Chapter 57
Maximisation and Optimisation of the Total Solar Radiation Reaching the Solar Collector Surfaces

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Abstract The performance of solar conversion systems is affected by its orientation and tilt angle with the horizontal plane. This is because both of these parameters change the amount of solar energy received by the collector surfaces.

In this study, the optimum tilt angle of solar systems is determined in order to increase the solar radiation amount reaching the collector plane. Mathematical models have been used for estimating the total (global) solar radiation on a sloped surface and to determine the optimum tilt and surface azimuth angles in Ghardaïa area, southern Algeria, on a daily basis, as well as for a specific period of the year. Then, these models are compared with experimental data for choosing the best one, through the statistical test calculation.

For this purpose, a database for one complete year of solar radiation components (global, diffuse and direct) has been used.

The results obtained show that the optimum angle for each month allows us to collect the maximum solar radiation; the annual tilt angle is approximately equal to the latitude of the site and the surface azimuth is due south. The loss in the amount of energy between a fixed plane of $32.6°$ and another one adjustable according to the monthly optimum tilt angle is around 10 %.

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Nomenclature

A_l	Anisotropy index in HDKR model
B_n	Beam solar irradiance W/m^2
D_h	Diffuse solar irradiance on horizontal surface W/m^2
f'	Modulating factor in HDKR model
F	Modulating function in Klucher model
f'_1	Circumsolar brightening coefficient in Perez model
f'_2	Horizon brightening coefficient in Perez model
G_h	Global solar irradiance on horizontal surface W/m^2
G_β	Global solar irradiance on inclined surface W/m^2
$G_{\beta,e}$	Estimated global irradiation on inclined surface kWh/m^2/day
$G_{\beta,m}$	Measured global irradiation on inclined surface kWh/m^2/day
I_{on}	Extraterrestrial solar irradiance W/m^2
MBE	Mean bias error
MPE	Mean relative percentage error
R_b	Ratio of the beam radiation on the tilted surface to that on horizontal surface
R_d	Ratio of the diffuse radiation on the tilted surface to that on horizontal surface
RMSE	Root mean square error

Greek Letters

A	Solar elevation angle $^\circ$
ρ	Ground albedo
β	Tilt angle $^\circ$
θ	Incidence angle $^\circ$
θ_z	Zenith angle $^\circ$
δ	Sun declination angle $^\circ$
Ø	Latitude $^\circ$
γ	Azimuth angle $^\circ$
ω	Hour angle $^\circ$

57.1 Introduction

In the recent years, the world is interested by the renewable energies more than before to reduce the greenhouse effect, caused by the absurd exploitation of conventional sources. This interest is resulted in the launch of several ambitious programs in order to develop cleaner energy sources and energy efficiency.

Measurements of global solar radiation reaching the surface of the earth and its two components direct and diffuse are essential in the most research fields of solar energy. The daily values as well as the monthly ones are needed to evaluate the

performance of existing solar devices, and to estimate the efficiency of the future installations [1].

The optimum design of photovoltaic or thermal solar systems for any application is important to determine their performance at the site of installation. The amount of power produced by a solar energy conversion system depends upon the amount of sunlight reaching the collector. So, more light means more power; the performance of a solar collector is highly influenced by its orientation and its angle of tilt with the horizontal. This is due to the fact that both the orientation and tilt angle change the solar radiation reaching the collector surfaces. The best way to collect maximum daily energy is the use of tracking systems, but in most practical cases, the tracker cannot be used due to its expensive cost and for the complication of some requested PV or thermal configuration systems.

In the literature, many researchers in different sites of the world have determined the optimum tilt angle and orientation of solar collectors. In general, the obtained results suggested that the yearly optimum tilt angle is found to be around the latitude of the site, while the optimum azimuth is full south in the hemisphere north (due north in the hemisphere south). A brief state of the art of some works regarding optimum tilt angle is cited below:

Benghanem [2] has optimised the tilt angle for solar panel in Madinah (Saudi Arabia); he has used measurements of daily global and diffuse solar radiation on a horizontal surface; the results gotten showed that the optimum tilt angle for each month allows to collect the maximum solar energy. He found too the loss in the amount of collecting energy when using the yearly average fixed angle of around 8 % compared with the monthly optimum tilt angle.

