A Hybrid Algorithm for Stochastic Multiobjective Programming Problem

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Abstract. The traditional approach in the solution of stochastic multiobjective programming problem involves transforming the original problem into a deterministic multiobjective programming problem. However, due to the complexity in practical application problems, the closed form of stochastic multiobjective programming problem is usually hard to obtain, and yet, there is surprisingly little literature that addresses this problem. The principal purpose of this paper is to propose a new hybrid algorithm to solve stochastic multiobjective programming problem efficiently, which is integrated with Latin Hypercube Sampling, Monte Carlo simulation, Support Vector Regression and Artificial Bee Colony algorithm. Several numerical examples are presented to illustrate the validity and performance of the hybrid algorithm. The results suggest that the proposed algorithm is very suitable for solving stochastic multiobjective programming problem.

Keywords: Stochastic programming · Multiobjective programming · Pareto efficient solution · Hybrid algorithm

1 Introduction

Many real-life problems require considering and optimizing multiple and conflicting objectives from the multiobjective optimization point of view, leading us into the area of multiobjective programming (MOP) probelm. The MOP problem in static environment with deterministic parameters has received much research interest [1][2][3]. However, since indeterminacy is inherent in most real cases, where observed phenomena are disturbed by indeterministic perturbations, the application of deterministic MOP methods to real-world problems

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often faces the difficulty that for a particular problem considered, the parameters involved take unknown or uncommensuarable values at the moment of making the decision. With the great improvement of probability theory, the probability distribution is widely adopted to depict such kind of indeterministic phenomenon in real-life MOP problem, which leads to the research field of stochastic MOP problem. The stochastic MOP modeling is widely used in many real-world decision making problems of management science, engineering, and technology, including distributed energy resources planning [4], network design [5], traveling salesman problem [6], capacitated arc routing problem [7], etc.

The review of these works shows that the stochastic MOP models can lead to very large scale problems, and the solution of such problems always involves introducing several equivalent deterministic models to remove the random ambiguity in original stochastic MOP problem, such as expected value model, minimum variance model, etc. In this paper, we use the valuation criteria of objective functions C in stochastic MOP problem to remove the random ambiguity and obtain the closed form of equivalent deterministic model, where C denotes the criteria of the specific valuation in practical application. Furthermore, we propose the definition of Pareto efficient solution in stochastic MOP problem based on criteria C.

However, due to the complex nature of real life problem, in most cases, the closed form of equivalent deterministic models in stochastic MOP problem is difficult to obtain. Under these circumstances, the methods based on approximation should be applied, such as Sample Average Approximation (SAA-N, where N is the sample size) method [8]. Though the sequence of SAA-N optimal values (N=1,2 ...) can converge almost surely to the true optimal value, it is prohibitively expensive when the problem to be solved is provided with complicated formulations and feasible set, such as NP-hard with many local extremums. In order to improve the computation efficiency, some hybrid algorithms using Monte Carlo simulation, artificial neural network (ANN) and genetic algorithm (GA) have been adopted for solving stochastic programming problems [9] [10]. However, since the traditional hybrid algorithms need to generate large-scale size of decision points through purely random sampling to obtain the desired precision in model approximation, and need long time to obtain the optimal solution in model optimization with GA, the computation cost of these hybrid algorithms is very time-expensive. A new powerful and efficient hybrid algorithm should be designed and applied to the stochastic MOP problem to reduce the computation cost and improve the computation accuracy. For this purpose, a new hybrid algorithm composed of Latin Hypercube Sampling (LHS), Monte Carlo simulation, Support Vector Regression (SVR) and Artificial Bee Colony (ABC) algorithm is built to obtain the Pareto efficient solutions in stochastic MOP problem in this paper.

In the hybrid algorithm presented, it is broken into four phases, that is, sample phase, simulation phase, approximation phase and optimization phase. The problem in sample phase is addressed using Latin hypercube sampling introduced by McKay et al [11], which is a very popular sampling method for use with computationally demanding models. It has been theoretically and experimentally proved that LHS is more precise and robust than traditional random sampling methods [12] [13]. The problem in simulation phase is addressed using Monte Carlo simulation method to calculate the specified valuation meanings of functions on the sample generated in sample phase. The problem in approximation phase is addressed using a new and very promising regression technique developed by Vapnik, Steven Golowich, and Alex Smola [14] in 1996, called support vector regression (SVR). The excellent performances of SVR in approximation have been obtained in [15]. The problem in optimization phase is addressed using ABC algorithm proposed by Karaboga in 2005 [16], which is a meta-heuristic bionic algorithm based on the intelligent foraging behavior of honey bees. It has been validated that its effectiveness and efficiency on algorithm performance are competitive to other optimization algorithms [17][18][19]. Since every phase in the hybrid algorithm is implanted with advanced methods, where using the LHS and Monte Carlo simulation for model data collection, SVR for model approximation, and ABC algorithm for model optimization, it is expected that it can reduce the computation cost and improve the computation accuracy greatly. The comparison result with traditional hybrid algorithm in a numerical example shows that this new algorithm is more precise and efficient.

