Using Indifference Information in Robust Ordinal Regression

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Abstract. In this paper, we propose an extension to Robust Ordinal Regression allowing it to take into account also preference information from questions about indifference between real and fictitious alternatives. In particular, we allow the decision maker to suggest a new alternative that is different from the existing alternatives, but equally preferable. As shown by several experiments in psychology of the decisions, choosing between alternatives is different from matching two alternatives since the two aspects involve two different reasoning strategies. Consequently,by including this type of preference information one can represent more faithfully the DM's preferences. Such information about indifference should narrow down the set of compatible value functions much more quickly than standard pairwise comparisons, and a first simple example at least indicates that this intuition seems to be correct.

1 Introduction

Multiple Criteria Decision Aiding (MCDA) (see [8,9]) aims to recommend the Decision Maker (DM) a decision that best fits her/his preferences when a plurality of criteria has to be taken into consideration. Typically, in MCDA, a set of alternatives $A = \{a_1, \ldots, a_n\}$ is described in terms of performances with respect to a coherent family of criteria $G = \{g_1, \ldots, g_m\}$ [24]. Without loss of generality, each criterion $g_j \in G$ can be considered as a real-valued function $g_j : A \rightarrow \mathcal{I}_j \subseteq \mathbb{R}$, such that for any $a, b \in A, g_j(a) \geq g_j(b)$ means that a is at least as good as b with respect to criterion g_j.

Given two alternatives $a, b \in A$ and considering their performances with respect to the m criteria belonging to G, very often a will be better than b for

A. Gaspar-Cunha et al. (Eds.): EMO 2015, Part II, LNCS 9019, pp. 205–217, 2015.
DOI: 10.1007/978-3-319-15892-1_14

some of the criteria while b will be better than a for the remaining criteria. For this reason, in order to cope with any multiple criteria decision problem, we need to aggregate the performances of the alternatives taking into account the preferences of the DM. The three most well known aggregation models are the following:

- MAVT - Multi-Attribute Value Theory (see [7,19]) assigning to each alternative $a \in A$ a real number representative of its desirability ,
- outranking methods (see [10,12,24]) building some outranking preference relations S on A, such that for any $a, b \in A$, aSb means that a is at least as good as b,
- decision rule models using a set of "if..., then..." decision rules induced from the DM's preference information through Dominance-based Rough Set Approach (DRSA, see [14,15,26,27]).

Such MCDA models have recently been integrated into Evolutionary Multi-objective Optimization (EMO) as a means to interact with the DM and focus the search to the part of the Pareto front most preferred by the DM [1,3].

In this paper we consider the first model and we take into consideration a value function $U : \prod_{j=1}^{m} \mathcal{I}_j \rightarrow \mathbb{R}$ such that for any $a, b \in A$, a is at least as good as b ($a \succsim b$) if $U(g_1(a), \ldots, g_m(a)) \geq U(g_1(b), \ldots, g_m(b))$. The simplest form of the value function is the additive form, defined as: $U(g_1(a), \ldots, g_m(a)) = \sum_{j=1}^{m} u_j(g_j(a))$, where $u_j(g_j(a))$ are non-decreasing functions of their arguments. In the following, for simplicity of notation, we shall use $U(a)$ instead of $U(g_1(a), \ldots, g_m(a))$ for all $a \in A$.

