Chapter 8
Basic Constraints Between Two Vectors

Abstract This chapter deals with the characterization of the basic constraints between two vectors. This issue plays a crucial role in the formulation of constraint equations for mechanical joints. In particular, relations between two parallel and two perpendicular vectors are derived. Moreover, formulation for a vector that connects two generic points is presented. The material described here is developed under the framework of multibody systems formulation for spatial systems.

Keywords Constraints · Parallel vectors · Perpendicular vectors

The main purpose of this section is to provide the fundamental concepts that are necessary to formulate the constraint equations associated with kinematic pairs. For instance, some kinematic constraints require that two vectors remain parallel or perpendicular. On the other hand, it is frequently necessary to define one or more points on bodies. Taking into consideration that points P_i and Q_i represented in Fig. 8.1 are located on body i by their local coordinates, then the corresponding global coordinates can be determined at any given instant by the following expressions (Nikravesh 1988; Schiehlen 1990; Flores et al. 2008)

$$\mathbf{r}_i^P = \mathbf{r}_i + \mathbf{A}_i\mathbf{s}_i'^P \tag{8.1}$$

$$\mathbf{r}_i^Q = \mathbf{r}_i + \mathbf{A}_i\mathbf{s}_i'^Q \tag{8.2}$$

Considering now that points P_i and Q_i are connected by a vector $\mathbf{s}_i$, as Fig. 8.1 shows, then this vector attached on body i can be computed as

$$\mathbf{s}_i = \mathbf{s}_i^P - \mathbf{s}_i^Q = \mathbf{A}_i\mathbf{s}_i'^P - \mathbf{A}_i\mathbf{s}_i'^Q = \mathbf{A}_i(\mathbf{s}_i'^P - \mathbf{s}_i'^Q) \tag{8.3}$$

It should be noted that vector $\mathbf{s}_i$ is only function of the rotational matrix of the body i. This vector has a constant magnitude in any system.

Figure 8.2 depicts a vector that links two points P_i and P_j located on different bodies. Knowing the local components of points P_i and P_j, then the corresponding global coordinates of vector $\mathbf{d}$ can be evaluated as

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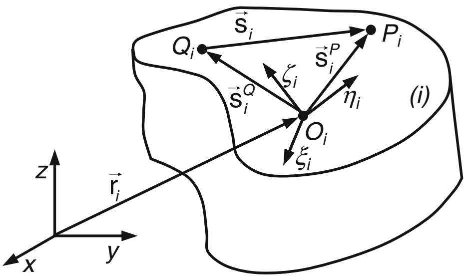

Fig. 8.1 Vector connecting two points P_i and Q_i located on body i

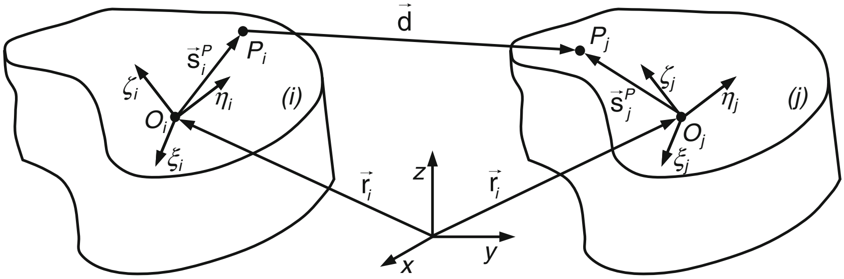

Fig. 8.2 Vector connecting two points P_i and P_j located on different bodies

$$\mathbf{d} = \mathbf{r}_j{}^P - \mathbf{r}_i{}^P = \mathbf{r}_j + \mathbf{A}_j\mathbf{s}_j{}'^P - \mathbf{r}_i - \mathbf{A}_i\mathbf{s}_i{}'^P \tag{8.4}$$

which, in the most general case, can have a variable length.

