Dynamical Characterization of Typical Mexican Colonial Churches

Fernando Peña and Julio Manzano

Abstract The dynamic characterization of ancient masonry churches built in the colonial era in Mexico is evaluated by means of a case study. Seismic assessment of old masonry churches is not an easy task, mainly due to the fact that the churches are large structures with complex geometries and built with materials with a highly nonlinear behavior. The case study concerns a typical church in Central Mexico. The aim of the characterization was to develop a methodology suitable for assessing the seismic vulnerability of this type of constructions. The study is based on the so-called complementary analysis approach. Thus, three steps were performed for the needs of assessment: (a) *preliminary studies*, which include the essential historical information, documentation of materials and geometry through survey of the structure and non-destructive testing; (b) *calibration and validation of numerical models*, based on the results obtained from the previous step; and (c) *dynamic characterization*. By combining the results of different types of models and analysis, this approach allows to overcome the complexity of the study of the seismic behavior of ancient masonry structures. The chapter is based on a case study but also provides a very relevant discussion on an issue that has received insufficient attention: the seismic analysis of masonry structures without a box behavior. From the results obtained, it was identified that roof system cannot be considered a rigid diaphragm. The belfry and the roof (dome and vaults) are the most critical elements. The church is very rigid, since the lateral wall drifts are very low. Thus out-of-plane and shear behavior in walls are not critical behaviors in this particular case.

Keywords Dynamical characterization · Colonial churches · Historical masonry · Seismic vulnerability · Seismic safety

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1 Introduction

Earthquakes have been one of the main causes of destruction of architectural built heritage. Ancient stone masonry structures frequently show extensive cracking after being subjected to earthquakes even of rather moderate intensity. As an example, after a magnitude 7.0 earthquake in central Mexico in 1999, 1542 colonial churches developed damage, in a radius of more than 100 km from the epicenter. In most cases the damage was rated as light, consisting mainly of flexural cracking in the vaults and bell towers.

Careful seismic assessment of the architectural heritage affected by severe earthquakes is one of the most effective ways to understand the structural weaknesses of those constructions. It is also necessary to assess the interventions made in the past, in order to identify those that developed successful behavior and may be used, from those that are unsafe. In this context, the dynamic characterization of structures is in general a useful tool for the evaluation of seismic safety.

Churches built in the colonial era in Mexico, between the 16th and the 18th centuries, vary in size and architectural styles. However, churches of a particular zone follow some basic typologies (both architectural and structural), which depend mainly on the seismic zone where they were built [14, 18]. An important factor which influenced the architectural style was the experience of the ancient builders owing to the seismic activity of the country. Generally, in the Pacific's coast the recurrent destruction of the first constructions caused an evolution of the churches towards a shorter height with big buttresses and not much outer ornamentation. On the other hand, in other regions where the seismic activity is smaller, the churches remained taller and more slender (see Fig. 1). Therefore, by case studies it is possible to extrapolate the results to other similar churches.

The seismic activity of Mexico can be divided in four big zones: A, B, C and D (Fig. 2; [4]). These zones are delimited based on the frequency of occurrence of the earthquakes in the different regions and the expected maximum ground acceleration in a century. Zone A is the region where no earthquakes have been reported in the last 80 years and ground accelerations are not expected to exceed 10 % of the acceleration of gravity. Zone D identifies the places where earthquake occurrence is very frequent and the ground accelerations can be higher than 70 % of the gravity acceleration. It is here where the historical big earthquakes have been registered. The B and C zones are intermediate regions where earthquakes are not registered frequently and the ground accelerations recorded are not greater than 70 % of gravity.

In 1999, the center and southwest of Mexico were affected by two earthquakes (15 June and 30 September) of intermediate magnitude (M = 7.0). The Tehuacan earthquake of June 15th was a deep-seated intra-plate earthquake, having its epicenter inland [20]; whereas the earthquake of September 30th (Oaxaca earthquake) was a normal faulting event and had its epicenter in the Pacific's coast [8]. These earthquakes caused significant damages in masonry structures, including more than 1800 historical monuments. Façades and bell towers were the most frequently

Fig. 1 Typical colonial churches built in: **a** low seismic risk zone (central Mexico), **b** high seismic risk zone (Pacific coast)

Fig. 2 Seismic zonation of Mexico [4]

damaged elements of churches and the belfries were the most common elements to collapse.

In this context, in recent years the study of the seismic behavior of old masonry churches becomes an important issue in Mexico [14, 18, 21]. By means of the dynamic characterization is the first step to understand the seismic behavior and

vulnerability of this type of construction. The dynamic characterization should be based on a theoretical (structural analysis) and experimental base (non-destructive test). But, in the first place, it should respect the integrity of the artistic and architectural values of the building. In this context, the methodology used must be compatible with the ISCARSAH principles [9, 10]. A specific methodology for historic buildings in each country is needed—in this light, the study of particular example cases will support the development of a methodology for Mexico.

The subject of the paper is the dynamic characterization of typical colonial churches of central Mexico. This typology corresponds to the churches built in particular in the region of Mexico city, located around 400 km from the subduction zone, but which suffers earthquakes frequently. The study was confined to the study of the "Saint Bartholomew—The Apostle" church. This church was chosen for its historical and architectural significance within the area of study.

1.1 Typical Colonial Churches

Thousands of churches were built in Mexico from 16th to 18th century, and survive to date in rather good condition. They vary in size and architectural sophistication, but follow some basic typologies. The simplest among them are rather small parochial churches which are found in every barrio of Mexican towns and villages. One important factor that has influenced the evolution of their architectural features has been the experience of damage suffered from earthquake activity. It is in the area of the Pacific Coast where the recurrent destruction of the early construction produced an evolution towards low rise, heavily buttressed buildings with scarce external ornamentation. In other regions the lower concern for earthquake failures favored taller and more slender constructions, as for example in the center of the country.

