

STRUCTURAL EQUATION ANALYSIS OF THREE METHODS
FOR EVALUATING CONJOINT MEASUREMENT RESULTS

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Abstract

This paper applies a structural equation methodology (LISREL) to the evaluation of the results obtained from conjoint measurement. Contrary to previous work, the results indicated that stress, sign, and prediction give consistent evaluations of the results of conjoint measurement. The solution given by the LISREL analysis was criticized; however, following a Spearman factor analysis, the same conclusion was drawn: stress, sign, and prediction appear to be measuring the same construct.

Introduction

Obviously the best evaluation of conjoint analysis results is the degree to which they are predictive of actual choice behavior. Such an evaluation, however, is not always practical; therefore users of conjoint analysis frequently rely on other methods--such as Kruskal's (1965) stress, counting the number of sign violations (Parker and Srinivasan 1976); Scott and Wright 1976), or rank prediction; i.e., the degree to which judgments are predicted in a holdout profile from part-worth functions obtained in an estimation sample (Acito and Jain 1980; Green 1974; Jain, Acito, Malhotra, and Mahajan 1979). Indeed, the above mentioned evaluation methods are not exhaustive; but, nevertheless they are the emphasis of this paper. The purpose of this paper is to assess the degree to which sign, stress, and rank prediction give consistent evaluation of the results of conjoint analysis.

Prior Research

Recent research by Acito and Jain (1980) was designed to compare the above mentioned evaluation methods: Kruskal's (1965) stress, sign violations, and a modified prediction method proposed by Green (1974). Hereafter, we will refer to these three evaluation methods as "stress", "sign", and "prediction". In their study 249 heads of household were given three conjoint tasks (identified as parts A, B, and C) related to health maintenance organization's HMO's. For each person on each task, the three evaluation measures were obtained. In Task A, respondents were presented 27 profiles that contained various combinations of convenience attributes related to health maintenance organizations. In Task B, respondents were presented 16 profiles that contained combinations of insurance coverage attributes; Task C presented 16 profiles that contained convenience, coverage, cost, types of facility, and attributes related to selecting physicians. During each task, each respondent was required to sort the profiles into three piles according to preference; then to rank order the profiles from least to most preferred. Additionally, the three tasks were presented sequentially--A, B, with C last. The "average" respondent took 10-15 minutes to complete one task, and the three tasks were part of a larger data collection.

Acito and Jain (1980, p. 106) argued that, "Comparison of the (three) methods is important because one needs to know whether one evaluation method will lead to the same

conclusion as another. If the correlations among the evaluation measures are high, the researcher can pick any convenient method. If the correlations are low, the researcher can use more than one method, or use different methods in different circumstances."

The results of their research revealed very low correlations among the three methods; hence, they concluded that stress, sign, and prediction ought to be used for different purposes. This conclusion implies that the three evaluation methods evaluate unrelated aspects of conjoint results; this conclusion, however, should be held with reservation; for, we do not know why the observed correlations were low. One reason for a low correlation between two measures is that they, indeed, do measure two unrelated attributes. However, one may also obtain a low correlation between two measures that measure highly related attributes if the measures are highly contaminated with measurement error. Had Acito and Jain (1980) obtained reliability coefficients for their measures, one could "correct for attenuation"; i.e., one could estimate the correlations between the evaluation methods in the absence of measurement error.¹ The purpose of this paper is to report a new analysis of this research; the results of this analysis suggest that stress, sign, and prediction may be highly correlated when measurement error is removed. This conclusion stems from a structural equation analysis of the correlation matrix reported in [Table 1](#).

A Structural Equation Analysis

The ideal measurement is one which captures true differences in the attribute one desires to measure. Frequently, however, the observed measure may be "contaminated" by both systematic and random error, (cf., Churchill 1979, p. 254; Nunnally 1967, p. 206). Systematic error may derive from a stable but biased measuring instrument or in other stable characteristics of the person. It is a constant source of error. Random measurement error, however, is neither stable nor constant. It may derive from transient influences from both the person being measured as well as the measuring instrument itself. The correlations reported in [Table 1](#) were obtained from observed measurements, hence, the reported correlations may be attenuated by both systematic and random error.² We show below that one source of systematic error is related with the conjoint tasks.

