Revisiting Default Description Logics – and Their Role in Aligning Ontologies

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Abstract. We present a new approach to extend the Web Ontology Language (OWL) with the capabilities to reason with defaults. This work improves upon the previously established results on integrating defaults with description logics (DLs), which were shown to be decidable only when the application of defaults is restricted to named individuals in the knowledge base. We demonstrate that the application of defaults (integrated with DLs) does not have to be restricted to named individuals to retain decidability and elaborate on the application of defaults in the context of ontology alignment and ontology-based systems.

1 Introduction and Motivation

The wide adoption of linked data principles has led to an enormous corpus of semantically enriched data being shared on the web. Researchers have been building (semi-)automatic matching systems [1,24] to build links (correspondences) between various conceptual entities as well as instances in the linked data. These systems are commonly known as ontology matching/alignment systems. The correspondences generated by these systems are represented using some standard knowledge representation language such as the web ontology language (OWL). However, due to the amount of heterogeneity present in the linked data and the web, OWL does not seem to be a completely suitable language for this purpose as we discuss in the following.

One key aspect of the web (or the world) is variety. There are subtle differences in how a conceptual entity and its relation to other entities is perceived depending on the geographical location, culture, political influence, etc. [15]. To give a simple example consider the concept of marriage, in some conservative parts of the world, marriage stands for a relationship between two individuals of opposite genders whereas in other more liberal places the individuals involved may have the same gender. Consider the axioms in Figure 1, let axioms (1) to (8) represent a part of ontology A (the conservative perspective) and axioms (9) to (13) represent a part of ontology B (the liberal perspective). It would be safe to assume that an ontology matching system would output the axioms (14) to (16) as the correspondences between these two ontologies. This however, leads to a logical inconsistency under OWL semantics when the two ontologies are

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$$a{:}hasWife \sqsubseteq a{:}hasSpouse \quad (1)$$
$$symmetric(a{:}hasSpouse) \quad (2)$$
$$\exists a{:}hasSpouse.a{:}Female \sqsubseteq a{:}Male \quad (3)$$
$$\exists a{:}hasSpouse.a{:}Male \sqsubseteq a{:}Female \quad (4)$$
$$a{:}hasWife(\text{a:john}, a{:}mary) \quad (5)$$
$$a{:}Male(a{:}john) \quad (6)$$
$$a{:}Female(a{:}mary) \quad (7)$$
$$a{:}Male \sqcap a{:}Female \sqsubseteq \bot \quad (8)$$

$$symmetric(b{:}hasSpouse) \quad (9)$$
$$b{:}hasSpouse(b{:}mike, b{:}david) \quad (10)$$
$$b{:}Male(b{:}david) \quad (11)$$
$$b{:}Male(b{:}mike) \quad (12)$$
$$b{:}Female(b{:}anna) \quad (13)$$

$$a{:}hasSpouse \equiv b{:}hasSpouse \quad (14)$$
$$a{:}Male \equiv b{:}Male \quad (15)$$
$$a{:}Female \equiv b{:}Female \quad (16)$$

Fig. 1. Running example with selected axioms

merged based on the given correspondences: From axioms (10), (11), (14), and (3), we derive *a:Female(b:mike)* which together with axioms (12), (15), and (8) results in an inconsistency as we derive *a:Male(b:mike)* and *a:Female(b:mike)* while axiom (8) states $a{:}Male \sqcap a{:}Female \sqsubseteq \bot$.

This drives the need for an alignment language which could handle such subtle differences in perspectives. We propose an extension of description logics based on defaults to be used as an ontology alignment language. Using the notion of defaults we could re-state axiom (14) to an axiom which would semantically mean: every pair of individuals in *b:hasSpouse* is also in *a:hasSpouse* (and vice versa) unless it leads to a logically inconsistency. And those pairs which lead to an inconsistency are treated as exceptions to this axiom. In such a setting the pair *(b:mike, b:David)* would be treated as an exception to the re-stated axiom and would not cause an inconsistency any more.