The typical colonial church in the central Mexico has a Latin-cross floor plan. It has one or two bell towers. Roof is constituted by a quadripartite vaulting system. A hemispherical dome is placed over the transept bay and it is supported by the drum. Small buttresses are placed along the main nave (Fig. 3). In the first bay there is an intermediate floor for the chorus. The façade is composed by the main wall, which is connected with the towers and the frontispiece. The façade has, generally, two openings: the main door and a window for illumination of the chorus, which can be rectangular or circular. The height of the door is between 5 and 8 m.

The bell tower can be divided in three different parts: the body of the tower or basement; the belfry which can be made with one, two or three bodies; and a small hemispheric dome at the top of the bell tower. In plan, the body of the towers is like a square pipe, which is attached to the walls of the façade and the nave. Small openings can be found in the basement, which have the function to illuminate and ventilate the indoor of the basement where the stairs to the belfry are located. In general, it is common that the belfries present two or three bodies, the top being

Fig. 3 Typical colonial
church of central Mexico
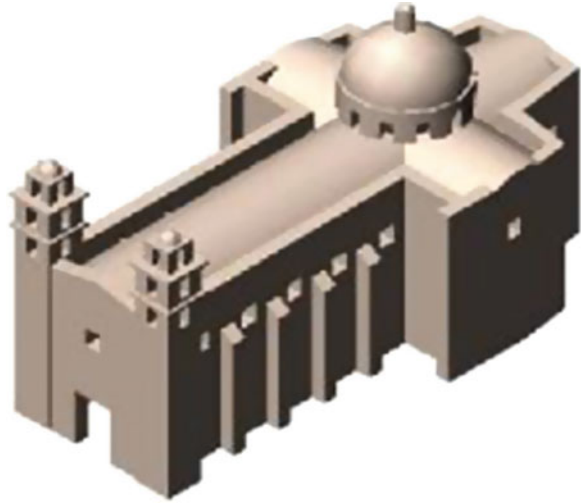
smaller than the others. Sometimes the third body is more similar to a lantern like element.

The main material of construction is a heterogeneous masonry constituting of stones connected with a lime-sand mortar (rubble masonry). Frequently, broken clay bricks or light-weight volcanic stones were added to the masonry. This heterogeneous masonry is a kind of pozzolanic concrete whose composition varies according to the structural element. This characteristic allowed building the structural elements as a monolithic, one leaf element. Thus no planes of weakness occur in the interior of the walls due to the different layers of the masonry (see Fig. 4). These characteristics allow modeling the material as homogeneous and isotropic [5, 15].

Walls comprise stone masonry, while vaults, domes and arches are generally built with bricks. Rarely, stone ashlars were used for structural elements. In few cases brick masonry was not only used in the roof, but also in the façade, columns, buttresses and basement. Adobe was widely used for early churches, but in most cases these buildings were soon destroyed by earthquakes and were subsequently replaced by stone masonry constructions.

1.2 Typical Earthquake-Damage Patterns

Observation and evaluation of seismic damage suffered by typical stone masonry historic churches have enabled researchers to identify the basic modes of failure and main characteristics of seismic response: behavior is governed by the low tensile strength of constituting materials, which makes it almost impossible to secure continuity within and between structural members, and leads to specific mechanisms for resisting the seismic actions.

Fig. 4 Detail of a typical colonial masonry wall and arch

Vaults, arches and domes are very efficient for resisting their self-weight, but they are very sensitive to opening at their support, which introduces tensile stresses and could produce extensive cracking. Due to the height of the temples, supporting walls and columns experience significant lateral displacements at the top, during strong ground shaking, thus generating longitudinal cracks in vaults and cracks along the meridians in domes (Fig. 5). In most cases, this cracking does not endanger the stability of the roof, giving rise only to a redistribution of stresses. Nevertheless, when large displacements take place, the shape of the elements could change to a less stable configuration and progressively lend to collapse, especially because of the cumulative effect of subsequent earthquake motions. In this regard, domes are more sensitive than vaults to motions of their supports, which can be enhanced by flexibility of their drums. Vertical ground accelerations, which are
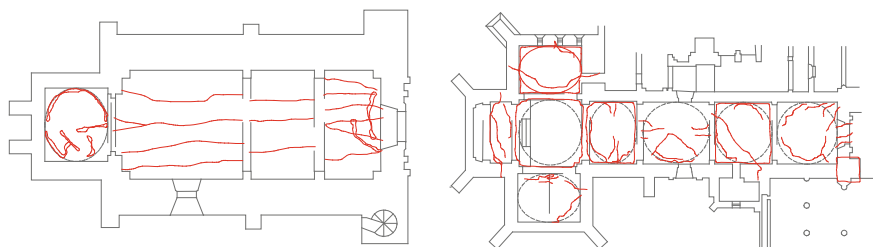