The paper is organized in the following manner. In Section 2, the mathematical formulation of stochastic MOP problem is introduced, and three equivalent deterministic models are presented, that is, expected value model, minimum variance model, and α-optimistic value model. In Section 3, a new powerful hybrid algorithm is built for solving the stochastic MOP problem more efficiently, and a numerical example with many stochastic local minimums is provided to illustrate the solution of stochastic MOP problem using the hybrid algorithm in Section 4. Finally, a brief summary is given and some open points are stated for future research work in Section 5.

2 Mathematical Formulation

In this section, the mathematical description of a general stochastic MOP problem is presented first, and then three equivalent deterministic models are proposed to remove the random ambiguity in original stochastic MOP problem.

2.1 Description of Stochastic Multiobjective Programming Problem

Let us consider the stochastic MOP problem as follows:

$$\min_{x \in D} f(x, \xi) = (f_1(x, \xi_1), f_2(x, \xi_2), \cdots, f_p(x, \xi_n)) \tag{2.1}$$

where $x \in D$ is a vector of decision variables of the problem; $\xi_1, \xi_2, \cdots, \xi_p$ are random vectors whose components are random variables, $\xi_i = (\xi_{i1}, \xi_{i2}, \cdots, \xi_{in})$, defined on the probability space $(\Omega, \mathcal{F}, \mathrm{Pr})$; and the set of feasible solutions $D \subset R^n$ is crisp, nonempty and compact.

Since the objective function in model (2.1) becomes dependent not only on the solution, but also on a random influence, i.e., it becomes a random variable. There are no methods can compare the random variables directly, it needs to remove the random ambiguity in it before comparison. Most frequently, the practical aim is then to propose several equivalent deterministic models to optimize the specific valuation of the objective functions in model (2.1).

2.2 Equivalent Deterministic Models

Here, the general equivalent deterministic model of original stochastic MOP problem is described as follows:

$$\min_{x\in D} C[f(x,\xi)] = (C[f_1(x,\xi_1)], C[f_2(x,\xi_2)], \cdots, C[f_p(x,\xi_n)]) \tag{2.2}$$

where C denotes the criteria of the specific valuation of the objective functions in model (2.1).

Different real-life problems call for different criteria of valuation to satisfy its need in practical application, when C denotes the expected value of the objective functions, the model (2.2) is called expected value model of stochastic MOP problem, and can be presented as follows:

$$\min_{x\in D} E[f(x,\xi)] = (E[f_1(x,\xi_1)], E[f_2(x,\xi_2)], \cdots, E[f_p(x,\xi_n)]) \tag{2.3}$$

When C denotes the variance of the objective functions, the model (2.2) is called minimum variance model of stochastic MOP problem, and can be presented as follows:

$$\min_{x\in D} V[f(x,\xi)] = (V[f_1(x,\xi_1)], V[f_2(x,\xi_2)], \cdots, V[f_p(x,\xi_n)]) \tag{2.4}$$

When C denotes the $\alpha-$optimistic value of the objective functions, the model (2.2) is called $\alpha-$optimistic value model of stochastic MOP problem, and can be presented as follows:

$$\min_{x\in D} f(x,\xi)_{sup}(\alpha) = (f_1(x,\xi_1)_{sup}(\alpha), f_2(x,\xi_2)_{sup}(\alpha), \cdots, f_p(x,\xi_n)_{sup}(\alpha)) \tag{2.5}$$

Though the random ambiguity is removed, the objectives are usually in conflict in stochastic MOP problem, there is no optimal solution that simultaneously minimizes all the objective functions. In this case, we have to introduce the concept of Pareto efficient solution in stochastic MOP problem, which means that it is impossible to improve any one of objectives without sacrificing on one or more of the other objectives.

