Chapter 11
Force Elements and Reaction Forces

Abstract In the present chapter some of the most relevant applied forces and joint reaction forces are introduced. There are many types of forces that can be present in multibody systems, such as gravitational forces, spring-damper-actuator forces, normal contact forces, tangential or frictional forces, external applied forces and moments, forces due to elasticity of bodies, and thermal, electrical and magnetic forces. However, only the first six types of forces are relevant in the multibody systems of common application.

Keywords Springer-damper forces · Contact forces · Reaction forces

Figure 11.1 illustrates a body i acted upon by a gravitational field in the negative z direction. The choice of the negative z direction as the direction of gravity is totally arbitrary. However, in the present work, the gravitational field will be considered to be acting in this direction unless indicated otherwise. If w_i is the weight of the body i, resulting from the product of mass of the body by the gravitational constant, then the contribution of this force to the generalized vector of forces of body i is given by (Nikravesh 1988; Shabana 1989)

$$\mathbf{g}_i^{(g)} = \{0 \quad 0 \quad -w_i \quad 0 \quad 0 \quad 0\}^T \tag{11.1}$$

Consider a single body force $\mathbf{f}_i$ acting on body i at point P_i, as shown in Fig. 11.2a. This force has three Cartesian components. In addition, a moment with respect to the body center of mass must be computed as (Jalón and Bayo 1994)

$$\mathbf{n}_i = \tilde{\mathbf{s}}_i^P \mathbf{f}_i \tag{11.2}$$

Thus, the contribution to generalized vector of forces of a single force is

$$\mathbf{g}_i^{(f)} \equiv \begin{Bmatrix} \mathbf{f}_i \\ \mathbf{n}_i \end{Bmatrix} \tag{11.3}$$

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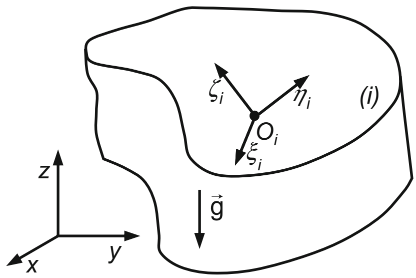

Fig. 11.1 Gravitational field acting on a body i

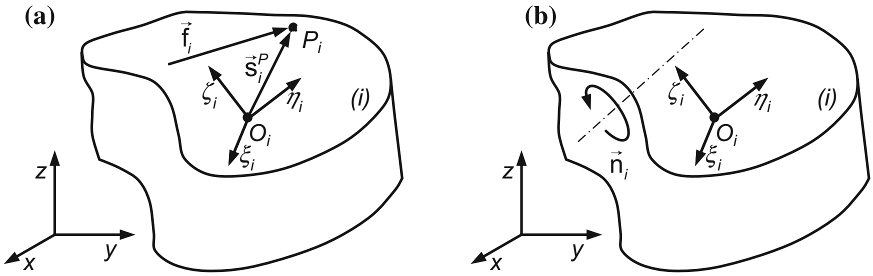

Fig. 11.2 A body i acted upon by **a** a single force **b** a pure moment

When a pure moment with magnitude n_i acts on a body i, as shown in Fig. 11.2b, its contribution to the vector of forces of body i is given by

$$\mathbf{g}_i^{(n)} \equiv \begin{Bmatrix} \mathbf{0} \\ \mathbf{n}_i \end{Bmatrix} \tag{11.4}$$

Figure 11.3 shows a spring-damper-actuator element connecting bodies i and j through two points of connectivity P_i and P_j. The vector $\mathbf{l}$ that connects the points P_i and P_j can be evaluated by

$$\mathbf{l} = \mathbf{r}_j^P - \mathbf{r}_i^P = \mathbf{r}_j + \mathbf{A}_j\mathbf{s}_j'^P - \mathbf{r}_i - \mathbf{A}_i\mathbf{s}_i'^P \tag{11.5}$$