As noted above, each person responded to three different conjoint tasks; hence is it possible that the task

¹Measurement error tends to obscure or "attenuate" the true correlation between two attributes. If the reliability of measures are known, however, then observed correlations may be "corrected" to estimate the true correlation (Nunnally 1976, p. 203-5).

²Although sign and prediction were logarithmically transformed, they are still observed measures; i.e., they may still contain both systematic and random error (see [Table 1](#)).

TABLE 1

Correlation Matrix For Sign, Stress, and Prediction For Three Conjoint Tasks^a

		A			B			C		
		Stress	Sign	Prediction	Stress	Sign	Prediction	Stress	Sign	Prediction
A	Stress	1.00								
	Sign ^b	.15	1.00							
	Prediction ^c	.22	.20	1.00						
B	Stress	.11	.23	.03	1.00					
	Sign	.05	-.01	-.08	.45	1.00				
	Prediction	.08	.08	.08	.25	.07	1.00			
C	Stress	.11	.12	.11	.21	.01	.00	1.00		
	Sign	.00	.07	-.01	.33	.16	.11	.30	1.00	
	Prediction	.14	.20	.02	.29	.05	.07	.29	.35	1.00

^aCorrelation matrix adopted from Actio and Jain (1980, p. 109).

^b"Sign" is the number of total sign violations; each total was transformed $y = \ln(1+x)$ before correlations were computed; maximum number of sign violations for task A = 8, task B = 6, and task C = 5.

^c"Prediction" is the absolute value of predicted minus actual rank; each difference score was transformed $y = \ln(1+x)$ before correlations were computed.

differences had some influence upon the observed measures of stress, sign, and prediction. Further, stress, sign, and prediction may be measuring different but related attributes. An observed measure of, say, stress may be influenced by a factor that is general to stress measurements; by a factor that is general to the conjoint task; and by random measurement error. These hypotheses may be tested by the following measurement model.

$$\begin{aligned}
 k_1 &= \lambda_1 K + \lambda_2 A + \epsilon_{k_1} \\
 k_2 &= \lambda_3 K + \lambda_4 B + \epsilon_{k_2} \\
 k_3 &= \lambda_5 K + \lambda_6 C + \epsilon_{k_3} \\
 s_1 &= \lambda_7 S + \lambda_8 A + \epsilon_{s_1} \\
 s_2 &= \lambda_9 S + \lambda_{10} B + \epsilon_{s_2} \\
 s_3 &= \lambda_{11} S + \lambda_{12} C + \epsilon_{s_3} \\
 p_1 &= \lambda_{13} P + \lambda_{14} A + \epsilon_{p_1} \\
 p_2 &= \lambda_{15} P + \lambda_{16} B + \epsilon_{p_2} \\
 p_3 &= \lambda_{17} P + \lambda_{18} C + \epsilon_{p_3}
 \end{aligned}$$

For this model, let k denote the observed stress score, s denote the observed sign score, and p denote the observed prediction score. Further, let K denote the general stress factor, S the general sign factor, and P the general prediction factor. Epsilon, ϵ , denotes random errors of measurement. The matrix structure of this model is as follows:

$$\begin{bmatrix} k_1 \\ k_2 \\ k_3 \\ s_1 \\ s_2 \\ s_3 \\ p_1 \\ p_2 \\ p_3 \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 & 0 & \lambda_2 & 0 & 0 \\ \lambda_3 & 0 & 0 & 0 & \lambda_4 & 0 \\ \lambda_5 & 0 & 0 & 0 & 0 & \lambda_6 \\ 0 & \lambda_7 & 0 & \lambda_8 & 0 & 0 \\ 0 & \lambda_9 & 0 & 0 & \lambda_{10} & 0 \\ 0 & \lambda_{11} & 0 & 0 & 0 & \lambda_{12} \\ 0 & 0 & \lambda_{13} & \lambda_{14} & 0 & 0 \\ 0 & 0 & \lambda_{15} & 0 & \lambda_{16} & 0 \\ 0 & 0 & \lambda_{17} & 0 & 0 & \lambda_{18} \end{bmatrix} \begin{bmatrix} K \\ S \\ P \\ A \\ B \\ C \end{bmatrix} + \begin{bmatrix} \epsilon_{k_1} \\ \epsilon_{k_2} \\ \epsilon_{k_3} \\ \epsilon_{s_1} \\ \epsilon_{s_2} \\ \epsilon_{s_3} \\ \epsilon_{p_1} \\ \epsilon_{p_2} \\ \epsilon_{p_3} \end{bmatrix} \quad (2)$$

