New Mode of Instability and Double Mach Reflection in Stationary Supersonic Gas Flow

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1 Introduction

It is known that there are two types of reflection of shock waves: regular (two-shock configuration) and Mach reflection (three-shock configuration). It is believed that in a steady supersonic flow only these two types of reflection are possible [1]. There is a region of double solution where it is possible to get a two-shock and three-shock configurations. This fact explains the so-called hysteresis phenomenon. It has been studied in numerous works, but only for a constant adiabatic index $\gamma = 1.4$ [1] [2] [3].

A new type of reflection, a three shock wave configuration with a negative angle of reflection, has been found in steady supersonic flow at γ less than 1.4 in studies of the dependence of the three shock configuration on the adiabatic indices [4] [5] [6] [7]. Figure 1 shows two configurations: one is the usually depicted one with a reflected wave that is directed upward relative to the free-stream flow. We call it the configuration with a positive angle of reflection (Fig. 1, a). The second, with the reflected wave, directed downward relative to the free-stream flow is the new configuration with a negative angle of reflection (Fig. 1, b).

This configuration has been obtained by the parametric study of the dependence of the location of waves in three shock configuration using three shock theory. The location of shock waves depends on three parameters: the Mach number of free-stream M_1, the incident angle ω_1 and the effective value of the adiabatic index γ. Calculations using this theory were held in a wide range of Mach numbers, angles of incidence and adiabatic indices. It has been shown that at high Mach numbers and small values of the adiabatic index the reflection angle ω_2 is negative. It has been obtained that γ should be less than 1.4. Figure 2 (a) shows the analytically obtained domains and boundaries of the configurations with a negative angle of reflection.

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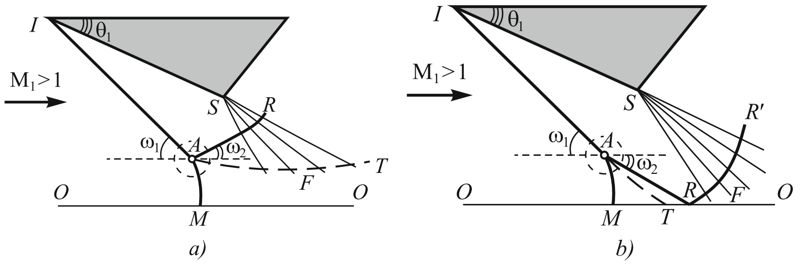

Fig. 1 Three-shock wave configuration at the entrance of an air intake with a positive $\omega_2 >$ (a) and negative $\omega_2 < 0$ (b) reflection angle. IA — incident shock wave, AR — reflected shock wave, AT — tangential discontinuity surface, AM — Mach wave, SF — rarefaction fan, θ_1 — wedge angle, ω_1 — angle of incidence, ω_2 — angle of reflection, A - triple point, OO — line of symmetry, M_1 — free-stream Mach number.

Figure 2 (b) represents the dependence of the reflection angle ω_2 on the incidence angle ω_1 at the constant adiabatic index $\gamma = 1.4$. It can be seen that the values of reflection angle ω_2 lie in the positive range for any free-stream Mach numbers M_1.

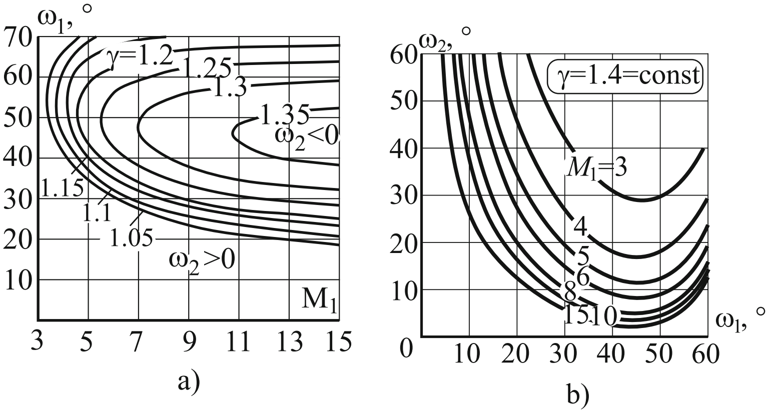

Fig. 2 (a)Boundaries of transition from Mach reflection with a positive reflection angle to irregular reflection with negative reflection angle as a function of Mach number M_1 and incident angle ω_1 at different adiabatic indices γ. Domains with a negative angle of reflection are inside the appropriate curves. (b) Calculated angles of reflection ω_2 as a function of angle of incidence ω_1 in a gas with $\gamma = 1.4$

Thus it has been found that the effective adiabatic exponent effects very strongly influence the structure of three shock wave configurations. The aim of the present paper is to find the dependence of boundaries between regular and Mach reflection on the value of the adiabatic indices, and to describe the general picture of all forms of reflection including the new form.