The magnitude of this vector is

$$l = \sqrt{\mathbf{l}^T\mathbf{l}} \tag{11.6}$$

The unit vector along the spring-damper-actuator element is defined as

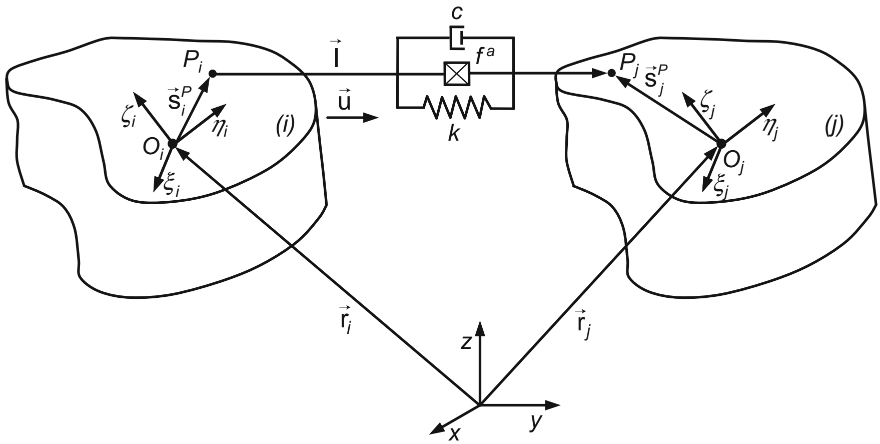

Fig. 11.3 Spring-damper-actuator element connecting bodies i and j

$$\mathbf{u} = \frac{1}{l}\mathbf{l} \tag{11.7}$$

The time rate of change of the damper length can be obtained by differentiating Eq. (11.6), yielding

$$\dot{l} = \frac{1}{l}\mathbf{l}^T\dot{\mathbf{l}} \tag{11.8}$$

where $\dot{\mathbf{l}}$, in turn, is found from Eq. (11.5)

$$\dot{\mathbf{l}} = \dot{\mathbf{r}}_j^P - \dot{\mathbf{r}}_i^P \tag{11.9}$$

Then, the resulting spring-damper-actuator force is evaluated as

$$f^{sda} = k(l - l^0) + c\dot{l} + f^a \tag{11.10}$$

where the first term on the right-hand side is the spring force, the second term represents the damper force and the third term denotes the actuator force. The spring stiffness is represented by k, l is the deformed length, l^0 is the undeformed or natural length of the spring, c is the damping coefficient of the damper and $\dot{l}$ is the time rate of change of the damper length.

The forces that act on the bodies i and j can be evaluated as

$$\mathbf{f}_i^{sda} = f^{sda}\mathbf{u} \quad \text{and} \quad \mathbf{f}_j^{sda} = -f^{sda}\mathbf{u} \tag{11.11}$$

Finally, the contribution to generalized vector of forces is given by

$$\mathbf{g}_i^{(sda)} \equiv \left\{ \begin{array}{c} \mathbf{f}_i^{sda} \\ \tilde{\mathbf{s}}_i^P \mathbf{f}_i^{sda} \end{array} \right\} \quad \text{and} \quad \mathbf{g}_j^{(sda)} \equiv \left\{ \begin{array}{c} \mathbf{f}_j^{sda} \\ \tilde{\mathbf{s}}_j^P \mathbf{f}_j^{sda} \end{array} \right\} \tag{11.12}$$

As it was presented previously, the joint reaction forces and moments are expressed in terms of the Jacobian matrix of the constraint equations and a vector of Lagrange multipliers and expressed by Eq. (10.10). Thus, for instance, for a spherical joint between bodies i and j, the vector of reaction forces is expressed as

$$\mathbf{D}^T \boldsymbol{\lambda} = \begin{bmatrix} -\mathbf{I} \\ \tilde{\mathbf{s}}_i^P \\ \mathbf{I} \\ -\tilde{\mathbf{s}}_j^P \end{bmatrix} \boldsymbol{\lambda} \tag{11.13}$$

Equation (9.5) has been considered in Eq. (11.13). For a spherical joint, $\boldsymbol{\lambda}$ is a 3-vector representing exactly the reaction force acting at point P_i. The same force but in the opposite direction acts at point P_j, as it is shown in Fig. 11.4. It must be noted that a spherical joint does not produce a reaction moment. However, when a reaction force is moved to the corresponding mass center, the moment associated with that force must be included in the rotational equations of motion. These reaction moments are automatically taken care of by the Jacobian matrix (Nikravesh 1988).