Definition 2.1 *A Pareto efficient solution x^* in model* (2.2) *is said to be C Pareto efficient to the stochastic MOP problem* (2.1)*, where the feasible solution x^* is said to be a Pareto efficient solution of model* (2.2) *if there is no feasible solution x such that*

$$C[f_j(x,\xi_j)] \le C[f_j(x^*,\xi_j)], j = 1, 2, \ldots, p$$

and $C[f_j(x,\xi_j)] < C[f_j(x^*,\xi_j)]$ for at least one index j.

3 Hybrid Intelligent Algorithm for Stochastic MOP Problem

To solve stochastic MOP problem with complicated closed form and feasible set, the most direct approach is to nest the iterative loops by performing complete Monte Carlo estimation (inner loop) for each optimization data request (outer loop). However, this can be prohibitively expensive, for this reason, the optimization techniques must be combined with hybrid algorithm. In this section, the Latin Hypercube Sampling (LHS), Monte Carlo simulation, Support Vector Regression (SVR) and Artificial Bee Colony (ABC) algorithm are integrated to design a powerful hybrid algorithm for solving stochastic MOP problem.

3.1 Design of Hybrid Intelligent Algorithm

(1) Generation of Sample Using LHS

The goal of sampling in stochastic MOP problem is to generate a matrix of experiments $X^n = (x_{ij})_{n\times k}$ from the feasible set where n is the number of experiments and k is the number of variables. As an extension of stratified-random procedure, Latin hypercube sampling has a long history and has shown its robustness capabilities in sample generation. The LHS involves sampling ns values from the prescribed distribution of each of k decision variables $X_1, X_2, \ldots, X_k$ in stochastic MOP problem. Unlike simple random sampling, LHS ensures a full coverage of the range of each variable by maximally stratifying each marginal distribution.As the information of design variables in stochastic MOP problem is always hard-available beforehand, it is usually assumed that all design variables follow uniform distribution.

(2) Computation Using Monte Carlo Simulation

Due to the complexity in stochastic MOP problem, it is hard to obtain the closed form of its deterministic objective functions, the Monte Carlo simulation is adopted to calculate the valuation of objective functions on the sample generated by LHS. In this paper, the Monte Carlo simulation is adopted to calculate the expected value, variation and $\alpha-$optimistic value of the objective functions respectively.

(3) Approximation Using ABC-SVR

After obtain the sample in feasible set and the corresponding valuation of objective functions, it needs to build a surrogate model to map the relationships between them, which can be considered as a regression process. Support Vector Regression (SVR) is a new regression method different from traditional/ statistical ones, it minimizes the generalized error bound instead of the observed training error, so as to achieve the generalized performance. In this paper, we start our study on the basis of the SVM toolbox—LIBSVM directly [20], rather than discuss about the principle and algorithm of SVR. LIBSVM is a library for SVM; its goal is to let users can easily use SVM as a tool. Since the control parameters of SVR are very sensitive to its performance, a successful parameter selection is very important, especially the parameter γ in kernel function and the parameter c of cost. The artificial bee colony (ABC) algorithm is used to find the optimal control parameters aiming at the best regression accuracy, called ABC-SVR, whose regression performance will be tested in Section 4. The main steps of the ABC-SVR application in the stochastic MOP problem can be summarized as follows:

Step 1: Generate train data and test data based on the sample and its corresponding valuation of objective functions;
Step 2: Normalize the train data and test data to improve the regression ability of SVR;
Step 3: Denote the parameter γ in kernel function and the parameter c of cost as a food position (c, γ), and the regression accuracy as the nectar amount;
Step 4: Adopt ABC algorithm to find the optimal control parameters of SVR (optimal food position) on the normalized data;
Step 5: Use the optimal control parameters to build a SVR model and train it according to the train data;
Step 6: Employ the test data to validate the accuracy of trained SVR model. If the regression accuracy does not meet, return to Step 3 to change the ABC options until the desired accuracy is met.