Figure 8.3 shows two vectors $\mathbf{s}_i$ and $\mathbf{s}_j$ attached to bodies i and j, respectively. In order to specify that these two vectors remain perpendicular at all times, it is necessary to define a constraint equation in the form

$$\Phi^{(n1,1)} \equiv \mathbf{s}_i{}^T\mathbf{s}_j = 0 \tag{8.5}$$

where the superscripts indicate that this is a normal type 1 constraint having 1 equation (Nikravesh 1988). If vector $\mathbf{d}$ in Fig. 8.4, which connects points P_i and P_j, is to remain perpendicular to $\mathbf{s}_i$, then it can be written that

$$\Phi^{(n2,1)} \equiv \mathbf{s}_i{}^T\mathbf{d} = 0 \tag{8.6}$$

which is a normal type 2 constraint having 1 equation (Nikravesh 1988).

For two vectors attached to different bodies to remain parallel, two constraint equations are required. The two constraint equations are derived by setting the cross product between them to zero. Yet, the cross product yields three algebraic equations, of which only two are independent. Thus, two of the equations can serve as

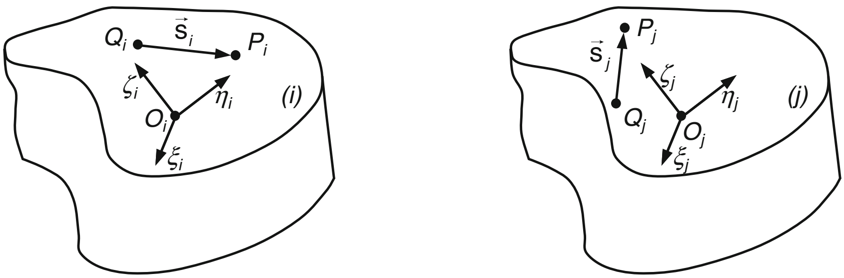

Fig. 8.3 Vectors $\mathbf{s}_i$ and $\mathbf{s}_j$ located on bodies i and j that remain perpendicular at all instants of time

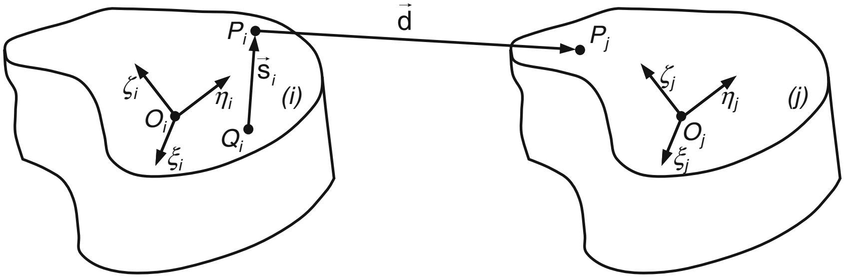

Fig. 8.4 Vector $\mathbf{s}_i$ located on body i that remains perpendicular to vector $\mathbf{d}$ connecting points P_i and P_j

the constraint equations. For two vectors $\mathbf{s}_i$ and $\mathbf{s}_j$ that are embedded in corresponding bodies, as Fig. 8.5 illustrates, the constraint equations imposing parallelism (parallel type 1) are

$$\boldsymbol{\Phi}^{(p1,2)} \equiv \tilde{\mathbf{s}}_i \mathbf{s}_j = \mathbf{0} \tag{8.7}$$

In turn, the condition for two parallel vectors, one fixed to one body and the other connecting two bodies, as it is represented in Fig. 8.6, is written in the following form

$$\boldsymbol{\Phi}^{(p2,2)} \equiv \tilde{\mathbf{s}}_i \mathbf{d} = \mathbf{0} \tag{8.8}$$