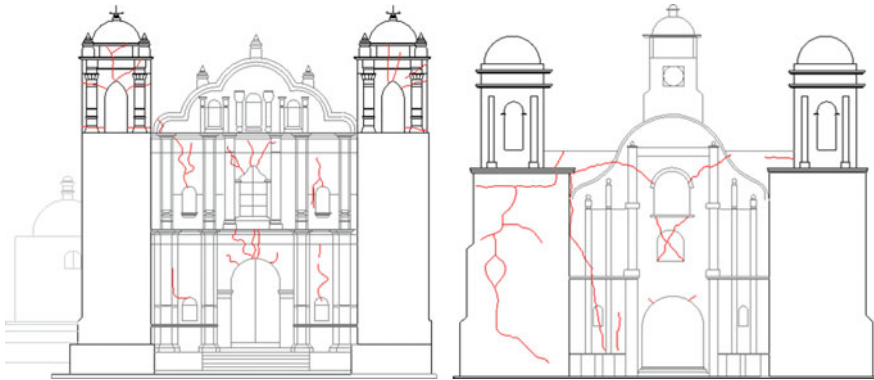**Fig. 5** Typical earthquake damages in domes and vaults

Fig. 6 Typical earthquake damages in Façades and towers

significant in epicentral areas, play an important role in the damage of the roof, mainly because they tend to increase the thrust of the roof elements at the supports, consequently aggravating their opening and condition out-of-plumb.

Bell towers are rather slender and weak elements, in which the ground motion is greatly amplified. Even if they are relatively low and sturdy, their failure is rather common in temples of the most seismically active areas, especially in the vertical elements and in the arches surrounding the belfry (Fig. 6). Additionally, their bending motion tends to separate them from the rest of the church or to generate shear cracking in the lower body of tower.

A façade is typically a tall and heavy wall connected to the rest of the temple. The key damages have shown that the main behavior of these elements is in the plane of the façade (Fig. 6). The out-of-plane behavior is generally less important and is only observed with the detachment of the façade from the nave, but without reaching the state of collapse [1]. This is mainly due to the presence of the towers, built integrally with the façade and not simply constructed in compliance. In fact, on the one hand these towers provide a constraint that reduces the movement out-of-plane, but also they increase the in-plane actions, and are also subjected themselves to a combination of bending and torsional distress. On the other hand, the rigidity of the vaults in the longitudinal direction coupled with the rigidity of the apse reduce the possible thrust effect of the vault over the façade. The slab of the chorus restrains the wall and reduces the façade slenderness. Thus, shear actions have a significant role in the damage patterns, especially in the joints between the façade and towers [3].

At the other end of the church, the apse has a shape that renders it generally stiffer and less vulnerable than the main façade; sometimes, it develops damage due to the thrust of the dome. Longitudinal walls of the temple do not commonly show damage due to in-plane forces. Their out-of-plane stiffness is greatly enhanced by rather heavy buttressing. Nevertheless, it is frequent to notice some out-of-plumb derived from the thrust of the roof during their vibration under seismic ground motion. Subsequent earthquakes produce a cumulative effect in this regard.

2 Description of the Methodology

Since the structural damage of a historic building has consequences not only in terms of cost or loss of life, but also as a loss of cultural and heritage significance, the study of the structural safety of this type of construction is vital. Thus, it is necessary that the structural design is based on the *full understanding* of the structural behavior of the building. Likewise, the diagnosis and assessment of safety should be based both on historical information and on appropriate criteria for the conservation of historic buildings. These criteria should explicitly recognize that conservation of architectural heritage should focus on three key points: the security of people, safeguarding the intrinsic value of the building and the current and future use of construction.

Moreover, as the aim of an intervention of a historic building is to safeguard the intrinsic values of the building and not only the structural safety of the construction, it is necessary that the structural analysis should not be based only on numerical analysis. It must be based on a more complete analysis that enables and highlights the understanding of the inner workings of the building, also accounting for the evidence regarding the behavior of the building during past events. Thus, the structural assessment should be based on historical information as well as on qualitative and quantitative methods. Qualitative methods are based on direct observation of structural damage and degradation of materials, as well as on historical research. Quantitative methods are mainly based on testing of materials and the overall structure, monitoring and structural analysis.

In this context, the proposed methodology is based on a stepped strategy named *Complementary Analyses Approach* (CAA) [17, 19]. This strategy helps to overcome the difficulties inherent to the analysis of historic structures, through a series of levels or steps. The user ought to bear in mind that there is no unique strategy and the steps must be selected according to the needs of the case study in hand. Therefore, the correct selection of the different steps must be based on the knowledge of the structure, the theoretical framework of restoration and on the cultural, social and economic aspects.

2.1 Intrinsic Values of a Historical Construction

The cultural, social and economic aspects to take into account in the selection of the steps are referred to as the *intrinsic values* of the structure. These are defined as the tangible and intangible values of the structure which characterize the importance, greatness and especially the uniqueness of the construction. They must be safeguarded in time and should not be modified by a conservation or intervention project. These intrinsic values are summarized below:

Architecture: It includes the architectonic style, space distribution, uses and changes made along the history of the building.

Art: The artistic value lie in the architecture, as in all the other elements that give an identity to the building, including: ceilings, columns and ornaments on walls, floors, etc; frescoes and wall paintings; furniture, art objects, etc.

History: This is an intangible value and does not refer only to the date of construction or the *age* of the structure. It must include all the events *lived* by the building (as for example: to be the house of a historical personage, a place where some treaty was signed, the building of the first hospital, etc.).

Economic: The historical buildings and centers are, in general, a main tourist attraction of the cities or countries. This creates direct or indirect economic benefits.

Engineering: The structural solution of an historical building is another value. These solutions highlight the ancient building techniques and construction materials used in the past. The structures of the monuments are, undoubtedly, an historical document of the abilities of the ancient builders.