(4) Optimization Using ABC Algorithm

Created by Karaboga [16], the artificial bee colony algorithm is a new population-based meta-heuristic method motivated by the intelligent foraging behaviors of honeybee swarm. There are three essential components in the basic ABC algorithm, respectively are, food source positions, nectar-amount, and three kinds of foraging bees (employed bees, onlookers, and scouts). Each food source position represents a feasible solution to the optimization problem being considered and the nectar-amount of a food source corresponds to the quality (fitness) of the solution being represented by the food source. Each kind of foraging bee performs one particular operation for generating new candidate food source positions. Employed bees are those bees which are searching the food around the food source in their memory currently; they are responsible for sharing the information about food sources with onlooker bees. Onlooker bees are those bees which are waiting in the hive for the information from the employed bees; they tend to choose good food source with more nectar-amount shared by the employed bees, and then further tap the foods around the selected food source. Scout bees are those bees which are carrying out random searches for discovering new food sources if the employed bees and onlookers cannot find a better neighboring food source. Thus, the ABC algorithm visualizes the employed and onlooker bees as performing the job of local search (exploitation), whereas the onlookers and scouts bees as performing the job of global search (exploration). Specifically, unlike real bee colonies, the ABC algorithm assumes that there is a one-to-one correspondence between the employed bees and the food sources, that is, the number of food sources (solution) is the same as the number of employed bees. The role conversion in the algorithm is activated when the bees cannot find a better food source, the employed bee of an abandoned food source becomes a scout bee, which will becomes an employed bee again after it finds a new food source.

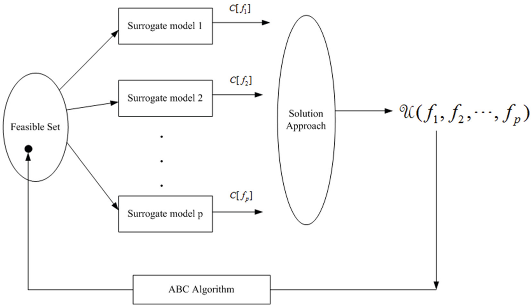

Fig. 1. Main idea of hybrid algorithm for stochastic MOP problem

Straightforwardly, the main idea of the proposed hybrid algorithm for stochastic MOP problem can be illustrated in Fig. 1. Firstly, the valuations of objective functions are obtained through the corresponding surrogate models built using ABC-SVR. Secondly, these valuations are integrated into one objective value using solution approaches which have been proved validated in deterministic MOP, such as linear weighted method, ideal point method, etc. Thirdly, the ABC algorithm finds the optimal solution in feasible set according to the integrated objective value, which is its optimization goal. According to Definition 2.1, the optimal solution ABC algorithm obtained is the Pareto efficient solution in stochastic MOP problem.

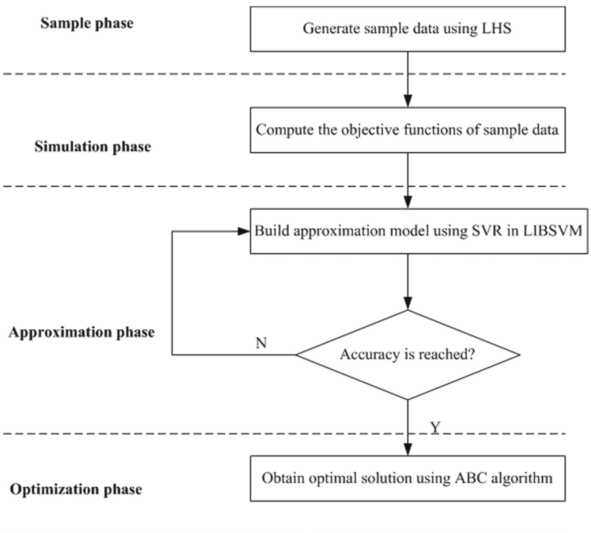

Fig. 2. Framework of hybrid algorithm for stochastic MOP problem

Specifically, the framework of the proposed hybrid algorithm is illustrated in Fig. 2, the main procedure can be summarized as follows:

Step 1: Generate sample data from feasible set as input data using LHS;
Step 2: Calculate corresponding objective values as the output data using Monte Carlo simulation;
Step 3: Build surrogate models for every objective function based on the approximation of input and output data using ABC-SVR;
Step 4: Send the employed bees to the food sources (solution), set it as the input data and determine the nectar amounts f_i (output data) using the SVR models built in Step 3 and the solution approaches such as linear weighted method, ideal point method, etc.;
Step 5: Calculate the fitness values of each solution fit_i and its corresponding probability values as follows:

$$fit_i = \begin{cases} 1/(1+f_i) & if\ f_i \geq 0 \\ 1+abs(f_i) & if\ f_i < 0 \end{cases}$$

$$p_i = fit_i / \sum_{i=1}^{SN} fit_i$$

where $i = 1, 2, SN$; SN is the number of food sources;
Step 6: Send the onlooker bees to their food sources according to the probability values;
Step 7: Send the scouts to the search area if a food source could not be improved through "limit" trials, and replace it with a new randomly produced solution if the new solution is better;
Step 8: Memorize the best food source (solution) achieved so far;
Step 9: If a stopping criterion is met, then output the best food source, otherwise, go back to Step 4.