Ghosh et al. [3] have used isotropic and anisotropic models for estimating the global solar radiation on an inclined plane. They found that the anisotropic model of Perez gives the best result based on a statistical test with an RMSE of 0.09. Thereafter, with this model they determined the optimum tilt and orientation angles for Dhaka (Bangladesh) site. Koray et al. [4] performed the optimum tilt angle in Izmir (Turkey), in order to maximise the solar radiation reaching solar collectors. He has employed the isotropic model of Liu and Jordan to calculate the total solar radiation on inclined surface. His obtained results indicated that the seasonal tilt angle should be $55.7°$ in winter, $18.3°$ in spring, $4.3°$ in summer and $43°$ in autumn. Murat et al. [5] have examined the optimum tilt angle and orientation of PV panels in Sanliurfa (Turkey); they found that the monthly optimum tilt angles change throughout the year with the minimum values obtained in June and the maximum ones achieved in December. Mohd et al. [6] have determined the collector optimum tilt angle and orientation in Brunei Darussalam; they revealed that changing tilt angle 12 times in a year maintains approximately the total solar radiation near the maximum value that is found by changing the tilt angle daily to its optimum value. The yearly gain is 5 % more than the case of collector fixed on a horizontal surface. Morcos [7] studied the optimum tilt angle and orientation in Assiut (Egypt); he noted that changing the tilt angle eight times in a year is necessary to maintain the global solar radiation near its maximum values. He has also found that the gain in global solar radiation is 6.85 % more than the case of collector fixed at the latitude of the site.

The goal of this study is the determination of the optimum tilt angle and orientation in order to increase the amount of solar radiation reaching the collector surfaces on a daily and on a monthly basis as well as for a specific period of the year. Four mathematical models have been tested to calculate the global solar radiation on inclined surface. A case study of a Ghardaïa site (south Algeria) has been taken as an example of application.

57.2 Description of the Site and Data Source

Ghardaïa site is considered an arid and dry area, located in the south of Algeria, about 600 Km south of the capital city (Fig. 57.1); it is framed by the following geographical coordinates: latitude of 32°36′ N, longitude of 3°48′ E and altitude of 450 m above mean sea level [8].

The measured data on solar radiation used in this study were collected by a radiometric station presented in Fig. 57.2 with high precision, installed on the roof of solar radiation laboratory. The station has two parts:

- A fixed part consists of two *EKO MS-64* pyranometers for the measurement of global solar radiation on a horizontal surface (its short-wave sensitivity is 7.0 $(mV/kW/m^2)$) and on inclined surface at the latitude of the site.
- A moving part, which is able to track the path of the sun from sunrise to sunset.

Fig. 57.1 Ghardaïa site location

Fig. 57.2 Radiometric station

The last one consists of an *EKO MS-101D* pyrheliometer with short-wave sensitivity of 6.71 $(mV/kW/m^2)$, which is pointed at the sun disc for measuring the DNI component. Another *EKO MS-64* pyranometer with short-wave sensitivity of 7.0 $(mV/kW/m^2)$ for the measurement of diffuse solar radiation on the horizontal plane is equipped with a shadow band for hiding the radiant flux coming directly from the sun. All solar components are made with an interval of five minutes for each one. Figure 57.3 illustrates the monthly average daily solar radiation components measured in Ghardaïa area.