2.2 Theoretical Framework of the Conservation

Intervention measures should be based on the causes and should aim to resolve the source of the problem. Thus it is not correct to only remedy the *symptoms*. Therefore, each intervention should be established according with the safety criteria and try to minimize the possible damage to the intrinsic values. The selection of a reinforcement technique should be based on the principles of conservation and modern criteria for analysis and restoration of historic structures, which are stipulated in the Venice Charter [9] and in ISCARSAH ICOMOS criteria [10]. These principles are:

1. **Respect for structural authenticity**. The structural design of historic buildings is part of their cultural value, so it is necessary to preserve both the original system and its materials.
2. **Minimum impact or minimum alteration**. Interventions that cause minimal alteration or impact to the original structural system should be preferred, but they must provide an adequate level of safety.
3. **Structural safety**. For valuable monuments, interventions should consider the artistic and cultural losses that the building may experience in case of structural damage.
4. **Compatibility**. The materials and technical devices used to repair or to reinforce must be compatible with the original materials. That is, their use should not result in an undesirable effect on the structure. The ancient materials should not

experience any kind of chemical deterioration when in contact with new materials (chemical compatibility), and new materials should not undergo chemical or physical phenomena that can cause damage to existing materials.

5. **Non-intrusiveness or non-invasiveness**. Only those repairs that that are the least-invasive should be preferred, as this helps to preserve the integrity of structures (first requirement). So, from among several alternatives, preference should be given to solutions that present minimal invasiveness.
6. **Reversibility and removability**. Whenever possible, measures should be reversible. Thus, it should be possible if necessary, to dismantle the original material or structure back to the state it was before, without damage or permanent alteration. A less rigid requirement is the removability limiting lasting deterioration in the original construction. The reversibility or removal opens the possibility to reset or change in the future, strengthening by a more appropriate or effective technique.
7. **Monitorability**. It must be possible to control the intervention during its execution. Measures that are impossible to control should not be allowed. Any proposal for intervention should be accompanied by a program of monitoring and control.

2.3 Selected Steps

Taking into account the intrinsic values of the church and the basic guidelines for conservation, the selected steps applied to the case study were as follows:

Step 1 **Preliminary studies**: At this step the preliminary studies of the selected temple were performed. The information collected included a historical study and the geometrical, architectural and structural surveys.

Step 2 **Studies for the calibration and validation of the structural models**: An ambient vibration tests were performed in order to obtain the dynamic characteristics of the temple. It was necessary to have a preliminary numerical model in order to design and perform correctly this test. With the numerical model it was possible to identify critical areas of the structure, as well as to define the correct position of the equipment used in the experimental tests. Also, the numerical models were calibrated and validated at this step with the results of the ambient vibration tests.

Step 3 **Dynamic characterization**: With the numerical models calibrated and validated the dynamic characterization of the temple was carried out. This included the identification of the mechanical characteristics of the materials and the seismic behavior, as well as the identification of critical elements such as appendices or weak elements.

3 Step 1. Preliminary Studies

In this step, all information available for further studies was collected. This included, among others: historical information, materials and geometry description, architectural and structural survey, preliminary studies, etc. Historical information was very important, because it allowed understanding of the structural behavior of the building through time. It was also necessary to have a good description of the temple; since in general, the geometry of historical buildings could be quite complex. Often there is not a clear distinction between structural and architectural elements.

The architectural and structural survey of the temple was most important information to be collected. With this information, it was possible to carry out preliminary numerical modelling of the structure based on the geometric information obtained from surveys and nominal properties for the materials found in the literature.

3.1 Historical Study

The St. Bartholomew the Apostle church is located at the north of Mexico city. In the colonial era, it was common to establish Indian towns around religious centers, in order to facilitate the missions. In this way, the village of St. Barth was founded, which soon became an important town. The date of construction of the church is not known, but there are some administrative records from 1679 [12]. This means that its construction should be between the late 16th and early 17th century. The church was founded by the Franciscan order and has a modest and simple architecture without great decorations, according to the canons of the Franciscan Order [2].

The church has been struck by several seismic events of moderate and high intensity. Due to this, the temple has been repaired and modified over time. Unfortunately, there are no detailed historical records about these changes. However, by means of a photographic record dating in the 20th century [11] was possible to identify some important changes.

For example, Figs. 7 and 8a show the façade now and in the middle of the 40s. Comparing both figures it is possible to observe significant changes in the façade. The right side window was expanded. The frontispiece is different, although there is evident effort to preserve the original style. The buttress on the right side of the main door is new. Figure 8b seems to be from the 50s of the past century. Compared with Fig. 8a, it is seen that the frontispiece is the same but without the pinnacles. The right side door and the buttress of the façade were already built but with a different architectural style.

It has been reported that in the second half of 1960, the temple underwent several damages. The main façade was at risk to collapse. Several structural and architectural elements partially collapsed. The frontispiece collapsed. Due to these

Fig. 7 Actual view of the church

Fig. 8 View of the Façade in the of the past century [11], decade of: **a** 40s; **b** 50s

collapses, the temple was reinforced and partially reconstructed. Thus, vaults were rebuilt with concrete and reinforced with concrete ribs. Several walls were reinforced, the belfry and the chapel and chorus slabs were rebuilt with concrete. The frontispiece and the portal were totally reconstructed with reinforced concrete elements.

3.2 Architectural and Structural Survey

The church has a Latin-cross floor plan, oriented in the East-West direction. The main nave has a length of 31.5 m and a width of 7 m. The transverse nave is 14×8 m^2. In the first bay there is an intermediate floor for the chorus. The roof is constituted by a barrel vaulting system with a thickness of 20 cm. A hemispherical dome is placed over the transept bay and it is supported by the drum. The average height of dome is 13.95 m. The church has a bell tower, 13.75 m high. The tower has a rectangular section of 3.5×4.2 m^2 in plan. The thickness of the walls varies from 0.85 to 1 m. Trapezoidal buttresses are placed along the longitudinal walls of the nave and in the apse. Figure 9 shows the photographic survey performed.