Following the notation set forth by Jöreskog (1966, 1967, 1969, 1973, 1977, and Jöreskog and Sörbom, 1978), we may write Equation 2 as the following:

$$\underline{y} = \underline{\Lambda}_y \underline{\eta} + \underline{\epsilon} \quad (3)$$

where $\underline{y}' = (k_1, k_2, k_3, s_1, \dots, p_3) = (y_1, y_2, \dots, y_9)$ is the vector of observed measures; the matrix $\underline{\Lambda}_y$ is the (9x6) measurement parameter matrix; $\underline{\eta}' = (K, S, P, A, B, C) = (\eta_1, \eta_2, \dots, \eta_6)$ is the vector of factors; i.e., unobserved or latent variables; $\underline{\epsilon}' = (\epsilon_{k_1}, \epsilon_{k_2}, \epsilon_{k_3}, \epsilon_{s_1}, \dots, \epsilon_{p_3}) = (\epsilon_1, \epsilon_2, \dots, \epsilon_9)$ is the vector of random measurement errors. Furthermore, it is assumed that $E(\eta) = 0$ and that the random errors of measurement, ϵ , are uncorrelated with η . It is also assumed that the observed measures, y , are expressed as deviations from their means so their expectations are also equal to zero.

Model Specification

Of particular concern in this analysis are the correlations among the following three factors: stress (K),

³Bagozzi (1980) presents an excellent discussion of structural equation (causal) models. An abundance of marketing examples is also given.

sign (S), and prediction (P). To estimate the measurement parameters as well as the correlations between the three factors, we employed Jöreskog and Sörbom's (1978) analysis of linear structural equations by the method of maximum likelihood (LISREL). The general LISREL model is written as:

$$B \eta = \Gamma \xi + \zeta, \quad (4)$$

where B and Γ are coefficient matrices, η is the vector of factors, as defined above; ξ is a vector of factors that act as independent or exogeneous latent variables, and ζ' = (ζ₁, ζ₂, ... ζ₆) is random vector of residuals; i.e., erros of equation. The vectors η and ξ are not observed; but rather, are derived from the observed measurements. For example, as noted above:

$$y = \Lambda_y \eta + \epsilon.$$

The model we propose does not contain exogeneous latent variables, ξ; hence Equation 4 reduces to:

$$B \eta = \zeta \quad (5)$$

Further, since a causal structure among the η factors is not specified, the coefficient matrix, B, may be set to the identity matrix; thus:

$$\eta = \zeta \quad (6)$$

The correlations between the η factors are then equal to the correlations among the ζ variables. The matrix of ζ correlations is called the Ψ matrix. Additionally, since our model states that η = ζ, Equation 3 may be written as:

$$y = \Lambda_y \zeta + \epsilon, \quad (7)$$

with the following covariance matrix (Jöreskog and Sörbom 1978, p. 40-43):

$$\Sigma_{yy} = \Lambda_y \Psi \Lambda_y' + \Theta_{\epsilon}. \quad (8)$$

The matrix Ψ is the correlation matrix of factors and Θ_ε is the diagonal matrix of unique measurement error variances. Jöreskog and Sörbom (1978, p. 40-43) have termed the above structural equation model a "confirmatory (restricted) factor analysis."

