Chapter 3
Global and Local Coordinates

Abstract This chapter described the global and local coordinate systems utilized in the formulation of spatial multibody systems. Global coordinate system is considered in the present work to denote the inertia frame. Additionally, body-fixed coordinate systems, also called local coordinate systems, are utilized to describe local properties of points that belong to a particular body. Furthermore, the process of transforming local coordinates into global coordinates is characterized by considering a transformation matrix. In the present work, Cartesian coordinates are utilized to locate the center of mass of each rigid body, as well as the location of any point that belongs to a body.

Keywords Global coordinates · Local coordinates · Transformation matrix

Displacements, velocities and accelerations are quantities frequently used to characterize the configuration and motion properties of the multibody systems (Shabana 1989). For this purpose, a proper system of coordinates must be adopted, which includes the global and local systems of coordinates. The expression global coordinate system, which is represented by three orthogonal axes that are rigidly connected at a point called origin of this system, is utilized to represent the global, absolute or inertial frame of reference. In the present work, the global coordinate system is denoted by xyz. In addition, a body-fixed or local coordinate system is considered to define local properties of points that belong to a body. This local system of coordinates is, in general, attached to the center of mass of the bodies and is denoted by $\xi\eta\zeta$. This local coordinate system translates and rotates with the body motion and, consequently, its location and rotation vary with time (Nikravesh 1988).

A free particle i moving in three-dimensional space can be located by three independent variables. Since a particle does not have dimension, it can be seen as a point the three-dimensional Euclidean geometry that can be described by a position vector $\mathbf{r}$ that contains three scalar values as

$$\mathbf{r}_i = \{ x_i \quad y_i \quad z_i \}^T \tag{3.1}$$

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Similarly, a free rigid body i in the same environment can be described by defining the position of one arbitrary point located on the body and the rotation of the body around that point. Therefore, it can be observed that there are infinite similar manners to define the spatial position of a rigid body, where some are more convenient than others (Schiehlen 1990; Jalón and Bayo 1994). The location and orientation of a body can be expressed with respect to any reference system. The generalized coordinates that describe the positions of the elements of a multibody system can be either with respect to each other or with respect to a common reference frame. The representation of a single body i with respect to a globally fixed xyz coordinate system, which is an absolute system, is shown in Fig. 3.1. The body possesses its own coordinate system, the body-fixed coordinate system denoted as $\xi_i\eta_i\zeta_i$. Vector $\mathbf{r}$, as expressed in Eq. (3.1), defines the location of the origin the local coordinate system. This vector represents the translational coordinates of the body. Another set of coordinates is needed to express the orientation of the body with respect to the global coordinate system.

The adequate chosen of the localization of the local coordinate system can simplify the definition of points and vectors on a body. For instance, point P_i located on body i in Fig. 3.1 can be defined from the origin of the local reference frame by vector $\mathbf{s}_i^P$, in such way that its direct location with respect to the global system can be expressed as (Flores et al. 2008)

$$\mathbf{r}_i^P = \mathbf{r}_i + \mathbf{s}_i^P \tag{3.2}$$

The same point can also be described with respect to the local coordinates system by $\mathbf{s}_i'^P$, that contains the local components of vector $\mathbf{s}_i^P$, and, therefore, $\mathbf{s}_i'^P$ is a constant vector for rigid bodies. In turn, the components of vector $\mathbf{s}_i^P$ vary if the body moves. The relation between $\mathbf{s}_i'^P$ and $\mathbf{s}_i^P$ is described by a transformation, which only depends on the relative orientation of the body with respect to the global frame in such way that

$$\mathbf{s}_i^P = \mathbf{A}_i \mathbf{s}_i'^P \tag{3.3}$$

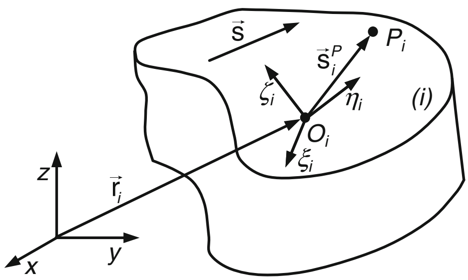

Fig. 3.1 Location of an unconstrained body i in the three-dimensional space

where $\mathbf{A}_i$ is a 3 × 3 rotation matrix that described the orientation of the local coordinate system with respect to the global frame. This rotation matrix introduces a general transformation from local coordinates to global coordinates. The global and local coordinates of the point P_i can be written as

$$\mathbf{s}_i^P = \begin{Bmatrix} x_i^P \\ y_i^P \\ z_i^P \end{Bmatrix} \quad \text{and} \quad \mathbf{s}_i'^P = \begin{Bmatrix} \xi_i^P \\ \eta_i^P \\ \zeta_i^P \end{Bmatrix} \tag{3.4}$$

Matrix $\mathbf{A}_i$ can also be utilized to transform components of vectors, such as vector $\mathbf{s}$ in Fig. 3.1, in a similar manner as

$$\mathbf{s}_i = \mathbf{A}_i \mathbf{s}_i' \tag{3.5}$$

in which $\mathbf{s}_i$ and $\mathbf{s}_i'$ have the following global and local components

$$\mathbf{s} = \begin{Bmatrix} s_x \\ s_y \\ s_z \end{Bmatrix} \quad \text{and} \quad \mathbf{s}_i' = \begin{Bmatrix} s_\xi \\ s_\eta \\ s_\zeta \end{Bmatrix} \tag{3.6}$$

Matrix $\mathbf{A}_i$ can be expressed in terms of direction cosines as

$$\mathbf{A}_i = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \tag{3.7}$$

where the columns of $\mathbf{A}_i$ contain components of three unit vectors $\mathbf{u}_{\xi i}$, $\mathbf{u}_{\eta i}$ and $\mathbf{u}_{\zeta i}$ projected onto the xyz axes, such as

$$\mathbf{u}_{\xi_i} = \begin{Bmatrix} a_{11} \\ a_{21} \\ a_{31} \end{Bmatrix}, \quad \mathbf{u}_{\eta_i} = \begin{Bmatrix} a_{12} \\ a_{22} \\ a_{32} \end{Bmatrix}, \quad \mathbf{u}_{\zeta_i} = \begin{Bmatrix} a_{13} \\ a_{23} \\ a_{33} \end{Bmatrix} \tag{3.8}$$

It should be noted that

$$\mathbf{u}'_{\xi_i} = \begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix}, \quad \mathbf{u}'_{\eta_i} = \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix}, \quad \mathbf{u}'_{\zeta_i} = \begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix} \tag{3.9}$$

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