The main material of construction is heterogeneous masonry constituting monolithic structure made of stones and lime-sand mortar. Frequently, broken clay bricks or light-weight volcanic stones were added to the masonry. This heterogeneous mass constitutes a kind of concrete whose composition varies according to the structural element; it is lighter than normal stone masonry, and has a tensile strength higher than brick masonry, mainly owing to the absence of planes of weakness that would normally be formed by mortar layers [15]. Walls are made of stone masonry, while vaults and the dome were rebuilt with concrete.
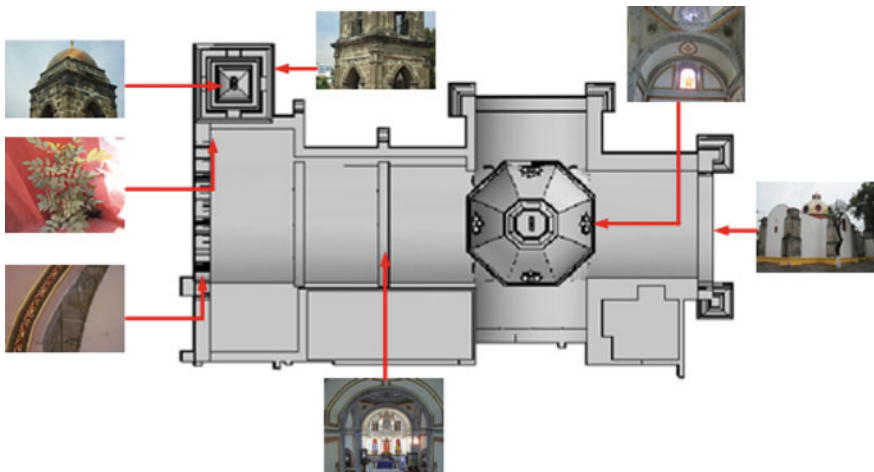

Fig. 9 Photographic survey

3.3 Numerical Model

Based on the information collected, a finite element model was assembled (see Fig. 10), using rectangular solid eight-noded elements. The computer model comprised 22,509 elements, 96,066 degrees of freedom and 33,650 nodes.

Three types of materials were considered in the model. Walls, columns, buttresses, tower base and arches are modelled with properties of stone masonry. The belfry was also modelled as masonry, but having different properties from the walls, in consideration of the fact that it encompasses embedded concrete and steel reinforcement. Finally, the vaults, the frontispiece, ribs and dome were considered of reinforced concrete. Table 1 shows the mechanical properties of the materials used in the model, which were obtained from the literature [5].

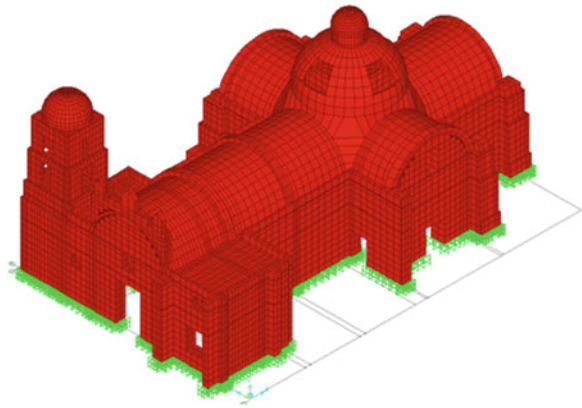**Fig. 10** Finite element model of the church

Table 1 Mechanical properties of the materials [13]

Material	Compressive strength (MPa)	Tensile strength (MPa)	Elastic modulus (MPa)	Density (kg/m^3)
Wall masonry	1.5	0.12	0.60	1600
Belfry masonry	3.0	0.24	0.25	1800
Concrete	20	2.10	21.7	2400

3.4 Preliminary Results

A modal analysis was performed. This analysis allows firstly to check the numerical model, but it may also be used to design the ambient vibration test (see Sect. 4.1). Table 2 shows the frequencies and percentages of modal mass for the first five modes. The structure is very rigid, as the first frequency is 3.66 Hz. The first two modes correspond to the vibration of the tower, which is the most flexible part of the church (Fig. 11a). The third mode is the first mode in the transverse direction with a frequency of 5.05 Hz (Fig. 11b). The fourth and sixth modes correspond to

Table 2 Dynamic properties

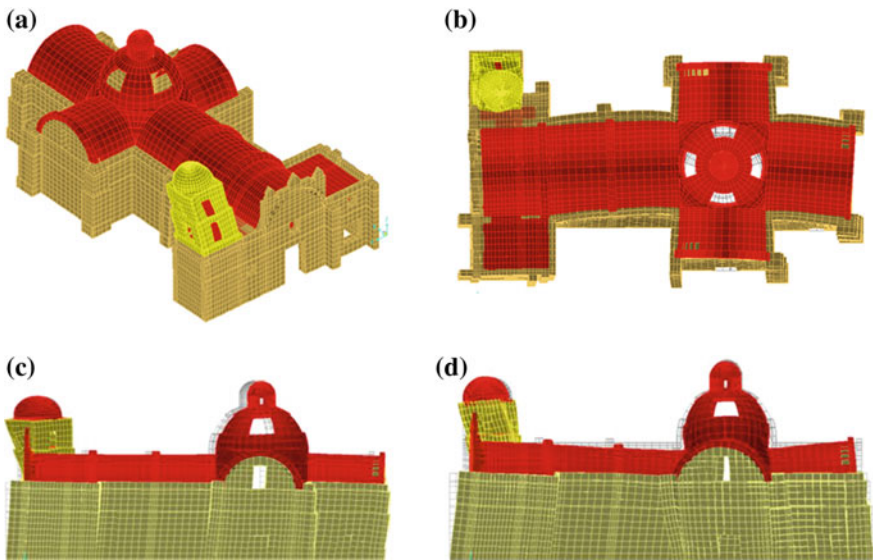| Mode	Frequency (Hz)	Percentage of modal mass (%)		
		Longitudinal	Transverse	Vertical
1	3.66	2.2	1.9	0.0
2	3.71	1.3	3.7	0.0
3	5.05	0.1	55	0.0
4	6.41	1.8	5.7	0.0
5	6.66	62	0.0	0.0
6	7.05	0.8	2.5	0.1
7	7.17	0.2	1.0	0.0
8	7.27	4.8	0.1	0.0
9	8.07	0.0	0.6	0.5
10	8.46	1.1	0.8	0.7