57.3   Global Solar Radiation on Inclined Surface (G_β)

Several models have been proposed by various researchers to calculate global solar radiation on tilt surfaces from the available data on a horizontal surface. The only difference among the models appears in the assessment of sky diffuse components; the global solar radiation on sloped surface is the sum of three components, direct, diffuse and reflected radiation (Fig. 57.4), and can be expressed as

$$G_\beta = (G_h - D_h)R_b + D_h R_d + G_h R_r \tag{57.1}$$
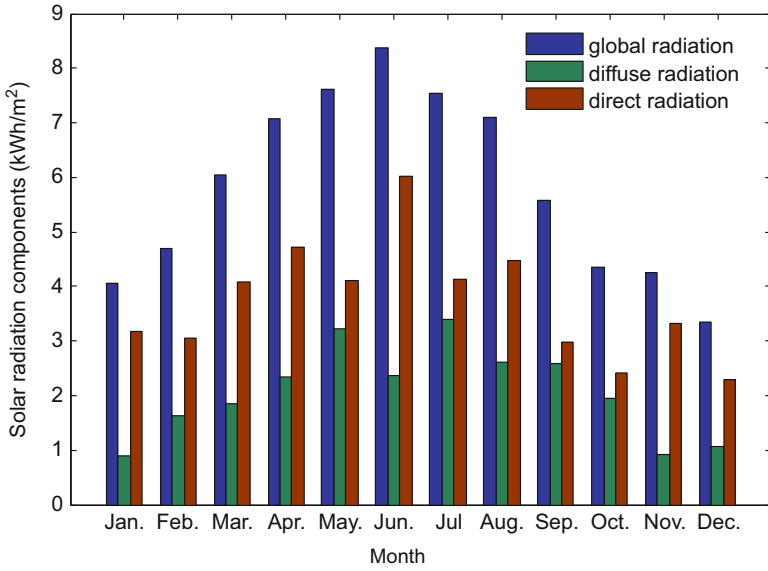
Fig. 57.3 Monthly average global, diffuse and direct solar radiation falling on horizontal surface measured in Ghardaïa site
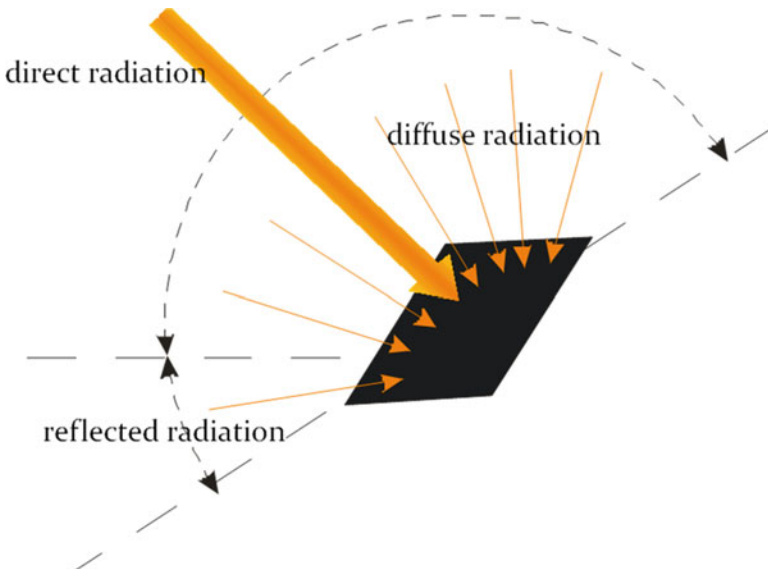

Fig. 57.4 Composition of the global solar radiation on a tilted surface

where R_r is ground reflection term

$$R_r = \rho \left(\frac{1 - \cos\beta}{2} \right) \qquad (57.2)$$

R_b is the ratio of the beam radiation on the tilted surface to that on horizontal surface calculated as

$$R_b = \frac{\cos\theta}{\cos\theta_z} \qquad (57.3)$$

θ and θ_z are calculated by the following equations:

$$\cos\theta = A + B \qquad (57.4)$$

$$A = \sin\delta\sin\varnothing\cos\beta - \sin\delta\cos\varnothing\sin\beta\cos\gamma + \cos\delta\cos\varnothing\cos\beta\cos\omega \qquad (57.5)$$

$$B = \cos\delta\sin\varnothing\sin\beta\cos\omega\cos\gamma + \cos\delta\sin\gamma\sin\beta\sin\omega \qquad (57.6)$$

$$\cos\theta_z = \sin\delta\sin\varnothing + \cos\varnothing\cos\delta\cos\omega \qquad (57.7)$$

Based on the suppositions made by Iqbal [9] and Duffie et al. [12], the estimation models can be classified into isotropic or anisotropic models.

