*(:x+1)) :t :k
make "l döny (5*(:x+1)) :t :k
pd setpos se :r :l pu setpc 5 setpos se (5*(:x+1)) :t pd
dönme :fonk :x+1 :k
end
```

```
to koş :fonk :x
op run se :fonk :x
end
to eksen
fd 400 bk 800 fd 400 rt 90 fd 400 bk 800 fd 400 lt 90
end
```

Piaget and Logo

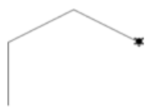


In Piaget's terms, when the individual is confronted with conflicts during mathematical activities, there are two possibilities for him/her: either she/he ignores the problem or accommodation process takes place with some modifications. This experience, therefore, enables the individual to conceptualize new situation from previous existing knowledge. Let us see how this occurs within a Logo-based environment:

Suppose that student's previous knowledge consists of writing small procedures in Logo. And also she/he knows the basic properties of square and equilateral-triangle (all sides are equal, all angles are equal and 60°. When we ask him to write a procedure for a square, he can write the following procedure and check it on the screen. Everything is going well.

```
to square
repeat 4 [fd 40 rt 90]
end
```

These are all his existing previous knowledge about drawing geometric figures in 2-D. After this experience, when we ask him to write a procedure for equilateral-triangle, this task is a new situation for him. By using his previous knowledge about Logo and triangle, probably he may just change only **repeat** line in the **square** procedure, and then write the following procedure:

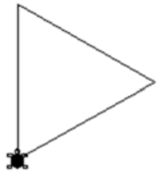


```
to equı-trıangle
repeat 3 [fd 40 rt 60]
end
```

When he run the procedure, he will see this figure.

This figure is entirely different what he expected to see.

This new situation is disequilibrium for him. In order to conceptualize this new situation, accommodation process needs to work. This process can work like this way; he can turn to the procedure and try to modify it, or he can put himself into the position of the turtle and traces the path of the turtle on the figure. When he realizes that the turtle turns according to the exterior angle rather than interior angle, it means that accommodation process is completed. Now student get new knowledge that drawing geometric shapes with Logo in 2-D space, we should use the sum of exterior angle of the shape which is 360°. After this adaptation, the procedure for an equilateral-triangle will be:



