Chapter 6
Vector of Coordinates, Velocities and Accelerations

Abstract This chapter describes the how the vector of coordinates are defined in the formulation of spatial multibody systems. For this purpose, the translational motion is described in terms of Cartesian coordinates, while rotational motion is specified using the technique of Euler parameters. This approach avoids the computational difficulties associated with the singularities in the case of using Euler angles or Bryant angles. Moreover, the formulation of the velocities vector and accelerations vector is presented and analyzed here. These two sets of vectors are defined in terms of translational and rotational coordinates.

Keywords Positions · Velocities · Accelerations

The configuration of a body free in the space is uniquely defined by six coordinates, whereas three coordinates are needed to specify the position of the body and additional three coordinates are required to describe its angular orientation. Throughout this work, translation is described in terms of Cartesian coordinates, while rotation is specified using the technique of Euler parameters. In general, the angular orientation of a body can be described with an arbitrary set of rotational coordinates, such as Euler angles or Bryant angles. However, since these alternative formulations may yield to singularities for critical configurations, Euler parameters are selected for formulating the constraint equations and the equations of motion presented later on in this work (Shabana 1989; Schiehlen 1990).

Thus, the position vector of a body i can be defined as

$$\mathbf{q}_i = \begin{Bmatrix} \mathbf{r}_i \\ \mathbf{p}_i \end{Bmatrix} \tag{6.1}$$

in which vector $\mathbf{r}_i$ contains the three translational coordinates of body i defined with respect to the global coordinates system as

$$\mathbf{r}_i = \begin{Bmatrix} x_i \\ y_i \\ z_i \end{Bmatrix} \tag{6.2}$$

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In turn, the vector $\mathbf{p}_i$ that includes the four Euler parameters can be written in the following form

$$\mathbf{p}_i = \begin{Bmatrix} e_0 \\ e_1 \\ e_2 \\ e_3 \end{Bmatrix} \tag{6.3}$$

For simplifying the formulation at the velocity and acceleration levels, the concept of angular velocity is employed, i.e., velocity vector specifying the translational and rotational velocity of a body i does not correspond to the time derivative of Eq. (6.1), but it is defined as (Jalón and Bayo 1994)

$$\mathbf{v}_i = \begin{Bmatrix} \dot{\mathbf{r}}_i \\ \boldsymbol{\omega}_i \end{Bmatrix} \tag{6.4}$$

where the vector of angular velocities is written as

$$\boldsymbol{\omega}_i = \begin{Bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{Bmatrix}_i \tag{6.5}$$

Accordingly, acceleration vector specifying the translational and rotational accelerations of a body i is defined as

$$\dot{\mathbf{v}}_i = \begin{Bmatrix} \ddot{\mathbf{r}}_i \\ \dot{\boldsymbol{\omega}}_i \end{Bmatrix} \tag{6.6}$$

which corresponds to the time derivative of Eq. (6.4).

It must that be highlighted that alternative arrays for velocity and acceleration equations can be defined as (Nikravesh 1988)

$$\dot{\mathbf{q}}_i = \begin{Bmatrix} \dot{\mathbf{r}}_i \\ \dot{\mathbf{p}}_i \end{Bmatrix}, \quad \mathbf{h}_i = \begin{Bmatrix} \dot{\mathbf{r}}_i \\ \boldsymbol{\omega}'_i \end{Bmatrix} \tag{6.7}$$

and

$$\ddot{\mathbf{q}}_i = \begin{Bmatrix} \ddot{\mathbf{r}}_i \\ \ddot{\mathbf{p}}_i \end{Bmatrix}, \quad \dot{\mathbf{h}}_i = \begin{Bmatrix} \ddot{\mathbf{r}}_i \\ \dot{\boldsymbol{\omega}}'_i \end{Bmatrix} \tag{6.8}$$

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