Application of any decision model requires the definition of its parameters which can be obtained by asking them directly to the DM or inferring them from preference information given by the DM. This second approach seems more practical because the DM can have some difficulty in realizing the exact meaning of the parameters in the preference model and, moreover, their direct elicitation requires a strong cognitive effort from the DM. The typical preference information considered in this case is the pairwise comparisons between alternatives on which the DM feels sufficiently confident. In this paper we propose a different type of preference information expressed in terms of indifference between two alternatives. More precisely, supposing that the DM declares that an alternative a is preferred to another alternative b, we ask the DM to indicate another alternative b^+, obtained by improving b on some criteria, so that alternative b^+ is indifferent to a. Another possible way to get a preference information in terms of indifference is the following. Supposing again that alternative a is preferred to alternative b, one can ask the DM to indicate an alternative a^-, obtained by deteriorating a on some criteria, so that a^- is indifferent to b. Yet another possibility is to consider an alternative a^{+-}, obtained from alternative a improving its performances on some criteria and deteriorating its performances on other criteria, so that a and a^{+-} are indifferent. The main advantage we expect from this type of preference information is that it should reduce the space of compatible value functions much more than usual information supplied in terms of preference pairwise comparisons. Indeed, from the mathematical point of view, the new preference information should be translated by equality constraints that, in case of a value function representing perfectly the preference of the DM, drastically will reduce the space of compatible value functions. In the following, to take into account a certain imprecision in the DM's preferences, we model the indifference information by imposing that the difference of the utilities of two indifferent alternatives, in absolute value, should be no greater than an indifference threshold. Anyway, even if we do not use equality constraints, the space of compatible value function is strongly reduced, especially if the considered indifference threshold is sufficiently small. Moreover, as proved by several experiments in psychology of the decisions [25,29], choosing between two alternatives is different from matching two alternatives since the two aspects involve two different reasoning strategies. Consequently, putting together usual preference information in terms of pairwise preference of one alternative over another with the new type of preference information we are introducing, permits to build a utility function representing the DMs preferences in a more faithful way. We think that this is beneficial also for the elicitation of preference information within the EMO algorithms, as it should allow a faster convergence of the interactive EMO algorithm to the part of the Pareto front most preferred by the DM.

The paper is organized as follows. In the next section we recall the basic concepts of ordinal regression and robust ordinal regression. In the third section we introduce the new type of preference information. In the fourth section we present a didactic example. Conclusions and perspective for future research are collected in the last section.

2 Ordinal Regression and Robust Ordinal Regression

2.1 Ordinal Regression

Each decision model requires the specification of some parameters. For example, using MAVT, the parameters are related to the formulation of the marginal value functions $u_j(g_j(a)), j = 1, \ldots, m$. Since, as explained previously, the indirect preference information is more applied in practice, within MCDA, many methods have been proposed to determine the parameters characterizing the considered decision model inducing the values of such parameters from some holistic preference comparisons of alternatives given by the DM. This indirect preference elicitation is the base of the ordinal regression paradigm.

The most well-known ordinal regression methodology is the UTA (UTilités Additives) method proposed by Jacquet-Lagrèze and Siskos [17], which aims at inferring one or more additive value functions from a given complete ranking of alternatives belonging to a reference set $A^R \subseteq A$. The method considers a piecewise additive value function $U(g_1(a), \ldots, g_m(a)) = \sum_{j=1}^{m} u_j(g_j(a))$ having marginal value functions $u_j(\cdot), j = 1, \ldots, m$, being piecewise-linear, with a predefined number of linear pieces. UTA uses linear programming to determine an additive value function compatible with the preference information provided by the DM. Technically, in order to check if there exists at least one additive function compatible with the preferences provided by the DM, one has to solve the following linear programming problem:

$$\left.\begin{array}{l}\varepsilon^* = \max\ \varepsilon,\ \text{s.t.}\\ U(a^*) \geq U(b^*) + \varepsilon \text{ if } a^* \succ b^*,\ \text{with } a^*, b^* \in A^R,\\ U(a^*) = U(b^*) \text{ if } a^* \sim b^*,\ \text{with } a^*, b^* \in A^R,\\ \left.\begin{array}{l}\sum_{j=1}^{m} u_j(\beta_j) = 1,\ \ u_j(\alpha_j) = 0,\ j = 1, \ldots, m,\\ u_j(g_j(a)) \geq u_j(g_j(b)) \text{ if } g_j(a) \geq g_j(b), \forall a, b \in A, j = 1, \ldots, m,\end{array}\right\} E\end{array}\right\} E^{A^R}$$

where

- β_j and α_j are the best and the worst considered values of criterion $g_j, j = 1, \ldots, m$,
- $\succ$ and $\sim$ are the asymmetric and the symmetric part of the binary relation $\succsim$ representing the DM's preference information, i.e., $a^* \succ b^*$ means that a^* is preferred to b^* while $a^* \sim b^*$ means that a^* and b^* are indifferent,
- here, as always in the following, ε is considered without any constraint on the sign.