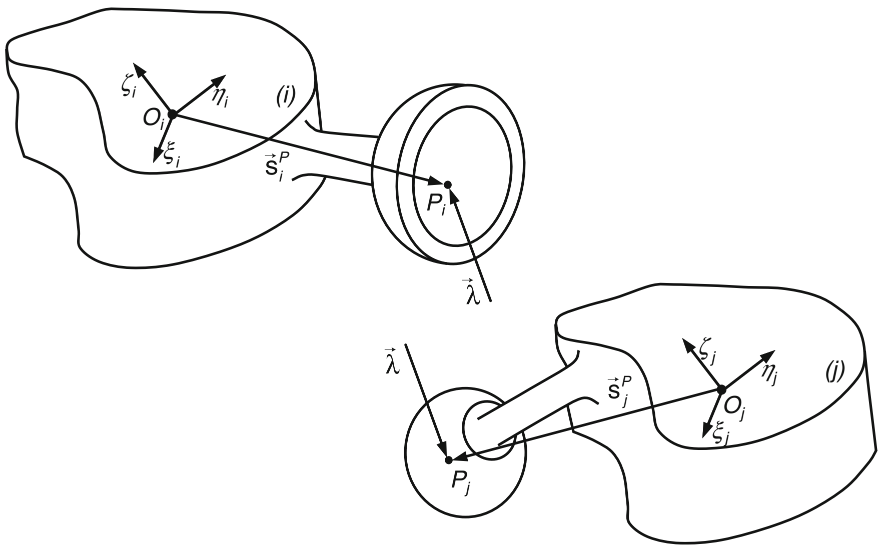

Fig. 11.4 Reaction forces associated with a spherical joint

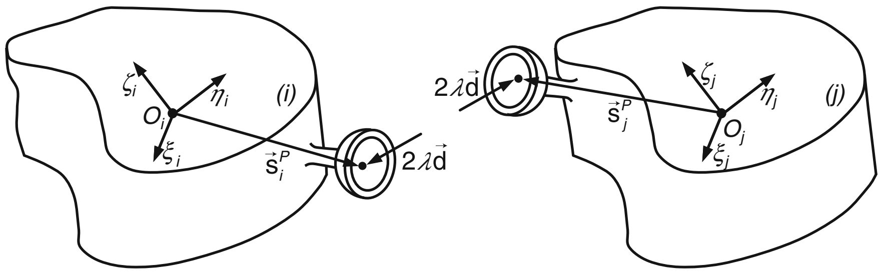

Fig. 11.5 Reaction forces associated with a spherical-spherical joint

For the case of a spherical-spherical joint, the reaction force can be expressed in the form

$$\mathbf{D}^T\boldsymbol{\lambda} = \begin{bmatrix} -2\mathbf{d}^T \\ 2\mathbf{d}^T\tilde{\mathbf{s}}_i^P \\ 2\mathbf{d}^T \\ -2\mathbf{d}^T\tilde{\mathbf{s}}_j^P \end{bmatrix}\boldsymbol{\lambda} = \begin{bmatrix} -\mathbf{I} \\ \tilde{\mathbf{s}}_i^P \\ \mathbf{I} \\ -\tilde{\mathbf{s}}_j^P \end{bmatrix} 2\mathbf{d}\lambda \tag{11.14}$$

in which Eq. (10.23) has been employed. This composite joint contains a single Lagrange multiplier which its value is proportional to the magnitude of the reaction force. This reaction force acts exactly along the axis of the link that defines the joint, as it is illustrated in Fig. 11.5. The reaction moments are the result of the reaction forces having arms with respect to their corresponding center of mass (Schiehlen 1990).

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