Fig. 11 Modal shapes of the uncalibrated model: **a** first mode; **b** third mode; **c** fifth mode; **d** eighth mode

torsional mode of the tower, while the fifth mode is the first mode in longitudinal direction. It should be noted that all modes have a torsional component, and there is no single mode with a pure form.

4 Step 2. Studies for the Model Calibration and Validation

The aim of the ambient vibration tests was to define the dynamic properties of the church, which included vibration frequencies, mode shapes and damping coefficients. These tests could also be used to assess the degree of connectivity between the various structural elements of the church. With the results of the test, the numerical models were calibrated. In this way, the mechanical properties of the materials were obtained from calibration of the frequencies and mode shapes of the analytical model with the results from the ambient vibration test.

4.1 Ambient Vibration Test

Based on the results of the finite element model and field survey, the points where the accelerometers should be placed for ambient vibration test were defined. Figure 12 shows schematically the points selected for the installation of the accelerometers. The signals obtained from the tests were processed and interpreted using a spectral analysis program. For each test, the average power spectra and the corresponding spectral ratios, phase angles and coherence between signal pairs were obtained. For these analyses, each record was processed in sections of 49.96 s, considering a 50 % of overlap between two consecutive segments and a Hanning window weighting type. The analysis of the spectra enabled obtaining several frequencies. Table 3 shows the significant frequencies of the tower, while Tables 4, 5 and 6 show the significant frequencies of the body of the church.
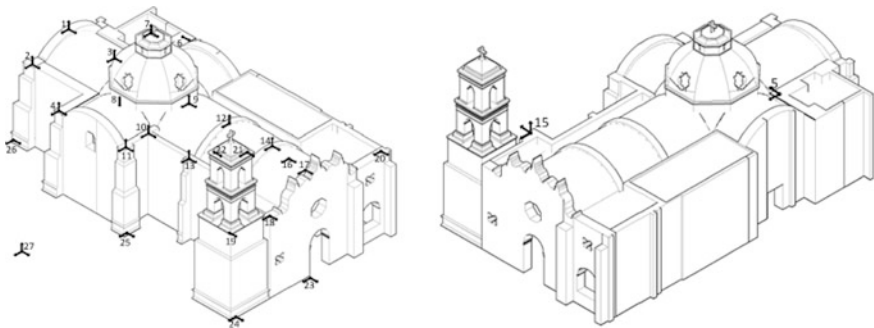

Fig. 12 Localization of the measured points

Table 3 Measured frequencies of the tower (Hz)

Mode	Longitudinal	Transverse	Vertical
1	3.15	3.47	7.9–8.0
2	7.7–7.9	8.5–8.8	–

Table 4 Measured frequencies of the church, longitudinal direction (Hz)

Mode	Nave	Façade	Cupola
1	5.6–6.3	–	–
2	8.0–10.3	–	8.2–8.7
3	12.0–15.0	13.5–15.5	–

Table 5 Measured frequencies of the church, transverse direction (Hz)

Mode	Apse	Nave	Façade	Cupola
1	–	3.47	–	–
2	–	4.3–4.8	4.5–4.8	–
3	5.7–7.3	5.7–7.3	5.7–7.3	–
4	6.1–7.1	6.1–7.1	–	–
5	–	–	8.3–9.3	–
5	–	–	10.9–12.7	14.8–15.7

Table 6 Measured frequencies of the church, vertical direction (Hz)

Mode	Nave vaults	Cupola
1	8.1–8.2	–
2	–	18.0–19.0

4.2 Calibration of the Numerical Model

The numerical model developed in the previous step was considered using properties for the materials postulated from values published in the literature. In order to have a reliable model, it was calibrated with the ambient vibration test results. In this particular case, the mechanical properties of the materials were the variables to be calibrated. The methodology proposed by Douglas and Reid [6] was used, where the frequencies of the structure can be estimated by means of:

$$f_i^D(X_1, X_2, \ldots, X_N) = C_i + \sum_{i=1}^{N} \left[A_{ik} X_k + B_{ik} X_k^N \right] \qquad (1)$$

where f^D is the frequency estimated, X_k are the variables to be calibrated, A, B and C are constants, and N is the number of frequencies used in the calibration. The variables are obtained by an optimization of the function J:

$$J = \sum_{i=1}^{m} w_i \varepsilon_i^2 \qquad (2)$$

$$\varepsilon_i = f_i^{EMA} - f_i^D(X_1, X_2, \ldots, X_N) \tag{3}$$

where f^{EMA} are the experimental values of the frequencies and w is a weight factor.