If the set of constraints E^{A^R} is feasible and $\varepsilon^* > 0$, then there exists at least one additive value function compatible with the DM's preferences. If there is no compatible value function, i.e., if the preferences of the DM cannot be represented by an additive value function with pre-defined number of linear pieces, [17] suggests either to increase the number of linear pieces in some marginal value functions, or to select the utility function U that gets the sum of deviation errors close to minimum and minimizes the number of ranking errors in the sense of Kendall or Spearman distance.

The ordinal regression paradigm has been applied within the two main MCDA approaches, that is those using a value function as preference model [4,17,18,23,28], and those using an outranking relation as preference model [21,22].

2.2 Robust Ordinal Regression

Usually, from among many sets of parameters of a preference model representing the preference information given by the DM, only one specific set is selected and used to work out a recommendation.

Since the selection of one of these sets of parameters compatible with the preference information given by the DM is rather arbitrary, *Robust Ordinal Regression* (ROR; [5,6,16]) proposes to take into account simultaneously all of them, in order to obtain a recommendation in terms of necessary and possible consequences of applying all the compatible preference models on the considered set of alternatives; the *necessary* weak preference relation holds for any two alternatives $a, b \in A$ ($a \succsim^N b$) if and only if a is at least as good as b for all compatible preference models, while the *possible* weak preference relation holds for this pair ($a \succsim^P b$) if and only if a is at least as good as b for at least one compatible preference model.

Although UTA^{GMS} [16] is the first method applying the ROR concepts, in the following, we shall describe the GRIP method [11] being its generalization. Then, we shall mention the other applications of the ROR that have been published later in several papers.

2.3 GRIP

In the UTA^{GMS} method [16], which initiated the stream of further developments in ROR, the ranking of reference alternatives does not need to be complete as in the original UTA method [17]. Instead, the DM may provide pairwise comparisons just for those reference alternatives (s)he really wants to compare. Precisely, the DM is expected to provide a partial preorder $\succsim$ on A^R. Obviously, one may also refer to the relations of strict preference $\succ$ or indifference $\sim$.

The transition from a reference preorder to a value function is done in the following way: for $a^*, b^* \in A^R$,

$$\left.\begin{array}{l} U(a^*) \geq U(b^*) + \varepsilon, \quad \text{if} \ \ a^* \succ b^*, \\ U(a^*) = U(b^*), \quad \text{if} \ \ a^* \sim b^*, \end{array}\right\} E_1$$

where ε is a (generally small) positive value.

Observe that $a^* \sim b^*$ can be represented as follows:

$$|U(a^*) - U(b^*)| \leq \delta, \tag{1}$$

i.e.

$$\left.\begin{array}{l} U(a^*) - U(b^*) \leq \delta, \\ U(b^*) - U(a^*) \leq \delta, \end{array}\right\} \tag{2}$$

where δ is a non-negative indifference threshold considered to take into account imprecision in the preference information.

Observe that the case $\delta = 0$ collapses to the constraints expressed as equality, i.e. $U(a^*) = U(b^*)$. It is apparent that if the indifference constraints are expressed in terms of equality, one can get a more precise inference of the utility function U (e.g. in case U is expressed as weighted sum and there are only two criteria, then a single indifference comparison formulated in terms of equality is enough to determine univocally the utility function). However, observe that this greater precision can be misleading because a certain imprecision is always implicit in the preference information given by the DM.

In some decision making situations, the DM is willing to provide more information than a partial preorder on a set of reference alternatives, such as "a^* is preferred to b^* at least as much as c^* is preferred to d^*". The information related to the intensity of preference is also accounted for by the GRIP method [11]. It may refer to the comprehensive comparison of pairs of reference alternatives on all criteria or on a particular criterion only. Precisely, in the holistic case, the DM may provide a partial preorder $\succsim^*$ on $A^R \times A^R$, whose meaning is: for $a^*, b^*, c^*, d^* \in A^R$,

$$(a^*, b^*) \succsim^* (c^*, d^*) \Leftrightarrow a^* \text{ is preferred to } b^* \text{ at least as much as } c^* \text{ is preferred to } d^*.$$