Parameter Estimation

The parameters of Equations 3 and 7 as well as the correlations between stress (K), sign (S), and prediction (P) may be defined in the following three matrices: Λ_y, Ψ, and Θ_ε. The structure of Λ_y is given above in Equation 2; the structure of symmetric Ψ is given below:

$$\Psi = \begin{matrix} & \begin{matrix} K & S & P & A & B & C \end{matrix} \\ \begin{matrix} K \\ S \\ P \\ A \\ B \\ C \end{matrix} & \begin{bmatrix} 1 & \Psi_{12} & \Psi_{13} & 0 & 0 & 0 \\ \Psi_{21} & 1 & \Psi_{23} & 0 & 0 & 0 \\ \Psi_{31} & \Psi_{32} & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{matrix}$$

The Θ_ε diagonal matrix contains the random measurement error variances.

Results

The correlations between factors as well as the obtained measurement parameters are presented in Figure 1. The correlation between the stress and sign factors is .99; between the sign and prediction factors is .94; between the stress and prediction factors is .62. These results suggest that stress, sign, and prediction correlate highly when systematic and random measurement error is removed.

Model Evaluation

One method of testing how well a structural equation model fits the observed correlation (or variance-covariance) matrix, is to compare the correlation (or variance-covariance) matrix generated from the model with the observed correlation (or variance-covariance) matrix. The test of the model is obtained by assessing whether the reproduced correlation (or variance-covariance) matrix is different from the observed correlation (or variance-covariance) matrix. A chi-square goodness-of-fit test is used; for the current model: χ² = 30.6, df = 21, p = .08. A significant chi-square (p < .05) is sufficient to reject the model. A non-significant chi-square (p > .05) indicates the model cannot be rejected (Jöreskog and Sörbom 1978, p. 37). The rationale and mathematics of the chi-square test is given in Jöreskog and Sörbom (1978, p. 13-15). Another indication of goodness of fit is the inspection of the residual matrix obtained from the difference between the observed correlation matrix and the reproduced correlation matrix.⁴ Most all of the residual correlations are near zero in magnitude; the residual matrix is given in Table 2. An additional measure of fit is given by Tucker and Lewis (1973). Their measure, p, is simply the amount of covariation explained by the model divided by the amount of possible covariation to be explained; the coefficient p = .72. The coefficient p is similar to an unadjusted R²; however, the sampling distribution of p is unknown, it is best treated as a descriptive rather than a test statistic (Burt 1973).

Conclusions and Implications for Conjoint Measurement

When systematic and random error are removed, this study demonstrated that stress, sign, and prediction are

⁴ Visual examination of the residual matrix may enable the researcher to determine those observed variables the structural equation model was unable to accurately predict. However, Costner and Schonenberg (1973) have demonstrated that the clues given by the residual matrix may be misleading in certain cases.

