Dynamics of Conflicting Beliefs in Social Networks

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Abstract. This paper analyzes two proposed models for simulating opinion dynamics in social networks where beliefs might be considered to be competing. In both models agents have a degree of tolerance, which represents the extent to which the agent takes into account the differing beliefs of other agents, and a degree of conflict, which represents the extent to which two beliefs are considered to be competing. In this paper, we apply different tolerance and conflict degrees to different groups in a network, and see how these groups affect each other. Simulations show that the groups having different tolerance degrees do not have significant effect upon each other in both Models I and II. On the other hand, the group perceiving a conflict causes more diversity in the agents based on Model I, but introduces a higher consensus level among agents when the fraction becomes larger in Model II.

Keywords: Opinion dynamics, Social network, Conflicting beliefs, Bounded confidence.

1 Introduction

Computer simulations have been employed successfully in the study of agent-based opinion dynamics in social networks for a long time from a wide range of perspectives, e.g., sociology, physics and philosophy [1,2,3,4,5,6]. In these models of opinion dynamics, a group of agents who hold beliefs about a given topic interact with each other to seek truth or reach consensus. Multidimensional opinion dynamics have recently become an active research area [7,8,9,10,11], where agents interact with each other based on their opinions on several topics, e.g., sports and politics. Following the ideas on multidimensional opinion dynamics, we have proposed two models for simulating the scenarios where the beliefs of agents about two (or more) topics may be perceived to be competing, e.g. two explanations of a given phenomenon [12].

The proposed models consider two competing beliefs, i.e., two dimensions, and they both consist of two updating steps. In the first step, the agents update their beliefs via network interaction by talking to their neighbors whose opinions are similar to theirs, and the similarity is decided by bounded confidence (tolerance degree) of each agent. The second step involves an internal update process allowing agents to update their beliefs based on the perceived conflict between beliefs. In the previous simulations, all of the agents in the network were assumed to have the same tolerance

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degrees with respect to two beliefs and the same conflict degree between the beliefs. In reality, however, the conflict and tolerance degrees of the agents may well differ from each other in most cases. It is therefore interesting and worthwhile to investigate how different groups of agents with different conflict and tolerance degrees affect each other during the belief update process.

The rest of this paper is structured as follows. We give an overview of the two belief update models in Section 2. Computer simulations and analysis are provided in Section 3 to investigate the impact of a fraction of the group having a particular conflict or tolerance degree on the belief update in the proposed models. Conclusions and discussions are presented in Section 4.

2 The Models

Assume that we have a network of n vertices, representing agents. Each agent holds two, possibly conflicting, beliefs about two topics, denoted as A and B, and the degrees of both beliefs may change along a set of discrete time points according to a given update mechanism. Both of the proposed models consist of two steps where the first step is to update the belief degrees of agents via network interaction and the second step involves an internal agent update process by taking the perceived conflict into consideration [12].

2.1 Network Update

For the first step (network update), we extend the well-known Hegselmann-Krause (HK) model [4,5,8,13] to include two-dimensional beliefs. The HK model involves a complete graph but the agents are only influenced by the neighbors who have similar opinions to theirs, where the similarity is decided by so-called bounded confidence. Suppose that $A_i(t)$ and $B_i(t)$ are the degrees of beliefs on two topics A and B of the ith agent at time t, where $A_i(t), B_i(t) \in [0, 1]$, with 0, 1, 0.5 corresponding to total disbelief, total belief, and indifference respectively, for all i and t, then the new belief degrees for agent i at time t+1 based on the HK model are

$$A_i(t+1) = |I_A(i,t)|^{-1} \sum_{j \in I_A(i,t)} A_j(t),$$

$$B_i(t+1) = |I_B(i,t)|^{-1} \sum_{j \in I_B(i,t)} B_j(t). \quad (1)$$

Here $I_A(i,t) = \{ j : |A_i(t) - A_j(t)| \le \varepsilon_A \}$ and $I_B(i,t) = \{ j : |B_i(t) - B_j(t)| \le \varepsilon_B \}$ are epistemic neighborhoods of agent i at time t with respect to A and B correspondingly, that is, the sets of agents whose belief degree in A or B at t is close to that of the corresponding belief of agent i at that time [8]. The parameters ε_A and ε_B, called tolerances [14], decide the bounded confidence intervals for the two beliefs, and $|I_A(i,t)|$ and $|I_B(i,t)|$ represent the cardinalities of the corresponding sets.

2.2 Internal Update

To consider conflict between the two beliefs, two models have been proposed at the internal update step, which represent different attitudes of people towards conflict between beliefs [12]. The degree of conflict is denoted as $c_i \in [0, 1]$, where 0 and 1 correspond to no perceived conflict and total conflict respectively.