A *default* is a kind of an inference rule that enables us to model some type of stereotypical knowledge such as "birds usually fly," or "humans usually have their heart on the left side of their chest." Default logic, which formalizes this intuition, was introduced by Ray Reiter [21] in the 80s, and it is one of the main approaches towards non-monotonic reasoning. In fact, it was the starting point for one of the primary approaches to logic programming and non-monotonic reasoning today, namely the stable model semantics [8] and answer set programming [20].

Reiter's approach is so powerful because exceptions to the default rules are implicitly handled by the logic, so that it is not left to the knowledge modeler to know all relevant exceptions and to take care of them explicitly, as is required in OWL-based ontology modeling. In fact, defaults in the general sense of Reiter still appear to be one of the most intuitive ways of formally modeling this type of stereotypical reasoning [11].

Alas, a paper by Baader and Hollunder [2], published almost 20 years ago, seemed to put an early nail into the coffin of default-extended description logics.

Therein, the authors show that a certain extension of the description logic $\mathcal{ALC}$[1] becomes undecidable if further extended with Reiter defaults. Since decidability was (and still is) a key design goal for description logics, this result clearly was a showstopper for further development of default-extended description logics. Of course, Baader and Hollunder also provided a quick fix: If we impose that the default rules only apply to known individuals (i.e., those explicitly present in the knowledge base), then decidability can be retained. However, this semantics for defaults is rather counter-intuitive, as it implies that default rules never apply to unknown individuals. In other words: to unknown individuals the defaults do, by default, not apply. Arguably, this is not a very intuitive semantics for defaults.

In this paper, we show that there is still a path of development for default-extended description logics, and that they may yet attain a useful standing in ontology modeling. In fact, we will present a way to extend decidable description logics with defaults which transcends the approach by Baader and Hollunder while retaining decidability: in our approach, defaults rules do apply to unknown individuals. We refer to the type of default semantics which we introduce as *free defaults*. Indeed, the contributions of this paper are (1) A new semantics for defaults (free defaults) in description logics and thereby OWL, such that the application of defaults are not limited to named individuals in the knowledge base, (2) We show that reasoning under this new semantics is decidable, which improves upon the results shown in [2], (3) Adding default role inclusion axioms also yields a decidable logic, and (4) We introduce the use of free defaults as a basis for a new language for ontology alignment and show some application scenarios in where defaults could play a major role and show how our approach covers these scenarios.

Let us briefly look at some of the related work published in recent years. There is in fact a plethora of publications on integrating description logics and non-monotonic formalisms, and it is not feasible to give all relevant proposals sufficient credit. We thus refer the interested reader to [18] for pointers to some of the most important approaches to date, including their relationships.

A series of work has recently been published which is related to typicality reasoning in description logics [9], in which the authors provide a minimal model semantics to achieve typicality. While our work is also based on preferred models, our goal is to follow some of the central ideas of default logic and to adapt it to provide a model-theoretic approach to defaults in DLs. Exact formal relationships between different proposals remain to be investigated. Other approaches that could be used to simulate defaults include circumscription [4,23], while again the exact relationship between the approaches remains to be investigated. Also [3] talks about defeasible inclusions in the tractable fragments of DL, which again follows a similar intuition. We understand our proposal and results as a contribution to the ongoing discussion about the best ways to capture non-monotonic reasoning for Semantic Web purposes.

[1] $\mathcal{ALC}$ is a very basic description logic which, among other things, constitutes the core of OWL 2 DL [13,14].

The remainder of this paper is organized as follows. Section 2 introduces preliminaries which form the basis required for the understanding of our work. In section 3, we discuss the semantics of free defaults for description logics, in the case of subclass defaults. In section 4 we give decidability results. In section 5 we show that adding free subrole defaults also retains decidability. In section 6 we discuss examples which illustrate the potential and relevance of free defaults to ontology modeling. We conclude in section 7.