2 Dependence of the Boundaries between Regular and Mach Reflection on an Adiabatic Index

Boundaries and domains of regular and irregular reflection have been determined by shock polars method [8]. The calculation has been carried out for a range of Mach numbers from 1 to 15, the incidence angles from 0 to 90 deg. and the adiabatic index from 1.05 to 1.66. The transition angle ω_D for the regular reflection has been determined (at greater angle regular reflection can not exist) in dependence on the Mach number of free-stream M_1 and the adiabatic index γ. For irregular reflection critical angle ω_N has been determined, below which the irregular reflection can not exist. The region of dual solution lies between these two curves. It is known that the domain of dual solutions for a constant adiabatic index $\gamma = 1.4$ increases rapidly with increasing Mach number [3]. In this paper it has been shown, that the same relationship exists for the other values of the adiabatic index. The effect of the adiabatic index (at constant Mach number) on the value of the domain of dual solutions has been studied. The calculations show, that with a decrease in the value of the adiabatic index the region of double solution increases.

The boundaries of the regular and Mach reflections are aligned with the boundaries of the existence of a negative angle of reflection. Figure 3 shows the total picture of all types of reflection at the constant adiabatic index $\gamma = 1.2$. It includes all three boundaries of existence: regular reflection (AB), irregular (DC) and the configuration with a negative angle of reflection (EGH). From this figure we can see that a new domain of double solutions BGH has been found except of the domain of double solution AGHCD, where either regular or Mach reflection are possible. In this region either regular reflection or irregular one with a negative angle of reflection are possible. Note that when the adiabatic index is greater than or equal to 1.4, there is only one domain of dual solutions ABCD since at the adiabatic index greater than or equal to 1.4, a configuration with a negative angle does not occur.

3 Numerical Study of the Irregular Reflection with Negative Reflection Angle

The software package STAR-CCM+ has been used for the numerical investigation. The program is based on the Reynolds-averaged Navier-Stokes equations (RANS) using the turbulence model Spalart-Allmaras. Calculations were carried out for a system of two symmetric wedges in supersonic gas flow.

It has been found that there are several forms of reflection with a negative angle of reflection. Analytical calculations of three-shock configuration have provide only the location of shock waves in the immediate vicinity of the triple point, i.e. the initial part of the reflected wave near the triple point in the configuration with a negative angle of reflection must be directed downward relative to the flow (Fig. 1, b). General view of the resulting configuration and the height of Mach stem will depend on the geometry of the system of wedges and the other conditions downstream.

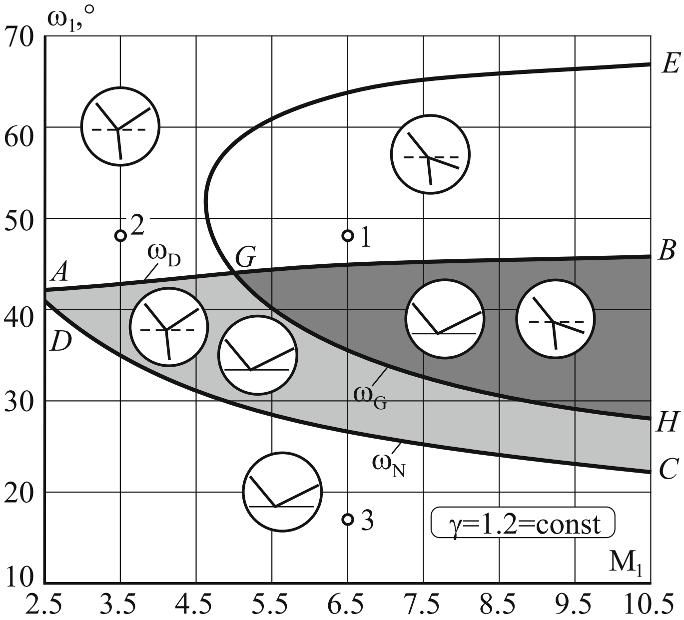

Fig. 3 The curves of the transition angles ω_N, ω_D and ω_G in dependence on the Mach number of flow for adiabatic index $\gamma = 1.2$. AGHCD — domain of dual solution, where both regular and irregular reflection may occur. GBH — domain in which it is possible as a regular reflection and irregular reflection with a negative angle of reflection.