It can be seen that the hybrid algorithm not only applies classic sample method to generate sample data in feasible set, but also adopts advanced regression procedure to build surrogate model, and powerful optimization algorithm for solution improvement.

3.2 Performance Test

To test the performance of the proposed hybrid algorithm, two numerical examples are presented. The first one is a stochastic single objective problem with available closed form of expected value, which is presented as follows.

$$\begin{cases} \min & f(x,\xi) = \xi_1 x_1^2 + \xi_2 x_2^2 + \xi_3 x_3^2 \\ \text{s.t.} & \\ & -1 \leq x_1, x_2, x_3 \leq 1 \end{cases} \tag{3.1}$$

where ξ_1, ξ_2, ξ_3 are random variables, and subject to uniform distribution $\mathscr{U}(0,2)$, normal distribution $\mathscr{N}(1,3)$, and exponential distribution $\mathcal{EXP}(1)$, respectively.

Due to the linearity characteristic of expected value in probability theory, we can deduce that

$$E[f(x,\xi)] = E[\xi_1]x_1^2 + E[\xi_2]x_2^2 + E[\xi_3]x_3^2$$

and the optimal solution is $(0,0,0)$, the corresponding optimal objective value is 0.

Numerically, let us solve (3.1) by using the hybrid algorithm proposed in this paper. The sample size in LHS is set as 500, and the number of expected value calculation cycle in Monte Carlo is set as 5000. Parameters set for the ABC algorithm are given in Table 1, and the maximum number of cycles in ABC is taken as 200. The optimal solution obtained is $(-0.0109, 0.0117, -0.0048)$, and the optimal approximated objective value is 2.1357E-4.

Table 1. Control parameters adopted in the ABC algorithm

Control parameters in ABC algorithm	
Colony size	40
Limit	100
Number of onlookers	Half of the colony size
Number of employed bees	Half of the colony size
Number of scouts	1

The convergence of hybrid algorithm in the solution (3.1) are shown in Fig. 3, from which it is easy to know that the value by simulation is almost equal to the true optimal value by performing over 450 sample data.

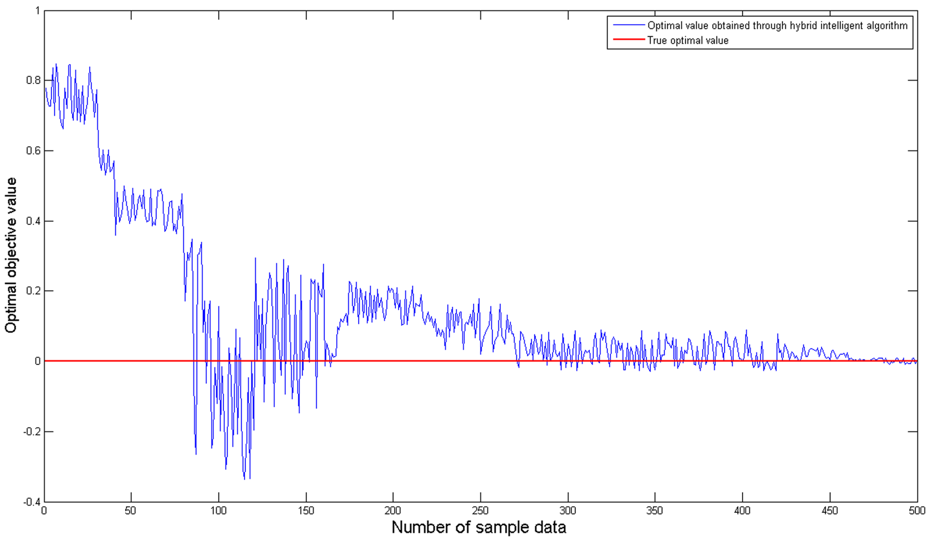

Fig. 3. Convergence to the true optimal value

The second one is a numerical example from [21], which has been solved by the traditional hybrid algorithm

$$\begin{cases} \min & E[\sqrt{(x_1-\xi_1)^2+(x_2-\xi_2)^2+(x_3-\xi_3)^2}] \\ \text{s.t.} & \\ & x_1^2+x_2^2+x_3^2 \le 10 \end{cases} \tag{3.2}$$