2 Preliminaries

2.1 Default Logic

Default logic [21] is a form of non-monotonic logic which allows us to add inference rules called default rules on top of the conceptual knowledge. A *default rule* (or simply *default*) is of the form

$$\frac{\alpha : \beta_1, \ldots, \beta_n}{\gamma},$$

where α, β_i, γ are first order formulae. α is called the *pre-requisite* of the rule, $\beta_1, \ldots, \beta_n$ are its *justifications* and γ its *consequent*. A default rule is *closed* if all the formulae that occur in the default are closed first order formulae, otherwise the default rule is called *open*. A *default theory* is further defined as a pair $(\mathcal{D}, \mathcal{W})$, where $\mathcal{D}$ is a set of defaults and $\mathcal{W}$ is a set of closed first order formulae. A default theory is *closed* if all the default rules in the set $\mathcal{D}$ are closed, otherwise it is called an *open* default theory.

The intuitive meaning of a default rule is that if α is true, and if furthermore assuming $\beta_1, \ldots, \beta_n$ to be true does not result in an inconsistency, then γ is entailed. The formal semantics of a default theory is defined in terms of a notion of extension. An extension of a default theory is a completion (i.e., closure under entailment) of a possibly incomplete theory. The following describes formally the notion of an extension, directly taken from [21].

Definition 1. *Let $\Delta = (\mathcal{D}, \mathcal{W})$ be a closed default theory, so that every default of $\mathcal{D}$ has the form*

$$\frac{\alpha : \beta_1, \ldots, \beta_n}{\gamma},$$

where $\alpha, \beta_1, \ldots, \beta_n, \gamma$ are all closed formulae of L (a first order language). For any set of closed formulae $S \subseteq L$, let $\Gamma(S)$ be the smallest set satisfying the following three properties:
- $\mathcal{W} \subseteq \Gamma(S)$
- $\Gamma(S)$ is closed under entailment.
- If $\frac{\alpha:\beta_1,\ldots\beta_n}{\gamma} \in \mathcal{D}$, $\alpha \in \Gamma(S)$, and $\neg\beta_1, \ldots, \neg\beta_n \notin \Gamma(S)$, then $\gamma \in \Gamma(S)$.
A set of closed formulae $E \subseteq L$ is an extension *of Δ if $\Gamma(E) = E$, i.e. if E is a fixed point of the operator Γ.*

The complexity of reasoning with (variants of) default logic is generally very high [10], and the same holds for most other non-monotonic logics, unless severe restrictions are put in place.[2]

In this paper we deal with a special type of defaults called *normal defaults*, and give it our own semantics which satisfies the intuition we intend to serve. We do this, rather than attempt to build on the more general approach by Reiter, because we strive for a simple but useful approach [23]. Normal defaults are those in which the justification and consequent are the same. We observe that these kinds of defaults have a variety of applications as we will see in section 6.

2.2 Description Logics

We briefly introduce the basic description logic (DL) $\mathcal{ALC}$, although our approach also works for more expressive description logics.

Let $\mathsf{N}_C, \mathsf{N}_R$ and N_I be countably infinite sets of concept names, role names and individual names, respectively. The set of $\mathcal{ALC}$ *concepts* is the smallest set that is created using the following grammar where $A \in \mathsf{N}_\mathsf{C}$ denotes an atomic concept, $R \in \mathsf{N}_\mathsf{R}$ is a role name and C, D are concepts.

$$C ::= \top \mid \bot \mid A \mid \neg C \mid C \sqcap D \mid C \sqcup D \mid \exists R.C \mid \forall R.C$$

An $\mathcal{ALC}$ *TBox* is a finite set of axioms of the form $C \sqsubseteq D$, called *general concept inclusion* (GCI) *axioms*, where C and D are concepts. An $\mathcal{ALC}$ *ABox* is a finite set of axioms of the form $C(a)$ and $R(a, b)$, which are called *concept* and *role assertion axioms*, where C is a concept, R is a role and a, b are individual names. An $\mathcal{ALC}$ *knowledge base* is a union of an $\mathcal{ALC}$ ABox and an $\mathcal{ALC}$ TBox