Numerical studies have shown that the forms of three-shock configuration with a negative angle of reflection will depend on the way of transition to this domain: from Mach reflection or from regular reflection. It has been found that if the transition from Mach reflection takes place than irregular reflection with the negative angle and a kink in the reflected wave occurs. In the case of transition from regular reflection the irregular reflection with the second triple point on the reflected wave occurs, i.e. double Mach reflection. Both configurations are unstable. The triple points move upstream. Figure 4 and 5 show the results of calculations of three shock configuration at the same geometry and the same flow parameters (Mach number – 6.5, adiabatic index – 1.2, incident wave angle – 48.3 °), in the domain of the existence of negative angle of reflection (point 1 in the domain EGB, in Fig. 3) . Two completely different results have been obtained. One can come to the point 1 in two different ways, depending on the initial conditions. If we first get stable configuration with a positive angle of reflection (point 2, Fig. 3) and if we change the conditions of the incoming flow so that a configuration with a negative angle of reflection occurs (point 1), then we get three shock configuration with the kink in the reflected wave. In another case, if at the initial time of the calculation there is a regular reflection (point 3) than for rapid transition from regular reflection (changing the angle of the incident wave ω_1) to a configuration with a negative angle of reflection (from point 3 to point 1), we get double Mach reflection (DMR).

Figure 4 shows the configuration with a negative angle of reflection and the kink in the reflected wave at the transition from Mach reflection with a positive angle of reflection to a configuration with a negative angle of reflection. The dots mark the position of the triple point. The triple point moves upstream.

Figure 5 shows the result of the transition from regular reflection to a configuration with a negative angle of reflection. As you can see, there is a double Mach reflection, which leads to disruption of steady state flow.

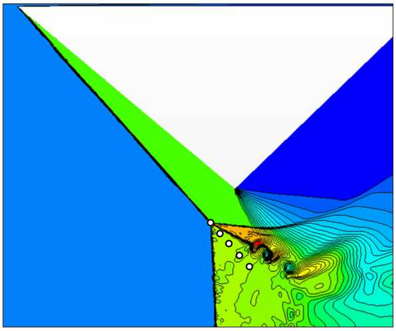

Fig. 4 Thee shock wave configuration with a negative angle of reflection and a kink in the reflected wave. Dots show the location of the sequence of position of the triple point. Mach number M_1 — 6.5, adiabatic index γ – 1.2, wedge angle θ_1 — 40°, incident wave angle ω_1 — 48.3°

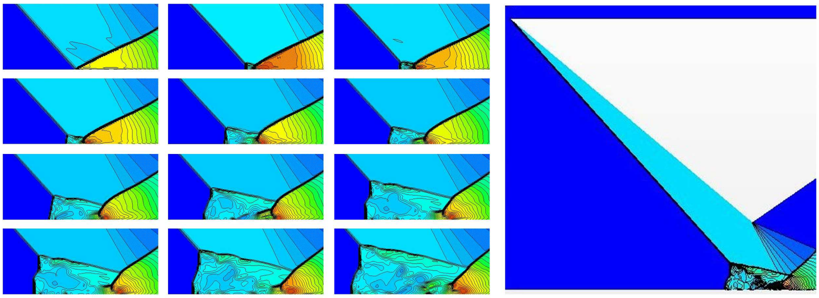

Fig. 5 Sequence of pictures of the unstable double Mach reflection. Right picture - the overall look. Mach number M_1 — 6.5, adiabatic index γ – 1.2, wedge angle θ_1 — 40°, incident wave angle ω_1 — 48.3°

4 Conclusion

Thus, in this paper the complete picture of the different types of reflections is given, depending on the value of the adiabatic index. It has been shown that the domain formerly known as the domain of dual solution, where there is a regular and Mach reflections, increases with the adiabatic index decreases. It was also has been found that at Mach numbers greater than 3 and the adiabatic index less than 1.4, there is another domain of dual solution. In this domain either the regular reflection is possible or configurations with a negative reflection angle.

It has been shown that the configuration with a negative angle of reflection may take different forms depending on the transition. In the transition from Mach reflection a three-shock configuration has a kink in the reflected wave. The reflection with a second triple point on the reflected wave (double Mach reflection) appears in the transition from regular reflection. The behavior of both configurations needs further investigation.

Double Mach reflection was observed previously only in quasi-steady reflection from plane wedges. It is noted that this kind of reflection does not exist in stationary gas flows [1]. The appearance of new configurations may lead to the disruption of the stationary flow pattern. This result may be useful in predicting emergency situations in aircraft flight and rocket engine operation.

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