where ξ_1, ξ_2, ξ_3 are random variables, subject to uniform distribution $\mathcal{U}(1,2)$, normal distribution $\mathcal{N}(3,1)$, and exponential distribution $\mathcal{EXP}(4)$, respectively. In the traditional hybrid algorithm, it needs to produce 2000 sample data to train the ANN, and 300 generations in the evolution of GA. While in the new algorithm we proposed, it just needs to produce 500 sample data, and go through 60 cycles. Programs are run independently for each algorithm in MATLAB R2010b (version of 7.11.0.584) on Intel(R) Core(TM) i3-2310M CPU @2.10GHz under Window XP environment. The obtained results are shown in Table 2. It is clear that the proposed algorithm is more precise and efficient than the traditional hybrid algorithm.

Table 2. Performance comparison

	Traditional algorithm	Proposed algorithm
Optimal solution	(1.1035,2.1693,2.0191)	(1.1469,2.3775,1.7375)
Minimum objective	3.56	3.34
Time cost	659 seconds	165 seconds

4 Application on A Theoretical Case

Here, a numerical example is provided to illustrate the proposed new hybrid algorithm. Assume that x_1, x_2 are two decision variables, and ξ_1, ξ_2 are random variables, subject to uniform distribution $\mathscr{U}(1,5)$ and normal distribution $\mathscr{N}(1,4)$, respectively. The problem under consideration is the following bi-objective programming problem involving random variables in the objective functions.

$$\begin{cases} \min\limits_{x_1,x_2} f_1(x,\xi_1,\xi_2) = \xi_1\sin^2(\xi_2 x_1)+\xi_1\cos^2(\xi_2 x_2) \\ \min\limits_{x_1,x_2} f_2(x,\xi_1,\xi_2) = \dfrac{(\xi_1 x_1+\xi_2)^2+(\xi_2 x_2+\xi_1)^2}{10} \\ \qquad \text{subject to :} \\ \qquad\qquad -2 \le x_1, x_2 \le 2 \end{cases} \tag{4.1}$$

To obtain Pareto efficient solution in problem (4.1), the linear weighted method and ideal point method are adopted. Using the linear weighted method, the equivalent deterministic model can be presented as follows:

$$\begin{cases} \min\limits_{x_1,x_2} C[f(x,\xi_1,\xi_2)] = \lambda_1 C[f_1(x,\xi_1,\xi_2)] + \lambda_2 C[f_2(x,\xi_1,\xi_2)] \\ \qquad \text{subject to :} \\ \qquad\qquad -2 \le x_1, x_2 \le 2 \end{cases} \tag{4.2}$$

where $\lambda_1, \lambda_2 > 0$, and $\lambda_1 + \lambda_2 = 1$.

Using the ideal point method, the equivalent deterministic model can be presented as follows:

$$\begin{cases} \min\limits_{x_1,x_2} C[f(x,\xi_1,\xi_2)] = \sqrt{(C[f_1(x,\xi_1,\xi_2)]-f_1^0)^2+(C[f_2(x,\xi_1,\xi_2)]-f_2^0)^2} \\ \qquad \text{subject to :} \\ \qquad\qquad -2 \le x_1, x_2 \le 2 \end{cases} \tag{4.3}$$

where f_1^0 and f_2^0 denote the optimal values of $C[f_1(x,\xi_1,\xi_2)]$ and $C[f_2(x,\xi_1,\xi_2)]$ without considering another objective, respectively.

Since C denotes the general meaning of valuation of random objective functions, three kinds of specific meaning and corresponding deterministic models are considered here, that is, the expected value model, the minimum variance model and the $\alpha-$optiministic value model.

The expected value model is solved first using the hybrid algorithm proposed in Section 3. Firstly, using LHS to generate sample in the feasible set. As the information of (x_1, x_2) is unknown beforehand, it is assumed that decision variables (x_1, x_2) follow uniform distribution $\mathscr{U}(-2, 2)$. The sample size is set as 500. Then Monte Carlo simulation is adopted to calculate the expected value of $E[f_1(x,\xi_1,\xi_2)]$ and $E[f_2(x,\xi_1,\xi_2)]$ on the 500 sample points generated by LHS. The number of expected value calculation cycle is set as 5000. The first ten sample points and its corresponding objective values are shown in Table 3.