The first model (Model I) suggests that if there is no perceived conflict or if $A_i(t) \le 0.5$ and $B_i(t) \le 0.5$, then the internal agent update will result in no change in both beliefs. Further, if one, or both of the belief degrees are greater than 0.5 and $c_i > 0$, then the perceived conflict will decrease the degree of the lesser held belief, but not increase the degree in the other. Specifically, if $c_i = 1$ then the lesser held belief should be rejected, i.e., its degree should be set to zero. It means that Model I represents the attitude of a group of people who incline to accept only one of the beliefs with larger degree but reject the other one if there is conflict between them. A rule for achieving this is

$$A_i^*(t) = \begin{cases} A_i(t), & \text{if } A_i(t), B_i(t) \le 1/2 \text{ or } A_i(t) > B_i(t), \\ \max(\min(A_i(t), B_i(t) - c_i), 0), & \text{if } A_i(t) < B_i(t), B_i(t) > 1/2, \\ \max(\min(A_i(t), B_i(t) - c_i), 0), & \text{if } A_i(t) = B_i(t), B_i(t) > 1/2 \end{cases} \quad (2)$$

with a corresponding rule for belief B, where the * superscript signifies an internal agent update. It is noted that the last rule contains the assignment at probability of p to prevent a 'stalemate' at equality, i.e., we randomly pick one of the beliefs to decrease. We usually set $p = 0.5$ based on the assumption that there is no bias between the two beliefs.

Different from Model I, which decreases the degree of the lesser held belief if there is a perceived conflict, the second model (Model II) tries to make the sum of the two belief degrees closer to 1, reaching unity when there is maximum conflict ($c_i = 1$). It also assumes that the beliefs will not change if there is no perceived conflict, i.e. $c_i = 0$. A rule that achieves this can be given as

$$A_i^*(t) = (1 - c_i)A_i(t) + c_i \frac{A_i(t)}{A_i(t) + B_i(t)}, \quad (3)$$

with a corresponding rule for belief B. This model is more appropriate for cases where the agent is unlikely to reject or accept both beliefs and might apply, for example, in contexts where an explanation is needed and there are only two plausible competing explanations.

The two proposed models represent two possible strategies for agents to update their beliefs when there is perceived conflict between them. The previous simulations have shown that, when there is a conflict between the two beliefs, Model I is more likely to partition the agents into several distinct groups with one of the beliefs being rejected, while Model II is highly likely to make the agents reach consensus in both beliefs.

3 Simulations and Results

The simulations are implemented in a complete network with a fixed number of 100 agents. The initial degrees of the two beliefs are both generated randomly (uniformly distributed) for each agent, as in most of the existing multidimensional models based on the assumption that there is no pre-defined bias between the two beliefs. Given randomly generated initial belief degrees, simulations might show variant results even with the same settings. We therefore implement 100 runs with all the other conditions being the same and study the average performance.

As a measure of consensus we use the average standard deviation. The standard deviation is calculated after each run with respect to the obtained belief degrees of agents, and the average standard deviation is then obtained across 100 runs. It is then not difficult to see that the larger the average standard deviation is, the more diverse the agents are, i.e., the lower consensus level the agents can achieve. The average standard deviation being zero means that there is a total consensus among the agents in the corresponding belief. We explore these two quantities in the simulations: the average degree of beliefs and the average standard deviation of beliefs.

3.1 Fraction of Tolerance

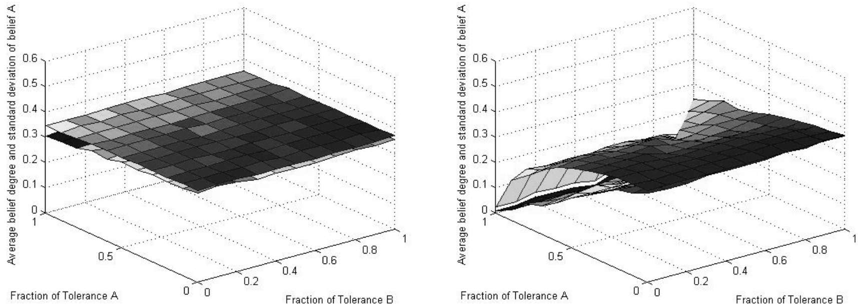
In previous work fixed tolerance degrees were used for two beliefs [12], i.e., a larger tolerance degree ($\varepsilon_A = 0.25$) for belief *A* and a smaller degree ($\varepsilon_B = 0.05$) for belief *B*. It was also assumed that all the agents hold the same tolerance degrees for the corresponding beliefs. Here, we divide the agents into two groups holding different tolerance degrees to see how the different groups affect each other during the belief update process.