57.3.1   *Isotropic model*

57.3.1.1   **Liu and Jordan model [9]**

This model assumes that the diffuse radiation is uniformly distributed over the sky dome; the radiation tilt factor (ratio of the diffuse radiation on inclined surface to that on horizontal surface) is given by

$$R_d = (1 + \cos\beta)/2 \qquad (57.8)$$

57.3.2   *Anisotropic Model*

These models assume that the anisotropy of the diffuse radiation is in the circumsolar region resulting from forward scattering of solar radiation and concentrated in the part of the sky, while it is isotropically distributed in the rest of the sky dome.

Figure 57.5 shows the schematic distribution of the diffuse solar radiation for both isotropic and anisotropic models.

Fig. 57.5 Isotropic and
anisotropic diffuse solar
radiation [9, 12]
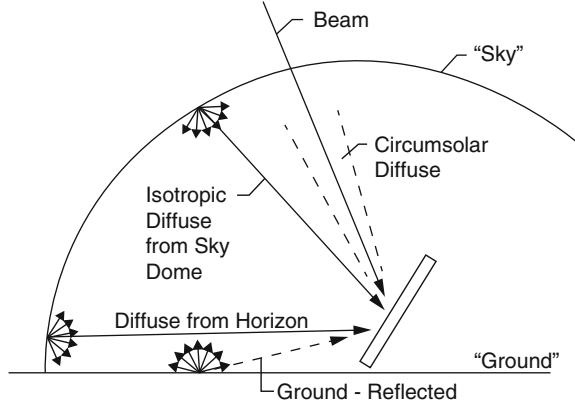
57.3.2.1 Klucher Model [10]

In this model, the condition of cloudiness of the sky has been taken into account; the
diffuse radiation tilt factor is obtained from the relation

$$R_{\mathrm{d}} = \frac{1}{2}(1 + \cos\beta)\left[1 + F\sin^3\left(\beta/2\right)\right]\left(1 + F\cos^2\theta_z\sin^3\left(\frac{\pi}{2} - \alpha\right)\right) \tag{57.9}$$

while

$$F = 1 - \left(\frac{D_{\mathrm{h}}}{G_{\mathrm{h}}}\right)^2 \tag{57.10}$$

57.3.2.2 Perez Model [11]

The diffuse tilt factor in this model is described as

$$R_d = \frac{1}{2}(1 + \cos\beta)\left(1 - F_1'\right) + F_1'\left(a/b\right) + F_2'\sin\beta \tag{57.11}$$

where F_1' and F_2' are circumsolar and horizon brightness coefficients and the values
are presented in a table given by Perez using data for nine stations having various
climatic conditions. a and b are terms that account for the angles of incidence of the
cone of circumsolar radiation on the tilted and horizontal surfaces
($a = \max(0, \cos\theta)$ and $b = \max(0.087, \cos\theta_z)$).

57.3.2.3 HDKR Model [12]

The diffuse tilt factor in the HDKR model is given as follows:

$$R_d = (1 - A_l)\left(\frac{1 + \cos\beta}{2}\right)\left(1 + f'\sin^3\left(\frac{\beta}{2}\right)\right) + A_l R_b \qquad (57.12)$$