When referring to a particular criterion $g_j \in G$, rather than to all criteria jointly, the meaning of the expected partial preorder $\succsim_j^*$ on $A^R \times A^R$ is the following: for $a^*, b^*, c^*, d^* \in A^R$,

$$(a^*, b^*) \succsim_j^* (c^*, d^*) \Leftrightarrow a^* \text{ is preferred to } b^* \text{ at least as much as } \\ c^* \text{ is preferred to } d^* \text{ on criterion } g_j.$$

In both cases, the DM is allowed to refer to the strict preference and indifference relations rather than to weak preference only. The transition from the partial preorder expressing intensity of preference to a value function is the following: for $a^*, b^*, c^*, d^* \in A^R$,

$$\left.\begin{array}{l} U(a^*) - U(b^*) \geq U(c^*) - U(d^*) + \varepsilon, \;\; \text{if } \; (a^*, b^*) \succ (c^*, d^*), \\ U(a^*) - U(b^*) = U(c^*) - U(d^*), \;\; \text{if } \; (a^*, b^*) \sim (c^*, d^*), \\ u_j(a^*) - u_j(b^*) \geq u_j(c^*) - u_j(d^*) + \varepsilon, \;\; \text{if } \; (a^*, b^*) \succ_j (c^*, d^*) \text{ for } g_j \in G, \\ u_j(a^*) - u_j(b^*) = u_j(c^*) - u_j(d^*), \;\; \text{if } \; (a^*, b^*) \sim_j (c^*, d^*) \text{ for } g_j \in G. \end{array}\right\} E_2$$

In order to check if there exists at least one model compatible with the preferences of the DM we solve the following linear programming problem:

$$\begin{array}{c} \varepsilon^* = \max \varepsilon \;\; \text{s.t.} \\ E \cup E_1 \cup E_2 = E^{DM} \end{array} \tag{3}$$

If the set of constraints E^{DM} is feasible and $\varepsilon^* > 0$, then there exists at least one additive value function compatible with the preference information provided by the DM, otherwise no additive value function is compatible with the provided information. In this case, the analyst can decide to check for the cause of the incompatibility [20] or can continue the decision aiding process accepting the incompatibility.

Denoting by $\mathcal{U}_{A^R}$ the set of value functions compatible with the preference information provided by the DM, in the GRIP method three necessary and three possible preference relations can be defined:

- $a \succsim^N b$ iff $U(a) \geq U(b)$ for all $U \in \mathcal{U}_{A^R}$, with $a, b \in A$,
- $a \succsim^P b$ iff $U(a) \geq U(b)$ for at least one $U \in \mathcal{U}_{A^R}$, with $a, b \in A$,
- $(a, b) \succsim^{*N} (c, d)$ iff $U(a) - U(b) \geq U(c) - U(d)$ for all $U \in \mathcal{U}_{A^R}$, with $a, b, c, d \in A$,
- $(a, b) \succsim^{*P} (c, d)$ iff $U(a) - U(b) \geq U(c) - U(d)$ for at least one $U \in \mathcal{U}_{A^R}$, with $a, b \in A$,

- $(a,b) \succsim_j^{*N} (c,d)$ iff $u_j(a) - u_j(b) \geq u_j(c) - u_j(d)$ for all $U \in \mathcal{U}_{A^R}$, with $a,b,c,d \in A$, $g_j \in G$,
- $(a,b) \succsim_j^{*P} (c,d)$ iff $u_j(a) - u_j(b) \geq u_j(c) - u_j(d)$ for at least one $U \in \mathcal{U}_{A^R}$, with $a,b \in A$, $g_j \in G$.