The division of agents is realized by a fraction of tolerance either in belief *A* or *B*. Suppose that the fraction is 0.6, this means that 60% of agents take the predefined tolerance degree in belief *A* or *B*, while the remaining 40% take a small tolerance degree 0.05, which means they are highly intolerant. We fix both the degree of conflict and the fraction of conflict to be 1 in this subsection to avoid confusion. The tolerance degrees of the two beliefs are assumed to be equal to each other and we consider two possible degrees of tolerance, 0.2 and 0.4.

Model I

Fig. 1 shows the simulation results using Model I for average degree and average standard deviation of belief *A* across different fractions of tolerances about two beliefs. It can be seen that the agents maintain a high level of diversity (large average standard deviation value) in belief *A* across the fraction of tolerances when the tolerance degrees are small (=0.2). This is caused by the nature of Model I accepting only one of the beliefs with larger degree but rejecting the other one when there is perceived conflict between them, and this makes the agents highly likely to partition into distinct groups. When the tolerance degrees are high enough (=0.4), the agents can reach consensus in belief *A* at the borderline where the fraction of tolerance of belief

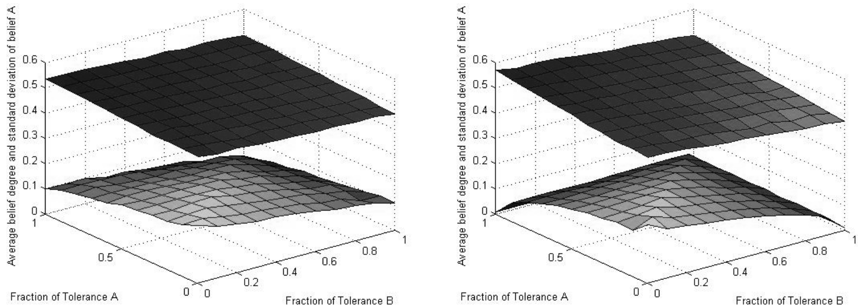
A is 1, but the diversity still remains high when the fraction of tolerance of belief *A* is low no matter what the fraction of tolerance of belief *B* is. The simulation results on belief *B* are symmetric to that of belief *A*, and are not included here.



(a) (b)

Fig. 1. Simulation results for Model I. Average belief degrees (upper surface) and average standard deviations (lower surface) of belief *A* with respect to fraction of tolerance with tolerance degrees being (a) 0.2, (b) 0.4

Model II



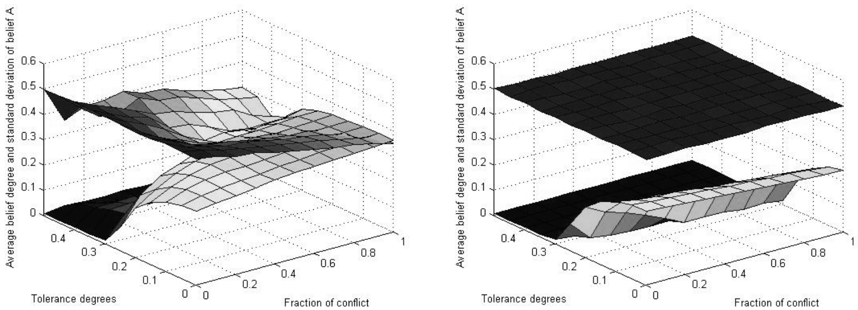
(a) (b)

Fig. 2. Simulation results for Model II. Average belief degrees (upper surface) and standard deviations (lower surface) of belief *A* with respect to fraction of tolerance with tolerance degrees being (a) 0.2, (b) 0.4

Similarly, Fig. 2 shows that the fraction of tolerance has little impact on consensus of agents in belief *A* based on Model II when the tolerance degrees are small (=0.2). This is because that the change of fraction of tolerance does not essentially change the fact that all the agents are intolerant given small tolerance degrees. When the tolerance degree becomes larger (=0.4), the agents are able to reach consensus at both the borderlines where either fraction of tolerance of belief *A* or *B* is 1 unlike Model I where this does not occur when the fraction of tolerance of belief *B* is 1. This is caused by the nature of Model II interpreting conflict in terms of the belief degrees summing to one and so when the agents can reach consensus in one of the beliefs the other will also achieve consensus. Hence, this is primarily the result of the impact of the conflict on consensus rather than the effect of the two groups upon each other, given the fact that the agents in the ‘intolerant’ group reject interaction with the other agents and maintain their beliefs. The simulation results on belief *B* are not included due to the fact that they are symmetric to that of belief *A*.

