Chapter 15
Demonstrative Example of Application

Abstract In this chapter a simple pendulum is considered as a demonstrative example of application of the methodologies described in the previous paragraphs. This example allows for the comparison of the different methods to solve the equations of motion in terms of accuracy and efficiency. Finally, the main concluding remarks of the material presented here are summarized and analyzed.

Keywords Application example · Constraints violation · Spatial system

The simple pendulum is made up of one rigid body, which is connected by revolute joint to the ground. This system is modeled with six coordinates and five kinematic constraints, which results in a system with one degree of freedom. Initially, the pendulum is resting in the *xy* plane position, as Fig. 15.1 shows. The system is then released from this initial configuration under the gravity action only, which is taken as acting in the negative *z* direction. The geometric dimension and inertia properties of the simple pendulum are listed in Table 15.1 (Flores et al. 2008).

Long time computational simulations are performed in order to test and compare the accuracy and efficiency of use different methods to solve the dynamic equations of motion. For this purpose, the four approaches are considered, namely the standard method based on the technique of Lagrange multipliers, the Baumgarte method, the penalty method and the augmented Lagrangian formulation. The quantitative measure of the efficiency of these approaches is drawn from the constraint violation as $\mathbf{\Phi}^T\mathbf{\Phi}$, as well as the number of function evaluations and the time consumed during the dynamic simulations. Table 15.2 gives the parameters used for the different models, required to characterize the problem, and for the numerical methods, required to solve the system dynamics. In the present case, there is no need for the initial conditions correction in the measure that the correct initial data can be easily determined.

Figure 15.2 shows that when the standard method is utilized the violation of the constraint equations grows indefinitely with time. In fact, this approach produces unacceptable results because the constraint equations are rapidly violated due to the inherent instability of the equations used and to the numerical errors that develop during computation. In sharp contrast, with other methods the behavior of the simple

P. Flores, *Concepts and Formulations for Spatial Multibody Dynamics*,
SpringerBriefs in Applied Sciences and Technology,
DOI 10.1007/978-3-319-16190-7_15

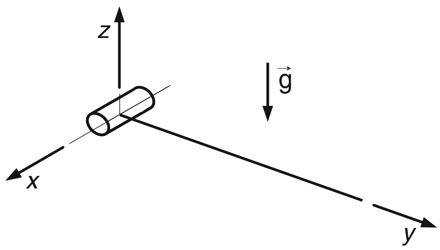


Fig. 15.1 Simple pendulum modeled by one rigid body and one revolute joint

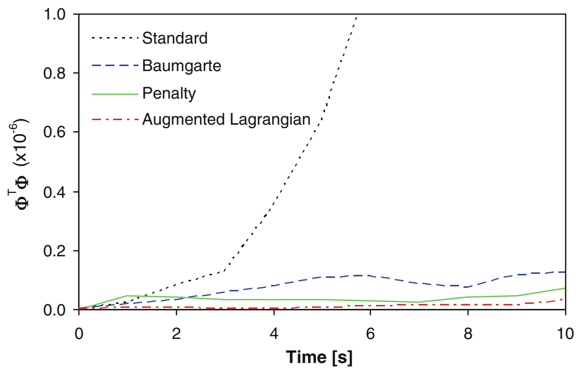
Table 15.1 Governing properties for the simple pendulum

Length (m)	Mass (kg)	Moment of inertia (kgm^2)		
		$I_{\xi\xi}$	$I_{\eta\eta}$	$I_{\zeta\zeta}$
1.0	7.02	0.5855265	0.0010530	0.5855265

Table 15.2 Parameters used for the dynamic simulations

Final time of simulation	10.0 s	Baumgarte-α	5
Integrator algorithm	ode45	Baumgarte-β	5
Reporting time step	0.02 s	Penalty-α	1×10^7
Relative tolerance	1×10^{-6}	Penalty-ω	10
Absolute tolerance	1×10^{-9}	Penalty-μ	1

Fig. 15.2 Constraints violation at the position level



pendulum is different in the measure that the level of the constraints violation is kept under control during the dynamic simulations. Indeed, Baumgarte approach, penalty method and augmented Lagrangian formulation, experience tells that the numerical result does not diverge from the exact solution, but oscillates around it. Magnitude and frequency of the oscillation depend on the values of penalty parameters. Table 15.3 lists the number of function evaluations and the time consumed during the

Table 15.3 Function evaluations and time consumed

Method	Function evaluations	Time consumed (s)
Standard	1639	6.3
Baumgarte	2401	10.7
Penalty	3769	17.2
Augmented lagrangian	3769	17.6

dynamic simulations for the cases described above, which can be used to have a rough idea about the computational efficiency of the different methods.