FIGURE 1

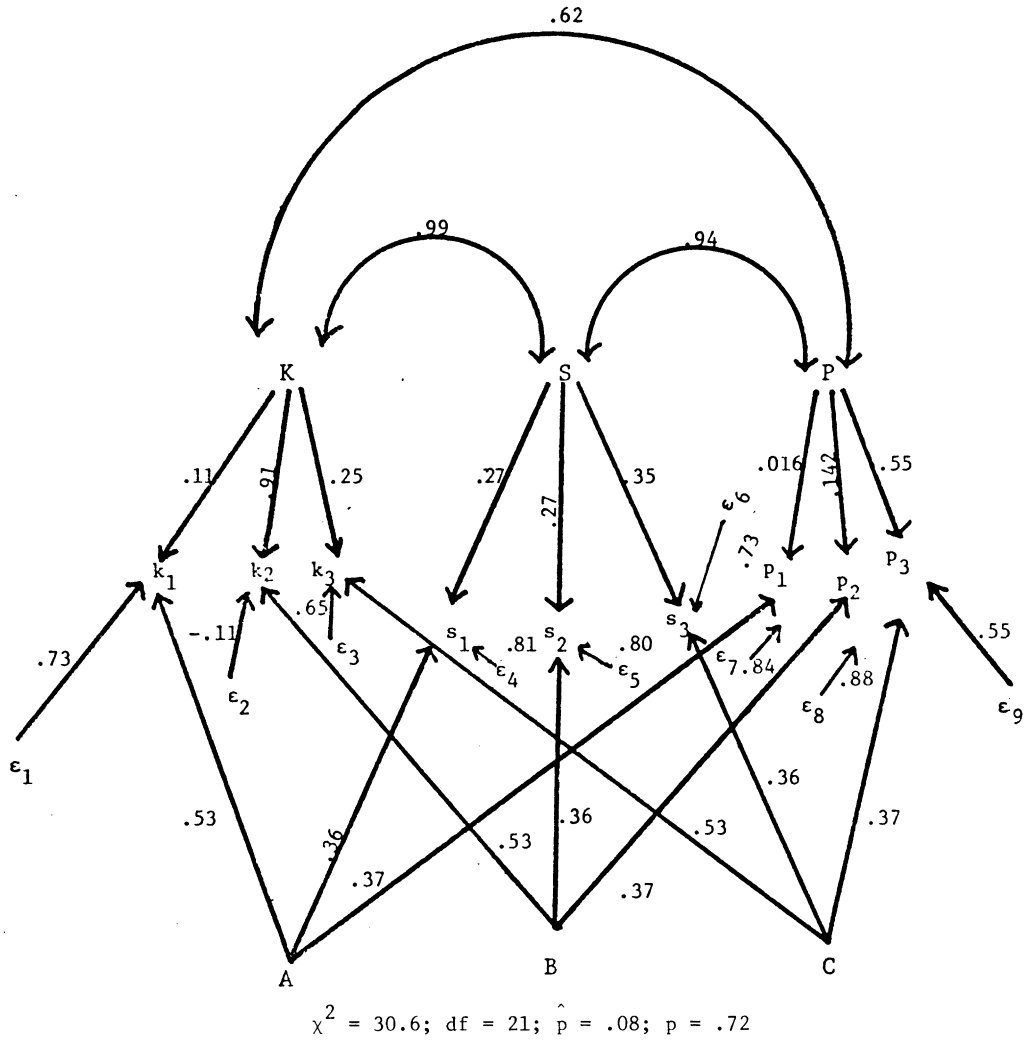


TABLE 2
RESIDUAL CORRELATION MATRIX^a

		A			B			C		
		Stress	Sign	Prediction	Stress	Sign	Prediction	Stress	Sign	Prediction
A	Stress	-.025								
	Sign	-.072	-.012							
	Prediction	.021	.062	.022						
B	Stress	.015	-.016	.021	.003					
	Sign	.022	-.083	-.084	.012	.000				
	Prediction	.071	.044	.078	-.027	-.100	-.034			
C	Stress	.084	.052	.107	-.017	-.058	-.022	.007		
	Sign	-.037	-.025	-.015	.010	.066	.063	.018	.014	
	Prediction	.104	.060	.011	-.020	-.089	-.008	.006	.034	.009

^aThe residual matrix is the difference between the observed matrix and the correlation matrix reproduced by the structural equation model.

highly correlated. Thus, it can be concluded that the low observed correlations given in Table 1 were obscured or "attenuated" by measurement error. Moreover, the original conclusion that stress, sign, and prediction evaluate unrelated aspects of conjoint results must be held with reservation. If the correlations between the stress, sign, and prediction factors, in Figure 1, were near zero--then one could conclude that stress, sign, and prediction were, indeed, measuring unrelated aspects of conjoint results. However, this was not the case.

More importantly, the new analysis presented in this study calls for a greater effort to minimize measurement error in the collection of conjoint data--this is especially important if the results serve as input into managerial decisions. Although measurement error can probably never be totally removed, Nunnally (1967, pp. 222-3) gives insight as to how it may be reduced:

Of course, doing everything feasible to prevent measurement error from occurring is far better than assessing the effects of measurement error after it has occurred. Measurement error is reduced by writing items clearly, making test instructions easily understood, and adhering closely to the prescribed conditions for administering an instrument. Measurement error because of subjectivity of scoring can be reduced by making the rules for scoring as explicit as possible and by training scorers to do their jobs.