Four variables were calibrated. Three elasticity moduli, one corresponding to each material (Table 1). The last variable was the mass density of the masonry of the belfry, since it was reinforced with concrete. Table 7 shows the base, limit and optimized values. Obviously, the base values correspond to the values used in the preliminary model (Table 1).

The variables were optimized by using five frequencies. The first two corresponded to the tower; the other frequencies corresponded to the first mode in transverse, longitudinal and vertical direction. Table 8 compares the calculated frequencies obtained from the calibrated numerical model with the values measured in the ambient vibration tests. It is noted that the errors are acceptable, their value being below 10 %.

5 Step 3. Dynamic Characterization

5.1 Analysis Under Self-weight

The first analysis performed was for the gravity loads of the church. The total weight of the structure is equal to 15,175 kN, corresponding to an average axial stress in the foundation equal to 0.13 MPa. Figure 13a shows the map of the vertical

Table 7 Values used in the optimization of the variables

Material	Variable	Base value	Upper limit	Lower limit	Optimized
Masonry	Elasticity modulus (GPa)	0.60	1.00	0.40	0.40
Belfry masonry	Elasticity modulus (GPa)	0.25	9.81	0.20	0.25
Belfry masonry	Mass density (kg/m³)	1800	2200	1600	1600
Concrete	Elasticity modulus (GPa)	21.72	25.70	16.82	16.82

Table 8 Frequencies used in the calibration (Hz)

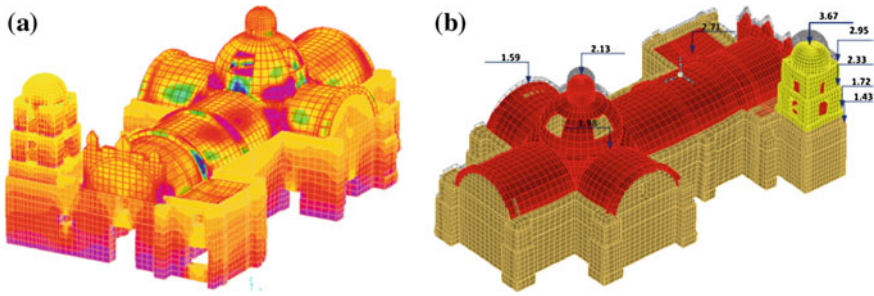| Mode	Direction	Calculated frequency	Measured frequency	Error (%)
1	1st tower	3.17	3.15	0.56
2	2nd tower	3.21	3.47	8.23
3	1st transverse	5.02	4.55	9.39
6	1st longitudinal	6.62	5.95	10.18
9	1st vertical	7.81	8.15	4.31

Fig. 13 Self-weight analysis: **a** map of vertical axial stresses; **b** deformed shape (in mm) 5.2 modal description

axial stresses due to the self-weight, whereas Fig. 13b shows the deformed model. It is clear that the maximum compressive stresses are presented at the base of the church. These stresses are low, around 10 % of the compressive strength of the masonry. This level of vertical stresses due to self-weight is common in the colonial churches in Mexico [7].

Table 9 lists the dynamic properties of the calibrated model. It is noted that the mode shapes are virtually the same as those of the original model, with the exception of mode 5 which was exchanged with mode 6. This means that the mode 6 is now the first longitudinal mode, while the mode 5 is a torsional mode of the tower. Likewise, it is observed that the vault cannot be considered as an infinitely rigid diaphragm. Note that the motion of the plane is not uniform as it can be observed in Fig. 14.

Table 9 Dynamical properties of the calibrated model

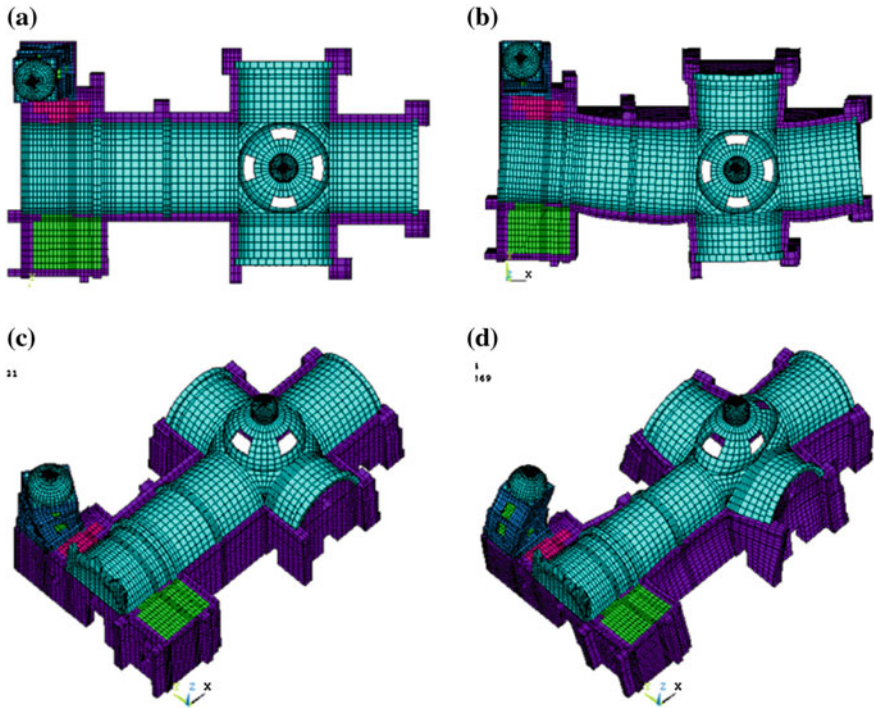| Mode	Frequency (Hz)	Percentage of modal mass (%)		
		Longitudinal	Transverse	Vertical
1	3.17	2.2	1.7	0.0
2	3.21	1.3	3.9	0.0
3	5.02	0.0	77.8	0.3
4	6.35	0.1	6.2	0.0
5	6.42	10.2	1.0	0.0
6	6.62	78.5	0.1	0.0
7	7.13	0.0	4.5	2.0
8	7.41	4.7	0.2	3.9
9	7.81	0.1	0.3	29.5
10	8.51	0.8	2.4	32.8