The fundamentals of the formulation for the dynamics of spatial multibody systems have been presented throughout this work. In first place, the definition of multibody system, made of interconnected bodies that undergo large displacements and rotations, was presented. In addition, basic concepts in multibody dynamics were also introduced. The main types of coordinates that can be used in the formulation of the equations of motion of constrained multibody systems were analyzed, in which their relative advantages and drawbacks were also discussed.

Displacements, velocities and accelerations are quantities frequently used to characterize the motion properties of the multibody systems. For this purpose, a proper system of coordinates must be adopted. In addition, special attention must be given to the selection of the angular coordinates used to describe the orientation of the bodies. In the study, the Euler angles, Bryant angles or Euler parameters were presented. However, due to the singularity phenomenon associated with the Euler angles and Bryant angles, the Euler parameters has been selected as the set of rotational coordinates utilized to define the orientation of the bodies (Nikravesh 1988).

From the mathematical point of view, Cartesian coordinates and Euler parameters are the supporting structure for all methodologies and dynamic analysis developed within the multibody systems methodologies. In the sequel of this concept, the constraint equations associated with the basic kinematic joints, as well as those related to the constraints between two vectors, were presented. In addition, their contributions to the Jacobian matrix of the constraints and to the right-hand side of acceleration constraint equations were studied (Shabana 1989; Schiehlen 1990; Jalón and Bayo 1994).

The formulation of multibody systems adopted in this work uses the generalized coordinates and the Newton-Euler approach to derive the equations of motion. This formulation results in the establishment of a mixed set of differential and algebraic equations, which are solved in order to predict the dynamic behavior of multibody systems. This approach is very straightforward in terms of assembling the equations of motion and providing all joint reaction forces. Additionally, to the standard method based on the Lagrange multipliers technique, three different approaches were presented and utilized to solve the equations of motion, namely the Baumgarte stabilization scheme, the penalty method and the augmented Lagrangian formulation (Baumgarte 1972; Jalón and Bayo 1994).

Under the framework of the multibody systems formulation, some of the fundamental forces that can act upon the bodies were also presented. These forces include the action of gravitational field and forces due to springs, dampers and actuators. Applied forces can be represented by force elements that act on the system components, modifying their dynamic response and the overall motion of the multibody systems. Moreover, the joint reaction forces, expressed in terms of the Jacobian matrix of the constraint equations and a vector of Lagrange multipliers, were analyzed.

In a simple way, the equations of motion for constrained multibody systems are expressed in the Hessenberg form. A set of initial conditions imposed on the positions and velocities is required to start the dynamic simulation. The selection of the appropriate initial conditions plays a crucial role in the prediction of the dynamic response of multibody systems. The subsequent initial conditions, for each time step in the simulation, are obtained from the final conditions of the previous time step. Then, from the initial values for positions and velocities, the equations of motion are solved for accelerations. The positions and velocities at the next time step are then obtained by integration of the velocity and acceleration vectors. This procedure is repeated until the final time of simulation is reached. The integration process can be performed by using a constant step size scheme, such as the fourth-order Runge-Kutta method, or a predictor-corrector algorithm with both variable step and order, such as the Gear method. The fundamental issues related to the numerical integrators used in dynamic analysis of multibody systems were briefly analyzed.

It was demonstrated that the numerical solution of the dynamic equations of motion requires a set of initial conditions on the positions and velocities. Moreover, this system of equations of motion does not use explicitly the position and velocity equations associated with the kinematic constraints. Hence, for moderate and long time simulations, the original constraint equations start to be violated due to the integration process and inaccurate initial conditions. Therefore, a special procedure to avoid this phenomenon was presented, which allows for the correction of the set of initial conditions. The described method is a simple, general and effective to correct the initial conditions at the position and velocity levels is presented. Finally, a simple pendulum was considered as a demonstrative example of application of the methodologies described in the present work. This example was utilized to quantify the accuracy and efficiency of the different methods presented to solve the dynamic equations of motion.

References

Baumgarte J (1972) Stabilization of constraints and integrals of motion in dynamical systems. Comput Methods Appl Mech Eng 1:1–16

Flores P, Ambrósio J, Claro JCP, Lankarani HM (2008) Kinematics and dynamics of multibody systems with imperfect joints: models and case studies. Lecture notes in applied and computational mechanics, vol 34. Springer, Berlin

Jalón JG, Bayo E (1994) Kinematic and dynamic simulations of multibody systems: the real-time challenge. Springer, New York

Nikravesh PE (1988) Computer-aided analysis of mechanical systems. Prentice Hall, Englewood Cliffs, New Jersey

Schiehlen W (1990) Multibody systems handbook. Springer, Berlin

Shabana AA (1989) Dynamics of multibody systems. Wiley, New York