Discussion of Results and Additional Comments

This discussion was written some time after the text reported above was written. Since then, we have learned to treat LISREL results with extreme caution. The above discussion of conclusions and implications assumed that the LISREL model was valid; however, this may not be the case. Despite the "good" chi-square result ($\chi^2 = 30.6$, $df = 21$, $p = .08$), a close inspection of the measurement parameters indicate an over-all weakness of the solution.

First of all, note the factor loadings for stress (K), sign (S), and prediction (P). Not only are they inconsistent (vary), but in general, they are not large: $K = .11, .91, .25$, $S = .27, .35$, and $P = .016, .142$, and $.55$. Second, note the negative error variance for $\epsilon_2 (= -.11)$. What is the meaning of a negative error variance? It is true that the square root of a negative variance gives an imaginary standard deviation; but, as far as we know, imaginary standard deviations are unacceptable to statisticians.

With these reservations in mind, the authors sought a "better" solution by modeling alternate structural models: First, we permitted the task factors (A, B, and C) to be correlated; then we relaxed the equality constraints for the task factor loading. Other alternate models were also tried; all produced the same startling result: The chi-square test indicated a near "perfect" fit; but the correlations between that factors (S, K, and P) were all greater than one! Some as high as twenty-five!

What can one conclude from this? First of all, sole reliance on the chi-square test appears to be insufficient when using LISREL for structural equation modeling. This fact has recently been illustrated by Fornell and Larcker (1981). Second, alternate tests of the solution should be pursued. Toward this end, we know of three attempts: One by Fornell and Larcker (1981; see Bagozzi 1981 for criticisms), a new computer package by Schonenberg, (1981) Multiple Indicator Linear Structural Models and a new version of LISREL, LISREL V by Jöreskog

and Sörbom.⁵ The validity of these approaches, however, remains to be demonstrated.

Evaluating The Results of Conjoint Measurement: Another Try

Our dissatisfaction with the outcome of the LISREL analysis prompted us to do a simple Spearman factor analysis (Hunter, 1980). To simplify the analysis, the correlation matrix presented in Table 1 was pooled over the three tasks: A, B, and C. Table 3 presents the pooled (average) correlations; from Table 3, we see that there is a consistent, but moderate, relationship between stress, sign, and prediction. Moreover, if

TABLE 3

Pooled Correlation Matrix and Factor Loadings For Stress, Sign, and Prediction

Measure	Measure			Factor Loading
	Stress	Sign	Prediction	
Stress	(.36)*	.30	.25	.60
Sign	.30	(.25)	.21	.50
Prediction	.25	.21	(.18)	.42

*Diagonal elements in parentheses are the indicator reliability coefficients.

there is a factor common to the three measures, then the following is true:

$$\begin{aligned}
 r_{sk} &= r_s^F r_k^F \\
 r_{sp} &= r_s^F r_p^F \\
 r_{kp} &= r_k^F r_p^F
 \end{aligned}
 \tag{9}$$

That is, the observed correlations may be decomposed as the product of their factor loadings, where r_s^F is the factor loading for stress, and F is the common factor. Equation 9 has a simple solution; the factor loadings may be obtained as:

$$r_k^F = \sqrt{\frac{r_{ks} r_{kp}}{r_{sp}}} = .60$$

$$r_s^F = \sqrt{\frac{r_{ks} r_{sp}}{r_{kp}}} = .50$$

$$r_p^F = \sqrt{\frac{r_{kp} r_{sp}}{r_{ks}}} = .42$$

⁵As of this writing we have learned that LISREL V has been recalled by National Educational Resources, Inc. due to "errors in the program."

We see, therefore, that stress, sign, and prediction are, to some extent, measuring the same thing. The reliability of measurement, however, is somewhat poor. The reliabilities of each measure are reported in the diagonal of [Table 3](#); the reliability for each measure was obtained following Hunter (1980):

$$\text{reliability of stress} = r_{kk} = r_k^F r_k^F = r_k^2 = .36$$

$$\text{reliability of sign} = r_{ss} = r_s^F r_s^F = r_s^2 = .25$$

$$\text{reliability of prediction} = r_{pp} = r_p^F r_p^F = r_p^2 = .18$$

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