The semantics is defined in terms of interpretations $\mathcal{I} = (\Delta^\mathcal{I}, \cdot^\mathcal{I})$, where $\Delta^\mathcal{I}$ is a non-empty set called the *domain* of interpretation and $\cdot^\mathcal{I}$ is an interpretation function which maps each individual name to an element of the domain $\Delta^\mathcal{I}$ and interprets concepts and roles as follows:

$$\top^\mathcal{I} = \Delta^\mathcal{I}, \quad \bot^\mathcal{I} = \emptyset, \quad A^\mathcal{I} \subseteq \Delta^\mathcal{I}, \quad R^\mathcal{I} \subseteq \Delta^\mathcal{I} \times \Delta^\mathcal{I},$$
$$(\neg C)^\mathcal{I} = \Delta^\mathcal{I} \setminus C^\mathcal{I}, \qquad (C_1 \sqcap C_2)^\mathcal{I} = C_1^\mathcal{I} \cap C_2^\mathcal{I}, \qquad (C_1 \sqcup C_2)^\mathcal{I} = C_1^\mathcal{I} \cup C_2^\mathcal{I},$$
$$(\forall r.C)^\mathcal{I} = \{x \in \Delta^\mathcal{I} \mid (x, y) \in r^\mathcal{I} \text{ implies } y \in C^\mathcal{I}\},$$
$$(\exists r.C)^\mathcal{I} = \{x \in \Delta^\mathcal{I} \mid \text{there is some } y \text{ with } (x, y) \in r^\mathcal{I} \text{ and } y \in C^\mathcal{I}\}$$

An interpretation $\mathcal{I}$ *satisfies* (is a *model* of) a GCI $C \sqsubseteq D$ if $C^\mathcal{I} \subseteq D^\mathcal{I}$, a concept assertion $C(a)$ if $a^\mathcal{I} \in C^\mathcal{I}$, a role assertion $R(a, b)$ if $(a^\mathcal{I}, b^\mathcal{I}) \in R^\mathcal{I}$. We say $\mathcal{I}$ *satisfies* (is a *model* of) a knowledge base K if it satisfies every axiom in K. K is *satisfiable* if such a model $\mathcal{I}$ exists.

The negation normal form of a concept C, denoted by $\mathsf{NNF}(C)$, is obtained by pushing the negation symbols inward, as usual, such that negation appears only in front of atomic concepts, e.g., $\mathsf{NNF}(\neg(C \sqcup D)) = \neg C \sqcap \neg D$.

We will occasionally refer to additional description logic constructs which are not contained in $\mathcal{ALC}$. Please refer to [14] for further background on description logics, and how they relate to the Web Ontology Language OWL [13].

[2] An exception is [17] for tractable description logics, but the practical usefulness of that approach for default modeling still needs to be shown.

3 Semantics of Free Defaults

In this section, we introduce the semantics of free defaults. We restrict our attention to normal defaults and show that reasoning in this setting is decidable in general when the underlying DL is also decidable. Normal defaults are very intuitive and we observe that there are many applications in practice where normal defaults can be very useful—see section 6. We also provide a DL syntax to encode default rules in the knowledge bases. For our purposes, a normal default rule is of the form $\frac{A:B}{B}$, where A and B are class names,[3] i.e., the justification and conclusion of the default rule are the same. For a description logic $\mathcal{L}$ we are going to represent the same rule in the form of an axiom $A \sqsubseteq_d B$, where A and B are $\mathcal{L}$-concepts and $\sqsubseteq_d$ represents (*free*) *default subsumption.* We refer to statements of the form $A \sqsubseteq_d B$ as (*free*) *default rules* or *default axioms.*

Definition 2. *Let KB be a description logic knowledge base, and let δ be a set of default axioms of the form $C \sqsubseteq_d D$, where C and D are concepts appearing in KB. Then we call the pair (KB, δ) a* default-knowledge-base.

The semantics of the default subsumption can be informally stated as follows: if $C \sqsubseteq_d D$, then every named individual in C can also be assumed to be in D, unless it results in a logical inconsistency. Also, if $C \sqsubseteq_d D$, then every unnamed individual in C is also in D, i.e., for unnamed individuals $\sqsubseteq_d$ behaves exactly the same as $\sqsubseteq$. Furthermore, we say a named individual a satisfies a default axiom $C \sqsubseteq_d D$ if (1) $a^\mathcal{I} \in C^\mathcal{I}, D^\mathcal{I}$ or (2) $a^\mathcal{I} \in (\neg C)^\mathcal{I}$. The intuition behind the semantics of free defaults is to maximize the sets of the named individuals that satisfy the default axioms while maintaining the consistency of the knowledge base.