Fig. 14 Modal shapes of the calibrated model: **a** first-mode; **b** third-mode; **c** fifth-mode; **d** eighth-mode

5.2 Modal Spectral Analysis

A modal spectral analysis was performed considering the design spectrum proposed by the Mexican building code [16]. The church was founded on rock soil, zone I, according with the building code. As the structure is considered as type A (historical structure), the stresses must be multiplied by the factor 1.5. Thus, the spectrum will have a maximum ground acceleration equal to 0.6 m/s^2, while the acceleration of the plateau is equal to 2.4 m/s^2. Since the plateau starts in a period equal to 0.2 s, only the first two modes (corresponding to the vibration of the tower) fall in the plateau, whereas the other modes lie in the ascending branch. The seismic behavior factor Q is equal to one, since the masonry is considered as non-ductile material. This means that the design spectrum was not reduced from the elastic values.

Figure 15 shows the deformed shape with the map of the vertical axial stresses, when the earthquake is applied in the transverse and longitudinal direction respectively. It is noted that the analyses consider the combination of the earthquake in the two directions (100 % for one direction plus 30 % in the other). It is observed that regardless of the direction of the earthquake, the deformed shape obtained is similar in both cases. The tower has a torsional behavior, which is concentrated in
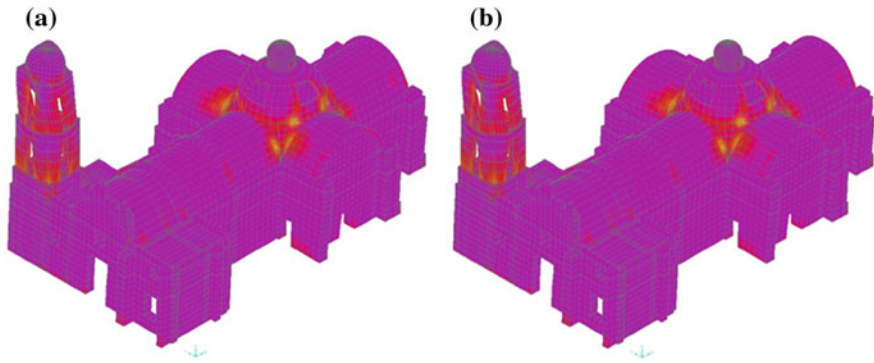
Fig. 15 Deformed shape and map of vertical axial stresses: **a** 100 % in lateral direction plus 30 % in the longitudinal; **b** 30 % in lateral direction plus 100 % in the longitudinal

the upper body of the belfry. Likewise, the dome has undergone a slight vertical movement, which produces a stress concentration at the base of the drum.

The vertical axial stresses are also concentrated in the pillars of the belfry and at the base of the walls and buttresses. A slight vertical stress concentration can be observed in the base of the ribs, which indicates that they are working due to vertical vibration of the vaults. Furthermore, the shear is concentrated in the body of the pillars of the belfry, in the drum and the ribs. It is interesting that the walls do not show important shear stresses. This is because the church has virtually no lateral or longitudinal distortions, whereas the motion is concentrated in the tower and roof (dome and vaults). These results show a seismic behavior consistent with historical record: *the critical elements of this church are the bell tower and the roof.*

6 Conclusions

The dynamic characterization of colonial Mexican churches was performed by means of a case study. Ambient vibration tests were performed in order to calibrate and validate a finite element model of the church. This study follows a stepped strategy in order to overcome the difficulties in the study of historical structures. The roof system cannot be considered as rigid diaphragm. The dynamical characterization shows that the belfry and the roof (dome and vaults) are the most critical elements. The church is very rigid, since the lateral wall drifts are so low. Thus out-of-plane and shear behavior in walls are not critical.

The key damages of the façade occurred due to the in-plane action. The out-of-plane behavior is generally less important and is only regarded with the detachment of the façade from the nave, but without reaching the state of collapse. This is mainly due to the confining influence of the towers, built in an integrated manner with the façade and not simply constructed in compliance. These towers provide a

constraint that reduces the movement out-of-plane, but also they increase the in-plane actions, while they are subjected to combined bending and torsional stresses. In the same way, the slab of the chorus restrains the wall and reduces the façade slenderness.

In order to safeguard the intrinsic values of a historic structure during an intervention, it is necessary that the structural design should be based on full understanding of the structural behavior of the building. Thus, damage-diagnosis and safety-assessment should be based on a comprehensive analysis of the structure. This analysis involves various aspects, in addition to a numerical analysis itself, such as the analysis of past conditions (historical analysis, damage analysis and interventions), analysis of the present conditions (geometry, materials, structural safety), and analysis of future conditions (possible changes and damages, seismic risk).

In this context, it is necessary that the engineer has a comprehensive understanding of the structure; which should include: history of the structure, assessment and diagnosis. Thus, the engineer must be supported by other disciplines, such as History and Architecture. Hence the multidisciplinary character of the project arises as the keystone of conservation. Based on this, in this paper a comprehensive analysis strategy is proposed, which is based on the Complementary Analyses Approach. This strategy overcomes the inherent difficulties of historical structures and incorporates the multidisciplinary work into the structural assessment.

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