```
to equı-trıangle
repeat 3 [fd 40 rt 60]
end
```

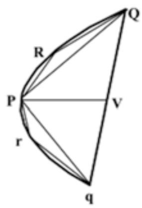
Dynamic Geometry Software

Doing mathematics in a dynamic geometry software environment is a process consisting of:

Making experimentation
Making conjecture
Proving the conjecture.

Let me give an example from the course of computer-based mathematics teaching which I have taught since 1996. I and my undergraduate students worked on the proposition of Archimedes by using CABRI:

Every segment bounded by a parabola and a chord Qq is equal to four-thirds of the triangle which has the same base as the segment and equal height.

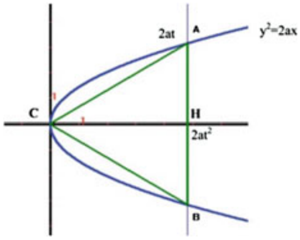


Making experimentation consists of three steps:

1. Specifying the proposition (Special Case)
2. Constructing the general form of the proposition (General Case)
3. Explaining the empirical findings

First, we constructed a **special case** as in the figure. In this case, we drew the largest triangle in the parabolic segment with Cabri.

In this case, the vertex is in the origin and the segment AB is perpendicular to the X-axis. Let A_T be the area of the triangle ABC, then $A(ABC) = 4a^2t^3$. The area of the half of the parabolic segment above X-axis will be:



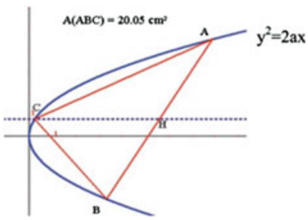
$$A_P = \int_0^{2at^2} \sqrt{2ax}\,dx = \frac{2}{3}\sqrt{2a}x^{\frac{3}{2}}]_0^{2at^2} = \frac{8}{3}a^2t^3 \Rightarrow 2A_P = \frac{16}{3}a^2t^3$$

$$\frac{A_P}{A_T} = \frac{\frac{16}{3}a^2t^3}{4a^2t^3} = \frac{4}{3}$$

2. Constructing the general form of the proposition

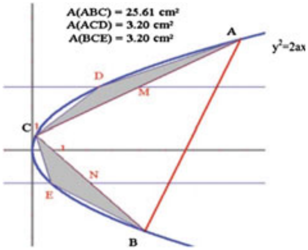
In order to expand our special case to the general case, we drew an arbitrary parabola with an arbitrary segment AB as in the figure in Cabri.

We construct triangle ABC with movable point C in order to search for the largest triangle ABC. Having located point C that maximized the area of triangle ABC, we marked the point C as the vertex. In the midst of this investigation we tried to find an answer to this question: Does the locus of the point C as a vertex have any geometrical property? The answer to this question would be the heart of the investigation. We observed that the point H is the midpoint of the segment AB and the line CH is parallel to X-axis.



(a) Making conjecture

After this observation, we conjectured that the locus of the highest point of the largest triangle in the parabolic segment is on the line passing through the midpoint of the base of the triangle and parallel to the X-axis. We continued to construct two second-tier triangles on the rest of the parabolic segment by using the same conjecture. We observed that the areas of two second-tier triangles are equal.



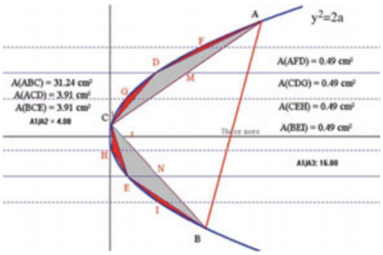
(b) Proving the conjecture

Similarly we continued to construct new triangles on the parabolic segment.

By stretching and shrinking the parabolic segment we got a series from the comparison of the areas of the triangles.

Let the area of the original triangle ABC be $A_T = a$ and the area of the parabolic segment be A_P. Then we got a series for the area of the parabolic segment as:

$$A_P = a[1 + \frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + \cdots + \frac{1}{4^{n-1}}] \Rightarrow A_P = \sum_{k=0}^{\infty} \frac{a}{4^k} = \frac{4}{3}a$$



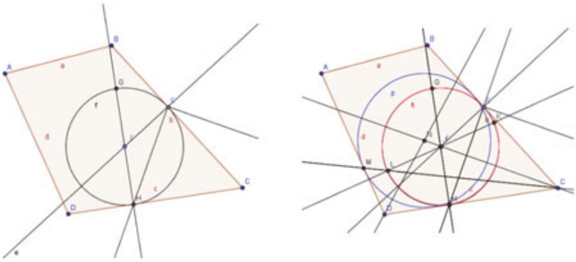
This concluded our proof.

Continuing with GeoGebra

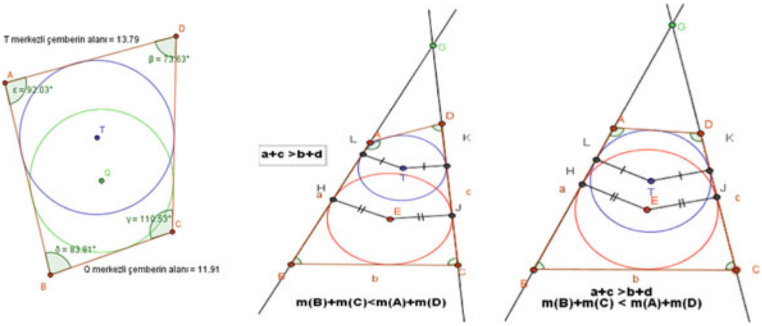
As a final example, I would like to share my individual exploration with GeoGebra as a final example of my presentation. The power of GeoGebra enables mathematicians to manipulate algebraic expressions and to construct geometric figures and drag them dynamically on the computer screen. Through using this powerful software work on the the problem dealing with the inscribing of the biggest circle in polygons. Although Euclidean geometry exists since two thousand years many interesting and challenging problems and theorems still remain to be explored by mathematicians. As a first step of the study I started with the problem stated that "*how to inscribe the biggest circle in a given regular polygon?*"

I easily solved this problem with GeoGebra. Successively, I checked whether if the solution of the initial problem is valid for all convex polygons.

Upon realizing that it is not valid for all convex polygons, then I re-stated the problem as "*how to inscribe the biggest circle in a given convex quadrilateral?*"

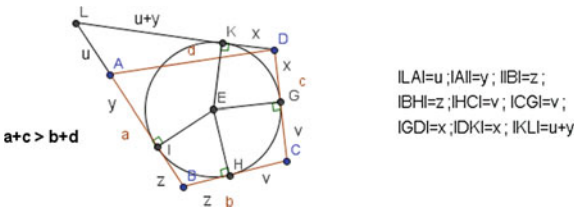


As a result of my investigation, we found that there is an original relationship between a non-regular quadrilateral and the biggest circle inscribed within it.



As a final step of the study I construct a formal proof of this relationship:

Proposition: Let ABCD be a convex quadrilateral, respectively a, b, c and d are length of IABI, IBCI, ICDI, IADI and a + c > b + d. In this case, circles are inside which are tangent to sides of AB, BC and CD or ABAD and CD.



Assert the contrary, let under the condition of $a + c > b + d \Rightarrow (y + z) + (x + v) > (z + v) + d \Rightarrow x + y > d \Rightarrow x + y + u > d + u$. This is a contradiction (according to triangle inequality).

In short, our journey does not end here; we continue to run towards new educational horizons.