Table 3. The first ten sample points and corresponding objective values

Sample Point	Sample Data	$E[f_1(x,\xi_1,\xi_2)]$	$E[f_2(x,\xi_1,\xi_2)]$
SP1	(0.0012,-1.2514)	1.5118	1.5695
SP2	(-0.2989,-1.2339)	2.4112	1.4701
SP3	(-0.8486,-0.9854)	3.0114	1.6675
SP4	(-1.7186,1.8216)	3.0121	6.3310
SP5	(-1.5538,0.8206)	3.0125	3.9422
SP6	(-0.9030,1.7695)	3.0114	4.4770
SP7	(-0.9130,0.5722)	3.0456	2.3637
SP8	(1.9990,-1.9940)	3.0096	7.6842
SP9	(-1.5418,-0.4542)	3.1798	2.9080
SP10	(-0.2729,0.3027)	2.8948	1.6806

After obtain the sample in feasible set and the corresponding expected values of objective functions, we use ABC-SVR method to build the surrogate model. We take the 1-450 sample data as the train data, and the 451-500 sample data as the test data to validate the surrogate model. The accuracy of output prediction is used to represent the regression performance, and compare the regression performance of ABC-SVR with normal SVR using grid search method and BP artificial neural network. In the ABC-SVR, parameters set for the ABC algorithm are given in Table 1. The mean square error (MSE) and coefficient of determination R^2 are adopted to be regression performance index of these three methods, and the comparison results are shown in Table 4 and 5.

As shown in Table 4 and 5, the regression performance of ABC-SVR is best, and has achieved an ideal precision.

After obtaining the two surrogate models of $E[f_1(x,\xi_1,\xi_2)]$ and $E[f_2(x,\xi_1,\xi_2)]$, the ABC algorithm is adopted to find the optimal solution of expected value model using linear weighted method and ideal point method. In the ABC algorithm, control parameters set for the ABC algorithm are the same as shown in Table 1.

Using linear weighted method, three scenarios are considered here, they are $\lambda_1 = 0.7, \lambda_2 = 0.3$, $\lambda_1 = 0.5, \lambda_2 = 0.5$, $\lambda_1 = 0.3, \lambda_2 = 0.7$, respectively. The results obtained are shown in Table 4.4.

Table 4. Regression performance comparison of three methods (ABC-SVR / SVR / ANN) about $E[f_1(x,\xi_1,\xi_2)]$

Method	Parameters	Regression Performance	
		MSE	R^2
ABC-SVR	bestc=1.5340, bestg=3.5834	0.00012	0.99875
SVR	bestc=0.4682, bestg=0.3356	0.00356	0.98499
ANN	hidden layer number=10	0.01647	0.96522

Table 5. Regression performance comparison of three methods (ABC-SVR / SVR / ANN) about $E[f_2(x,\xi_1,\xi_2)]$

Method	Parameters	Regression Performance	
		MSE	R^2
ABC-SVR	bestc=0.7462,bestg=0.164	0.00065	0.9982
SVR	bestc=2.6284,bestg=0.1864	0.0017	0.9819
ANN	hidden layer number=10	0.0028	0.9712

Table 6. Results obtained in expected value model using linear weighted method

Scenarios	Results	
$\lambda_1 = 0.7, \lambda_2 = 0.3$	$E[f_1(x,\xi_1,\xi_2)]$	1.3993
	$E[f_2(x,\xi_1,\xi_2)]$	2.3562
	Optimal solution	(-0.0157,-2)
$\lambda_1 = 0.5, \lambda_2 = 0.5$	$E[f_1(x,\xi_1,\xi_2)]$	1.5006
	$E[f_2(x,\xi_1,\xi_2)]$	1.9588
	Optimal solution	(-0.0200,-1.7071)
$\lambda_1 = 0.3, \lambda_2 = 0.7$	$E[f_1(x,\xi_1,\xi_2)]$	1.9742
	$E[f_2(x,\xi_1,\xi_2)]$	1.3857
	Optimal solution	(-0.0236,-1.5080)

As shown in the Table 6, the results obtained are different in three scenarios, this is due to that different weights denote different importance of the objectives, higher weight implies higher importance. Therefore, the optimal solutions obtained are different.