The modulating factor f' accounts for the effect of the cloudiness, while the coefficient A_l is the anisotropy index and it defines the portion of horizontal diffuse radiation to be treated as circumsolar:

$$A_l = \frac{B_n}{I_{on}} \qquad (57.13)$$

$$f' = \sqrt{\frac{B_h}{G_h}} \qquad (57.14)$$

57.4 Statistical Test

The accuracy of the estimated models will be judged by the statistical indicators, such as the mean bias error (MBE), root mean square error (RMSE) and mean relative percentage error (MPE). These indicators are usually applied in the comparison of solar radiation models. The mean square error provides information about the performance of correlations, which enable comparison of the real differences between the estimated values and the measured ones; a low RMSE is desirable. The MBE provides the long-term performance of the model; in general, the positive MBE shows overestimation while the negative MBE indicates underestimation. The expression of each statistical indicator is given below:

$$RMSE = \left[\frac{1}{n}\sum_{i=1}^{n}\left(\frac{G_{\beta,e} - G_{\beta,m}}{G_{\beta,m}}\right)^2\right]^{1/2} \qquad (57.15)$$

$$MBE = \frac{1}{n}\left(\sum_{i=1}^{n}\frac{G_{\beta,e} - G_{\beta,m}}{G_{\beta,m}}\right) \qquad (57.16)$$

$$MPE = \left[\left(\frac{G_{\beta,m} - G_{\beta,e}}{G_{\beta,m}}\right)\right]100 \qquad (57.17)$$

57.5 Results and Discussion

Using the equations described above, the global solar radiation falling on an inclined surface has been calculated for different angles of inclination. A program written in Matlab is used for the calculation of G_β for the isotropic and anisotropic models as well as the statistical errors. Measurements of global and diffuse solar radiation on a horizontal surface and the direct normal components with an interval of 5 min for the whole year are used as input parameters. Table 57.1 summarises the results of the statistical comparison obtained from the four tested models. It is clear that the anisotropic *HDKR* model gives least errors monthly and annually. Hence, this model is used therefore for determining the different tilt angles.

In order to determine the yearly optimum tilt angle in Ghardaïa site, the tilt angle has been varied from 0° to 90° in steps of 1°, when the azimuth angle has been varied from −90 to +90 covering all orientations. The optimum tilt angle is corresponding to the maximum of energy; the obtained results show that the yearly angle is around the latitude of the site ($\beta = 32.6°$) and the azimuth angle is due full south ($\gamma = 0°$); this result indicates an optimum fixed tilt throughout a year. Figure 57.6 illustrates the contour of global solar radiation on a sloped surface for

Table 57.1 Statistical comparison for the four studied models

		Statistical indicators	
Model	RMSE	MBE	MPE
Liu and Jordan	0.021	-0.0189	2
HDKR	*0.0216*	*0.0013*	*0.212*
Klucher	0.0344	0.0284	2.69
Perez	0.109	0.0518	5.54
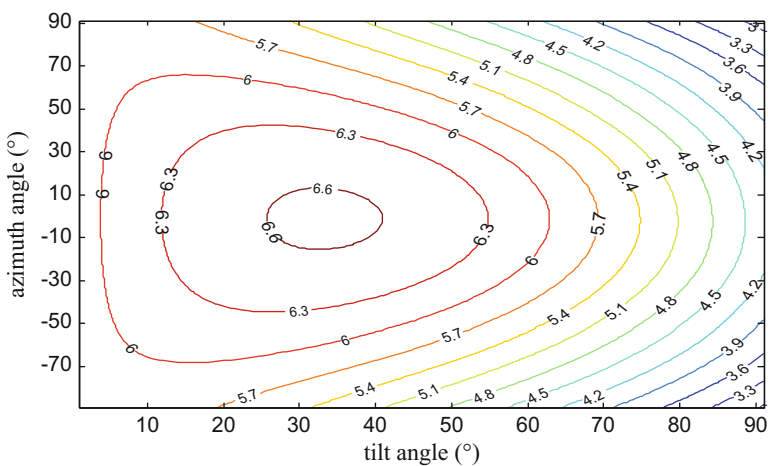
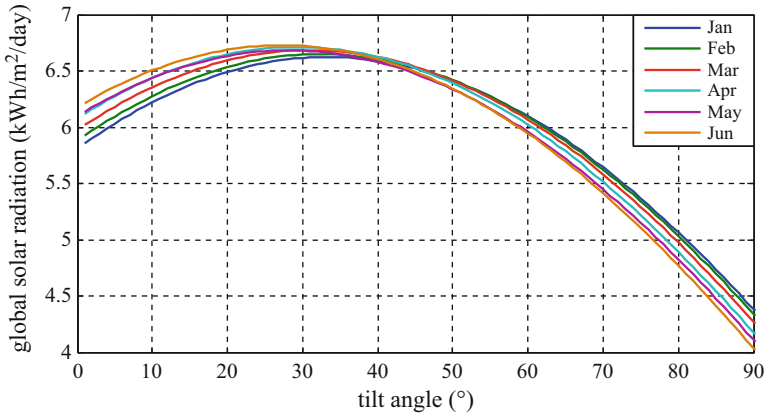**Fig. 57.6** Contour of global solar radiation (kWh/m²) falling on inclined surface