The following notations will be needed to formalize this intuition for the semantics of free defaults.

Definition 3. *For a default-knowledge-base (KB, δ), we define the following.*
- Ind_{KB} *is the set of all the named individuals occurring in KB.*
- $\mathsf{P}(\mathsf{Ind}_{KB})$ *is the power set of* Ind_{KB}.
- $\mathsf{P}^n(\mathsf{Ind}_{KB})$ *is the set of n-tuples obtained from the Cartesian product:* $\mathsf{P}(\mathsf{Ind}_{KB}) \times \ldots_{\ n\ times} \times \mathsf{P}(\mathsf{Ind}_{KB})$, *where n is the cardinality of δ.*

The notion of interpretation for the default-knowledge-bases (KB, δ) remains the same as that of the underlying DL of the knowledge base KB.[4] Additionally, given an interpretation $\mathcal{I}$, we define $\delta^\mathcal{I}$ to be the tuple $(X_1^\mathcal{I}, \ldots, X_n^\mathcal{I})$, where each $X_i^\mathcal{I}$ is the set of interpreted named individuals that satisfy the i^{th} default $C_i \sqsubseteq_d D_i$ in the sense that $X_i^\mathcal{I} = (C_i^\mathcal{I} \cap D_i^\mathcal{I} \cap \Delta_{Ind}^\mathcal{I}) \cup ((\neg C_i)^\mathcal{I} \cap \Delta_{Ind}^\mathcal{I})$ with $\Delta_{Ind}^\mathcal{I} = \{a^\mathcal{I} \mid a \in \mathsf{Ind}_{KB}\} \subseteq \Delta^\mathcal{I}$ being the set of interpreted individuals occurring in the knowledge base. We now need to define a preference relation over the interpretations such that we can compare them on the basis of the sets of named individuals satisfying each default.

[3] We will lift this to roles in section 5.

[4] See section 2.

Definition 4. *(Preference relation $>_{KB,\delta}$) Given a knowledge base KB and a set of default axioms δ. Let $\mathcal{I}$ and $\mathcal{J}$ be two interpretations of the pair (KB, δ), then we say that $\mathcal{I}$* is preferred over *$\mathcal{J}$ or $\mathcal{I} >_{KB,\delta} \mathcal{J}$ if all of the following hold.*
1. $a^{\mathcal{I}} = a^{\mathcal{J}}$ *for all* $a \in \mathsf{N}_I$
2. $X_i^{\mathcal{I}} \supseteq X_i^{\mathcal{J}}$ *for all* $1 \leq i \leq |\delta|$, *where* $X_i^{\mathcal{I}} \in \delta^{\mathcal{I}}$ *and* $X_i^{\mathcal{J}} \in \delta^{\mathcal{J}}$.
3. $X_i^{\mathcal{I}} \supset X_i^{\mathcal{J}}$ *for some* $1 \leq i \leq |\delta|$, *where* $X_i^{\mathcal{I}} \in \delta^{\mathcal{I}}$ *and* $X_i^{\mathcal{J}} \in \delta^{\mathcal{J}}$.

The concept of a model under the semantics of free defaults would be the one which is maximal with respect to the above relation.

Definition 5. *(d-model) Given (KB, δ), we call $\mathcal{I}$ a* d-model *of KB with respect to a set of defaults δ, written $\mathcal{I} \models_d (KB, \delta)$, if all of the following hold.*
1. *$\mathcal{I}$ satisfies all axioms in KB.*
2. $C_i^{\mathcal{I}} \setminus \Delta_{Ind}^{\mathcal{I}} \subseteq D_i^{\mathcal{I}}$, *for each* $(C_i \sqsubseteq_d D_i) \in \delta$.
3. *There is no interpretation $\mathcal{J} >_{KB,\delta} \mathcal{I}$ satisfying conditions 1 and 2 above.*

Furthermore, if (KB, δ) has at least one model, then the default knowledge base is said to be d-satisfiable.

The following proposition is obvious from the definition of d-model.