Using ABC algorithm and ideal point method, we can obtain the minimum values of $E[f_1(x,\xi_1,\xi_2)]$ and $E[f_2(x,\xi_1,\xi_2)]$ on the feasible set without considering other objectives. They are 1.1196, 1.2716, respectively. Then, we can solve the

Table 7. Results obtained in expected value model using ideal point method

	Results
$E[f_1(x,\xi_1,\xi_2)]$	1.6053
$E[f_2(x,\xi_1,\xi_2)]$	1.7532
Optimal solution	(-0.0236,-1.5080)

Table 8. Results obtained in minimum variance model using linear weighted method

Scenarios	Results	
$\lambda_1 = 0.7, \lambda_2 = 0.3$	$V[f_1(x,\xi_1,\xi_2)]$	1.1581
	$V[f_2(x,\xi_1,\xi_2)]$	2.3017
	Optimal solution	(0.1854,0.3297)
$\lambda_1 = 0.5, \lambda_2 = 0.5$	$V[f_1(x,\xi_1,\xi_2)]$	1.1843
	$V[f_2(x,\xi_1,\xi_2)]$	1.1596
	Optimal solution	(-0.1419,0.2198)
$\lambda_1 = 0.3, \lambda_2 = 0.7$	$V[f_1(x,\xi_1,\xi_2)]$	1.2104
	$V[f_2(x,\xi_1,\xi_2)]$	0.7798
	Optimal solution	(-0.1242,-0.1821)

Table 9. Results obtained in minimum variance model using ideal point method

	Results
$V[f_1(x,\xi_1,\xi_2)]$	1.1643
$V[f_2(x,\xi_1,\xi_2)]$	0.8005
Optimal solution	(-0.1668,-0.2527)

Table 10. Results obtained in $\alpha-$optiministic value model using linear weighted method

Scenarios	Results	
$\lambda_1 = 0.7, \lambda_2 = 0.3$	$f_1(x,\xi_1,\xi_2)_{sup}(\alpha)$	0.2470
	$f_2(x,\xi_1,\xi_2)_{sup}(\alpha)$	0.4366
	Optimal solution	(0.0012,-1.2514)
$\lambda_1 = 0.5, \lambda_2 = 0.5$	$f_1(x,\xi_1,\xi_2)_{sup}(\alpha)$	0.7220
	$f_2(x,\xi_1,\xi_2)_{sup}(\alpha)$	0.2138
	Optimal solution	(-0.2534,-1.6625)
$\lambda_1 = 0.3, \lambda_2 = 0.7$	$f_1(x,\xi_1,\xi_2)_{sup}(\alpha)$	1.2862
	$f_2(x,\xi_1,\xi_2)_{sup}(\alpha)$	0.1168
	Optimal solution	(-0.6105,-1.4748)

Table 11. Results obtained in $\alpha-$optiministic value model using ideal point method

	Results
$f_1(x,\xi_1,\xi_2)_{sup}(\alpha)$	0.3876
$f_2(x,\xi_1,\xi_2)_{sup}(\alpha)$	0.3672
Optimal solution	(-0.0988,-1.2830)

expected value model using ideal point method with the same way in the solution of expected value model using linear weighted method. The results obtained are shown in Table 7.

Following the same procedure in the solution of expected value model illustrated above, the results obtained in the minimum variance model and the $\alpha-$optiministic value model are shown as follows, where α is set as 0.8.

As shown in the Table 6-11, the Pareto efficient solutions obtained in three deterministic models are different from each other, this is due to that the meanings applied to remove the random ambiguity are different. Therefore, in the practical application of stochastic MOP, it needs to specify the meanings of random objective functions first, then remove the random ambiguity in objectives using the meanings specified to obtain the equivalent deterministic MOP problem, and finally generate the Pareto efficient solutions under the meanings specified. Additionally, in the same deterministic MOP model, the solutions obtained using linear weighted method and ideal point method are Pareto efficient, this indicates that the solution approach and the new hybrid algorithm proposed in this paper are valid.

5 Conclusions

The general purpose of this study is to propose a powerful hybrid algorithm to address the difficulty that the closed form of converted deterministic model in practical stochastic MOP problem is usually hard to obtain, which is integrated with Latin Hypercube Sampling (LHS), Monte Carlo simulation, Support Vector Regression (SVR) and Artificial Bee Colony (ABC) algorithm. A numerical example was provided to illustrate the validity of the solution approach and the performance of the hybrid algorithm. Our study shows that, different criteria of the equivalent deterministic MOP model can result in different solutions obtained, and the hybrid algorithm built in this paper is efficient to solve stochastic MOP problem. In our view, to be studied in future, a new solution method is needed for treating stochastic MOP problem on the Pareto front directly.

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