Given alternatives $a,b,c,d \in A$, and the sets of constraints

$$\left.\begin{array}{l} U(b) \geq U(a) + \varepsilon \\ E^{DM} \end{array}\right\} E^N(a,b), \qquad \left.\begin{array}{l} U(a) \geq U(b) \\ E^{DM} \end{array}\right\} E^P(a,b),$$

$$\left.\begin{array}{l} U(c) - U(d) \geq U(a) - U(b) + \varepsilon \\ E^{DM} \end{array}\right\} E^N(a,b,c,d),$$

$$\left.\begin{array}{l} U(a) - U(b) \geq U(c) - U(d) \\ E^{DM} \end{array}\right\} E^P(a,b,c,d),$$

$$\left.\begin{array}{l} u_j(c) - u_j(d) \geq u_j(a) - u_j(b) + \varepsilon \\ E^{DM} \end{array}\right\} E_j^N(a,b,c,d),$$

$$\left.\begin{array}{l} u_j(a) - u_j(b) \geq u_j(c) - u_j(d) \\ E^{DM} \end{array}\right\} E_j^P(a,b,c,d),$$

we get that:

- $a \succsim^N b$ iff $E^N(a,b)$ is infeasible or if $E^N(a,b)$ is feasible and $\varepsilon^N(a,b) \leq 0$, where $\varepsilon^N(a,b) = \max \varepsilon$, s.t. $E^N(a,b)$;
- $a \succsim^P b$ iff $E^P(a,b)$ is feasible and $\varepsilon^P(a,b) > 0$, where $\varepsilon^P(a,b) = \max \varepsilon$, s.t. $E^P(a,b)$;
- $(a,b) \succsim^{*^N} (c,d)$ iff $E^N(a,b,c,d)$ is infeasible or if $E^N(a,b,c,d)$ is feasible and $\varepsilon^N(a,b,c,d) \leq 0$, where $\varepsilon^N(a,b,c,d) = \max \varepsilon$, s.t. $E^N(a,b,c,d)$;
- $(a,b) \succsim^{*^P} (c,d)$ iff $E^P(a,b,c,d)$ is feasible and $\varepsilon^P(a,b,c,d) > 0$, where $\varepsilon^P(a,b,c,d) = \max \varepsilon$, s.t. $E^P(a,b,c,d)$;
- $(a,b) \succsim_j^{*^N} (c,d)$ iff $E_j^N(a,b,c,d)$ is infeasible or if $E_j^N(a,b,c,d)$ is feasible and $\varepsilon_j^N(a,b,c,d) \leq 0$, where $\varepsilon_j^N(a,b,c,d) = \max \varepsilon$, s.t. $E_j^N(a,b,c,d)$;
- $(a,b) \succsim_j^{*^P} (c,d)$ iff $E_j^P(a,b,c,d)$ is feasible and $\varepsilon_j^P(a,b,c,d) > 0$, where $\varepsilon_j^P(a,b,c,d) = \max \varepsilon$, s.t. $E_j^P(a,b,c,d)$;

As to properties of $\succsim^N$ and $\succsim^P$ on A, let us remind after [16] that:

- $\succsim^N$ is a partial preorder on A,
- $\succsim^N \subseteq \succsim^P$,
- $a \succsim^N b$ and $b \succsim^P c \;\Rightarrow\; a \succsim^P c$, $\forall a,b,c \in A$,
- $a \succsim^P b$ and $b \succsim^N c \;\Rightarrow\; a \succsim^P c$, $\forall a,b,c \in A$,
- $a \succsim^N b$ or $b \succsim^P a$, $\forall a,b \in A$.

The above properties are the minimal ones characterizing $\succsim^N$ and $\succsim^P$ [13]. Other interesting properties of $\succsim^N$ and $\succsim^P$ are the following [16]:

- $\succsim^P$ is strongly complete and negatively transitive,
- $\succ^P$ is complete, irreflexive and transitive.

3 Preference Information in Terms of Pairwise Indifference Comparisons

In this section we introduce a new type of preference information expressed in terms of indifference between alternatives. Of course, this new type of preference information is supposed to be added to the type of preference information already considered within GRIP and, more in general, within the ROR methods. Even more, as explained in the following, the new type of preference information is very often based on some preference information expressed in terms of strict preference already considered within ROR.