Fig. 57.7 Monthly average daily global solar irradiations for different tilt angles (January–June)
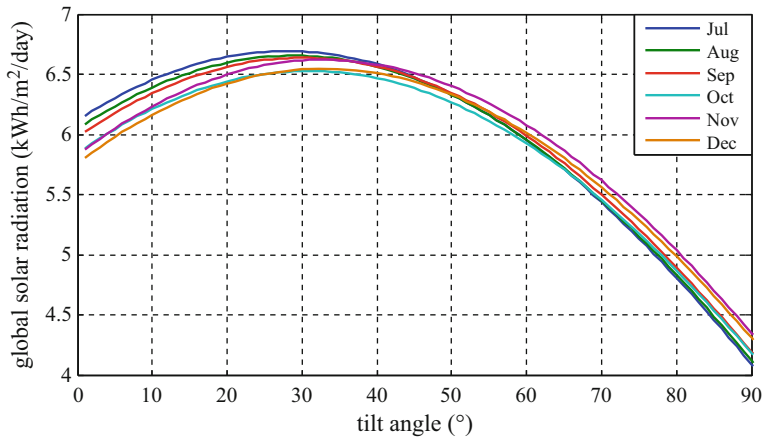

Fig. 57.8 Monthly average daily global solar irradiations for different tilt angles (July–December)

different tilt and azimuth angles; the area of energy peak is found around the latitude tilt angle and the maximum yearly energy in this tilt angle is 6.6 (kWh/m²/day).

Figures 57.7 and 57.8 present the monthly average daily global solar irradiation on a south-facing surface. It is clear from the figures that a unique optimum tilt angle exists for each month of the year that corresponds to the maximum point of each curve. Optimum tilt angle and monthly average daily total solar radiation on an optimum tilted surface are given in Table 57.2. The optimum tilt angle was changing between a minimum of 1° in June and a maximum of 61° in December throughout a year. The loss in the amount of energy between a fixed collector tilted at the latitude of the site and another one adjustable according to the monthly

Table 57.2 Optimum tilt angle and daily average total solar radiation for each month of the year in Ghardaïa site

Month	β (°)	G_β (kWh/m²/day)	G_h (kWh/m²/day)
January	60	7.27	4.07
February	52	7.24	4.69
March	38	7.16	6.04
April	20	7.4	7.08
May	6	7.88	7.61
June	1	8.31	8.38
July	4	7.7	7.68
August	14	6.7	7.17
September	28	6.34	6.06
October	43	5.39	4.36
November	58	7.08	4.24
December	61	5.93	3.34
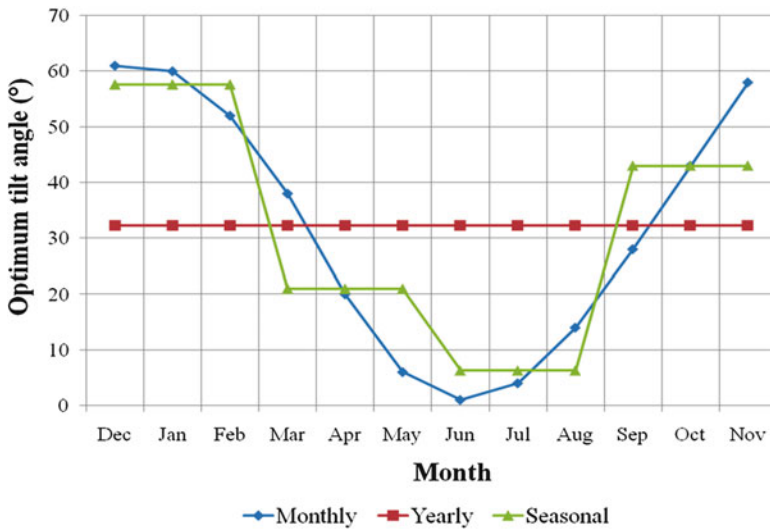
Fig. 57.9 Variation of optimum tilt angle for Ghardaïa site