3.2 Fraction of Conflict

It is also worthwhile to investigate how the groups holding different conflict degrees affect each other during the belief update process. To make the situation simpler, we fix the conflict degree to be 1, and apply this to a fraction of the agents, which divides the agents into two groups where one group holds total conflict and another holds no conflict. The fractions of tolerances with respect to both beliefs are also fixed to be 1 so that all agents have the same tolerance degrees. We consider two situations where both the tolerance degrees change equally from 0 to 0.5 in the first situation, and in another situation where there is a larger tolerance degree for one belief than the other.

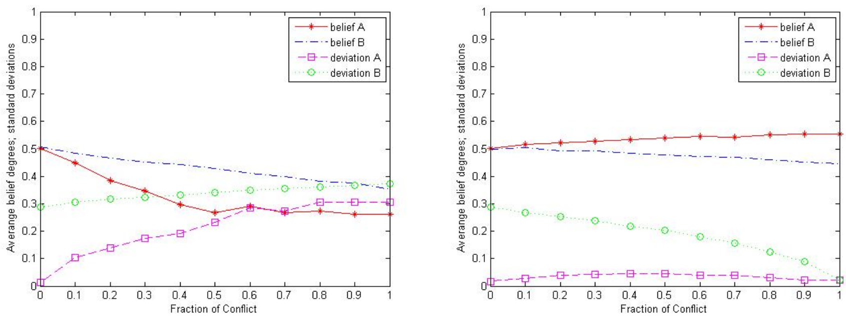


(a) (b)

Fig. 3. Simulation results for (a) Model I and (b) Model II. Average belief degrees (upper surface) and standard deviations (lower surface) of belief *A* with respect to fraction of conflict and tolerance degrees

It can be seen from Fig. 3 (a) that, given larger tolerance degrees, the agents become more likely to reach consensus when the fraction of conflict decreases. This can be explained as due to the way conflict is represented in Model I since it usually makes the agents accept only the belief with the larger degree of belief but reject the other one. Thus introducing more agents holding perceived conflict between beliefs makes the agents more likely to form multiple groups with belief *A* or *B* being rejected. On the other hand, Fig. 3 (b) shows that the fraction of conflict has little impact on Model II when the tolerance degrees are larger than 0.3 or smaller than 0.2, while the increase of fraction of conflict makes the agents become more likely to reach consensus when the tolerance degrees are between 0.2 and 0.3. In other words, introducing more agents holding conflicting beliefs in Model II lowers the consensus threshold from around 0.3 to 0.2.

For another scenario where the tolerance degree of belief *A* is fixed at 0.3, and the tolerance degree of belief *B* is 0.05, Fig. 4 show the results of the impact of fraction of conflict in the two models. It can be seen from Fig. 4 (a) that the increase of the fraction of conflict from 0 to 1 in Model I decreases the consensus in the belief with larger tolerance degree (belief *A*), and this is also the case for the belief with smaller tolerance degree (belief *B*) although the decrease is less dramatic. On the other hand, Fig. 4 (b) shows that introducing more conflicting agents in Model II has little impact on the belief with larger tolerance degree (belief *A*), but makes the agents increase consensus in the belief with the smaller tolerance degree (belief *B*). These results further verify the natures of the two models on conflict between beliefs. When there is perceived conflict between the two beliefs, the agents in Model I are more likely to accept only one of the beliefs but reject another, while Model II makes the agents to reach consensus in both beliefs if they can reach consensus in one of the beliefs.



(a) (b)

Fig. 4. Simulation results for (a) Model I and (b) Model II. Average belief degrees and average standard deviations of both beliefs with respect to fraction of conflict with tolerance degree of belief *A* being 0.3 and that of belief *B* being 0.05

4 Conclusions

Based on the two proposed models on two-dimensional opinion dynamics when there is perceived conflict between the two beliefs, this paper has examined the effect of varying the fraction of the population having given tolerance and conflict degrees to investigate group behavior. Simulation results show that the groups having different tolerance degrees do not have significant effect upon each other in both Models I and II, because one of the groups is 'intolerant' whose agents reject interaction with the other agents. On the other hand, the fraction of the group holding perceived conflict causes more diversity in the agents based on Model I, but introduces a higher consensus level among agents when the fraction becomes larger in Model II.

This paper considers two competing beliefs, but the ideas contained herein are generalizable to cases where there are a larger set of beliefs. The current paper considered the case that the agents only update their beliefs according to the beliefs of their neighbors. In future work this will be extended so that the agents can take reported information, external to the network, into consideration when updating their beliefs. Different network structures will also be explored to see the impact of network topology on the conflicting opinion dynamics.

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