We shall present three typical types of preference information expressed in terms of indifference pairwise comparisons:

- suppose that the DM has already declared that a^* is preferred to b^*. In this case one can ask the DM to indicate a new alternative b_H^{*+} obtained from b^* improving the performances on criteria from $H \subseteq G$ such that a^* is indifferent to b_H^{*+}. This preference information will be represented as follows:

$$|U(a^*) - U(b_H^{*+})| \leq \delta, \tag{4}$$

  i.e.

$$\left.\begin{array}{l} U(a^*) - U(b_H^{*+}) \leq \delta, \\ U(b_H^{*+}) - U(a^*) \leq \delta \end{array}\right\} E(a^*, b_H^{*+})$$

  where δ is a non-negative indifference threshold considered to take into account imprecision in the preference information;
- suppose again that the DM has already declared that a^* is preferred to b^* and let us ask the DM to indicate a new alternative a_K^{*-} obtained from a^* deteriorating the performances on criteria from $K \subseteq G$ such that a_K^{*-} is indifferent with b^*. This preference information will be represented as follows:

$$|U(a_K^{*-}) - U(b^*)| \leq \delta, \tag{5}$$

  i.e.

$$\left.\begin{array}{l} U(a_K^{*-}) - U(b^*) \leq \delta, \\ U(b^*) - U(a_K^{*-}) \leq \delta \end{array}\right\} E(a_K^{*-}, b^*);$$

- let us consider a reference alternative a^* and let us ask the DM to indicate a new alternative $a_{H,K}^{*+-}$ obtained from a^* by improving the performances on criteria from H and deteriorating the performances on criteria from K with $H, K \subseteq G, H \cap K = \emptyset$ such that $a_{H,K}^{*+-}$ is indifferent to a^*. This preference information will be represented as follows:

$$|U(a_{H,K}^{*+-}) - U(a^*)| \leq \delta, \tag{6}$$

  i.e.

$$\left.\begin{array}{l} U(a_{H,K}^{*+-}) - U(a^*) \leq \delta, \\ U(a^*) - U(a_{H,K}^{*+-}) \leq \delta \end{array}\right\} E(a^*, a_{H,K}^{*+-}).$$

Besides the above three typical types, other preference information expressed in terms of indifference pairwise comparisons can be the following: supposing again that a^* is preferred to b^*,

- $a^{*+-}_{H,K}$ is indifferent with b^*, with the related constraint denoted by $E(a^{*+-}_{H,K}, b^*)$,
- a^* is indifferent with $b^{*+-}_{R,S}$, with the related constraint denoted by $E(a^*, b^{*+-}_{R,S})$,
- $a^{*+-}_{H,K}$ is indifferent with $b^{*+-}_{R,S}$, with the related constraint denoted by $E(a^{*+-}_{H,K}, b^{*+-}_{R,S})$,

with $H, K, R, S \subseteq G, H \cap K = \emptyset, R \cap S = \emptyset$.

ROR methodology proceeds as explained before, simply adding constraints $E(a^*, b^{*+}_H), E(a^{*-}_K, b^*)$,$E(a^*, a^{*+-}_{H,K})$, $E(a^{*+-}_{H,K}, b^*), E(a^*, b^{*+-}_{R,S})$ and $E(a^{*+-}_{H,K}, b^{*+-}_{R,S})$ to set of constraints E^{DM}.

Let us observe that the new type of preference information is translated by inequalities such as the classical preference information as introduced in all ROR methods and, therefore, the recommendations obtained by the new model can be considered appropriate and consistent for the decision problem at hand. Moreover, as already observed in [25,29], choosing between two alternatives is different from matching two alternatives since the two aspects involve two different reasoning strategies. Consequently, we think that putting together these types of preference information can represent more faithfully the DM's preferences.

4 Didactic Example

In order to illustrate the proposed methodology, in this section we shall provide a didactic example. Let us suppose that 8 alternatives are evaluated on 4 criteria that should be maximized. The evaluations of the alternatives on the considered criteria are shown in Table 1 and, for the sake of simplicity, we shall suppose that the evaluation criteria can assume 5 discrete values only $(1, \ldots, 5)$.