optimum tilt angle is around 10 %, while the loss is 37 % if the fixed collector tilted at the latitude of the site is compared with another one placed on a horizontal surface, mainly in the winter months.

The seasonal optimal tilt angle was calculated by the same method as the monthly tilt, by looking for the maximum energy for the four seasons of the year. This requires that the collector angle must be changed four times a year, as shown in Fig. 57.9. In winter the optimum tilt angle should be 57.6°, in spring 21°, in summer 43° and in Autumn 6.3°. Figure 57.10 gives the average solar energy collected when the angle of the tilt is optimum, and when the seasonal and yearly average angles are used.
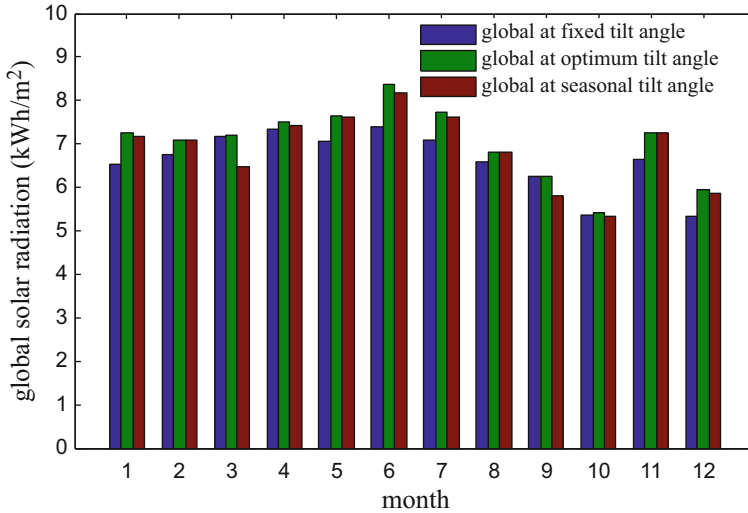
Fig. 57.10 Monthly average daily global solar radiation for optimum, seasonal and yearly tilt angles

57.6   Conclusions

In this study, the optimum tilt angle has been determined in order to increase the amount of total solar radiation reaching solar collectors. Four models have been tested for calculating the global solar radiation on inclined surfaces. The statistical test allowed identifying the HDKR model for the calculation of global solar radiation on inclined surface with the least errors monthly and annually.

The yearly optimum tilt angle is found to be 32.6° and the optimum azimuth is full south which permit to collect the yearly solar energy of 2,424.5 (kWh/m^2/day). The monthly optimum tilt angle has a minimum value of 1° in June and the monthly average total solar radiation at this angle is 8.34 (kWh/m^2/day). Then, the optimum tilt angle reaches its maximum in winter months and has a value of 61° in December which collects 5.93 (kWh/m^2/day) of solar energy monthly; the average optimum tilt angle for the summer months is 6.3° and for the winter months 57.6°. With the seasonally adjusted tilt angles, the yearly collected solar energy was 2,539.69 (kWh/m^2/day).

In the light of the obtained results, users are advised to set their collectors optimally to improve efficiency and get better output of solar collectors.

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