It must be highlighted that Eqs. (8.7) and (8.8) provide three equations each. However, the sets of equations each have only two independent equations. There exists a critical case that is associated with selection of two equations from (8.7) and (8.8). This critical case occurs when two vectors become parallel to one of the global axes. To circumvent this issue, the dot product can be used twice. In first place, two perpendicular vectors to $\mathbf{s}_j$, and also to each other, are defined, as it is illustrated in the representation of Fig. 8.7. Then, the two constraint equations can be written in the following form (Nikravesh 1988)

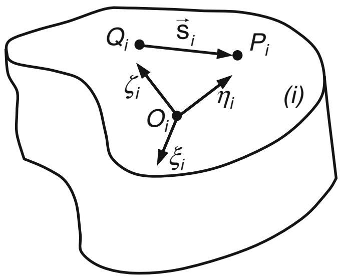

Fig. 8.5 Vectors $\mathbf{s}_i$ and $\mathbf{s}_j$ located on bodies i and j that remain parallel

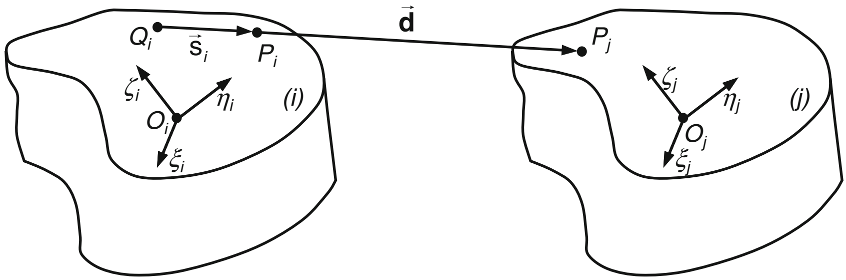

Fig. 8.6 Vector $\mathbf{s}_i$ located on body i that remains parallel to vector $\mathbf{d}$ connecting points P_i and P_j

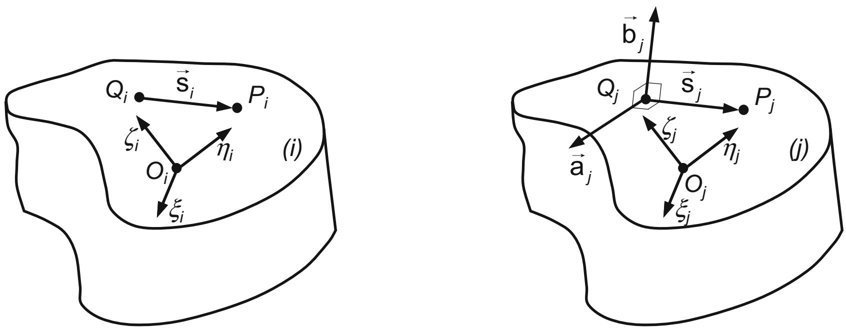

Fig. 8.7 Vectors $\mathbf{s}_i$ and $\mathbf{s}_j$ located on bodies i and j that remain parallel

$$\mathbf{\Phi}^{(p1,2)} \equiv \begin{cases} \mathbf{s}_i{}^T \mathbf{a}_j = 0 \\ \mathbf{s}_i{}^T \mathbf{b}_j = 0 \end{cases} \tag{8.9}$$

Finally, it must stated that the constraints that keep two vectors perpendicular or parallel are the basic constraints between two vectors. These constraints remove relative degrees of freedom between adjacent bodies. For instance, the constraints

$n1$ and $n2$ each remove one DoF from a multibody system. In general, there is no need to use $p1$ and $p2$ constraints, therefore, $n1$ and $n2$ are utilized instead by defining some other appropriate vectors. These basic constraints will be used to construct the constraint equations for a variety of kinematic joints, as it will be seen in the next section. The form of the basic constraints remains the same regardless of the choice of rotational coordinates, in the measure that they only affect the process of evaluating the rotational transformation matrix **A** for each body (Shabana 1989; Schiehlen 1990; Jalón and Bayo 1994).

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