Let us observe that the dominance relation on the set of alternatives A is empty because no alternative dominates another alternative.
In a first moment, let us suppose that the DM provides the following preference information:

$$a \succ f, \quad c \succ h, \quad b \succ e, \quad c \succ d, \quad d \succ f, \quad e \succ h$$

Using this preference information, we get the following necessary preference relation:

$$\succsim^N = \{(a, f), (b, e), (b, h), (c, d), (c, f), (c, h), (d, f), (e, h)\} \cup \{(x, x) : x \in A\}.$$

Table 1. Alternatives' evaluations

Alternative / Criterion	g_1	g_2	g_3	g_4
a	5	5	1	5
b	5	3	5	1
c	4	1	5	5
d	4	4	4	2
e	4	4	2	4
f	5	2	3	2
h	4	2	3	4
l	5	5	3	1

Let us suppose now that dealing with the same decision problem the DM provides the following preference information:

- $a \succ f, \quad c \succ h, \quad b \succ e,$
- $a \sim f^{+}_{\{2,4\}},$
- $c^{-}_{\{1,3\}} \sim h,$
- $e \sim e^{+-}_{\{3\},\{4\}},$

where $f^{+}_{\{2,4\}} = (5,4,3,4)$, $c^{-}_{\{1,3\}} = (3,1,4,5)$ and $e^{+-}_{\{3\},\{4\}} = (4,4,4,2) = d$. After considering the new set of constraints

$$\left.\begin{array}{l}
U(a) \geq U(f) + \varepsilon, \\
U(c) \geq U(h) + \varepsilon, \\
U(b) \geq U(e) + \varepsilon, \\
U(a) - U(f^{+}_{\{2,4\}}) \leq \delta, \\
U(f^{+}_{\{2,4\}}) - U(a) \leq \delta, \\
U(c^{-}_{\{1,3\}}) - U(h) \leq \delta, \\
U(h) - U(c^{-}_{\{1,3\}}) \leq \delta, \\
U(e^{+-}_{\{3\},\{4\}}) - U(e) \leq \delta, \\
U(e) - U(e^{+-}_{\{3\},\{4\}}) \leq \delta,
\end{array}\right\} E^{DM}_{*}$$

translating the preference information provided by the DM where $\delta = 10^{-4}$, we solve the linear programming problems shown in Section 3, getting the following necessary preference information:

$$\succsim^{N} = \{(a,e),(a,f),(a,h),(b,d),(b,e),(c,h),(d,f),(d,b),(d,e)\} \cup \{(x,x) : x \in A\}.$$

For example, by solving the optimization problem

$$\varepsilon^* = \max \varepsilon, \ \text{ s.t.}$$
$$\left.\begin{array}{l} U(e) \geq U(a) + \varepsilon, \\ E_*^{DM} \cup E \end{array}\right\}$$

where E is the set of normalization and monotonicity constraints defined in Section 2, we get $\varepsilon^* = 0$ and, consequently, $a \succsim^N e$.

Observe that in this second case we get a slightly richer preference relation in terms of pairs of alternatives from A for which necessary preference holds (9 non-trivial pairs in this second case vs 8 non-trivial pairs in the first case), with a smaller cognitive effort in terms of number of alternatives from A considered in the preference information (6 - a, b, c, e, f, h - in the second case vs 7 - a, b, c, d, e, f, h - in the first case).

5 Conclusions

In this paper we introduced new types of preference information in Robust Ordinal Regression. More precisely we considered pairwise indifference comparisons between real or fictitious alternatives. We believe that this new type of preference information could permit to get a more precise induction of the DM value function with a smaller cognitive effort. Moreover, the introduction of the new type of preference information makes the obtained value function more faithful because, according to the evidence of a certain number of experiments in Psychology of the decision, choosing between two alternatives (corresponding to the usual preference information) is different from matching two alternatives (corresponding to the new type of preference information) since the two aspects involve two different reasoning strategies. The results of a very first didactic example presented in this paper seem promising, but a lot of work remains to be done. In particular we envisage the following perspectives for the future research:

- we have to measure the advantages in terms of smaller cognitive effort and better results of the MCDA procedure offered by the new type of preference information;
- we have to discuss how to manage the selection of criteria to be modified in order to get indifference in the considered pairwise comparisons between alternatives;
- we have to verify how beneficial can be the use of the new type of preference information in EMO procedures based on preferences, especially those procedures based on ROR (e.g. [1–3].

Acknowledgments. This work has been partly funded by the "Programma Operativo Nazionale" Ricerca & Competitivitá "2007-2013" within the project "PON04a2 E SINERGREEN-RES-NOVAE".

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