Proposition 1. *If $\mathcal{I}$ is a d-model of the default-knowledge-base (KB, δ), then $\mathcal{I}$ is a classical model of KB.*

For default theories two types of entailments are usually considered: credulous and skeptical [21]. A logical formula is a credulous entailment if it is true in at least one of the extensions of the default theory. Skeptical entailment requires the logical formula to be true in all the extensions. We follow the skeptical entailment approach as it fits better to the description logic semantics.[5]

Definition 6. *(d-entailment) Given a default-knowledge-base (KB, δ) and DL axiom α, α is* d-entailed *by (KB, δ) if it holds in all the d-models of (KB, δ).*

4 Decidability

In this section we show that the tasks of checking for d-satisfiability and d-entailment for default-knowledge-bases are decidable in general. Let (KB, δ) be a default-knowledge-base where *KB* is encoded in a decidable DL $\mathcal{L}$ which supports nominal concept expressions. We show that finding a d-model for (KB, δ) is also decidable. For some $\mathcal{P} = (X_1, \ldots, X_n) \in \mathsf{P}^n(\mathsf{Ind}_{KB})$, let $KB_{\mathcal{P}}$ be the knowledge base that is obtained by adding the following axioms to *KB*, for each $C_i \sqsubseteq_d D_i \in \delta$:
1. $\overline{X_i} \equiv (C_i \sqcap D_i \sqcap \{a_1, \ldots, a_k\}) \sqcup (\neg C_i \sqcap \{a_1, \ldots, a_k\})$, where $\overline{X_i}$ is the nominal expression $\{x_1, \ldots, x_m\}$ containing exactly all the named individuals in X_i, and $\{a_1, \ldots, a_k\} = \mathsf{Ind}_{KB}$.
2. $C_i \sqcap \neg\{a_1, \ldots, a_k\} \sqsubseteq D_i$, where $\{a_1, \ldots, a_k\} = \mathsf{Ind}_{KB}$.

[5] Whether credulous entailment is useful in a Semantic Web context is to be determined.

The first step in the above construction is useful to identify the sets of default-satisfying individuals. The extensions of the $\overline{X_i}$s represent those sets. The second step ensures all the unnamed individuals satisfy the default axioms. Notice that $KB_{\mathcal{P}}$ as constructed using the above rewriting steps makes it fall under the expressivity of the DL $\mathcal{L}$, and construction of $KB_{\mathcal{P}}$ requires only a finite number of steps since δ is a finite set. Furthermore, we can compute an order $\succ$ on the set $\mathsf{P}^n(\mathsf{Ind}_{KB})$ based on the $\supseteq$-relation, defined as follows: Let $\mathcal{P}_1, \mathcal{P}_2 \in \mathsf{P}^n(\mathsf{Ind}_{KB})$, then $\mathcal{P}_1 \succ \mathcal{P}_2$ iff

1. $X_{1i} \supseteq X_{2i}$ for each $X_{1i} \in \mathcal{P}_1$ and $X_{2i} \in \mathcal{P}_2$ and
2. $X_{1i} \supset X_{2i}$ for some $X_{1i} \in \mathcal{P}_1$ and $X_{2i} \in \mathcal{P}_2$.

Lemma 1. *Given a default-knowledge-base (KB, δ), if $KB_{\mathcal{P}}$ is classically satisfiable for some $\mathcal{P} \in \mathsf{P}^n(\mathsf{Ind}_{KB})$, then (KB, δ) has a d-model.*

Proof. Let $\mathcal{P}_1 \in \mathsf{P}^n(\mathsf{Ind}_{KB})$ such that $KB_{\mathcal{P}_1}$ has a classical model $\mathcal{I}$. Then there are two possible cases.

In the first case there is no $\mathcal{P}_x \in \mathsf{P}^n(\mathsf{Ind}_{KB})$ such that $\mathcal{P}_x \succ \mathcal{P}_1$ and $KB_{\mathcal{P}_x}$ has a classical model. In this case $\mathcal{I}$ satisfies all the conditions of a d-model: (1) $\mathcal{I}$ satisfies all axioms of KB since $KB \subseteq KB_{\mathcal{P}}$. (2) $\mathcal{I}$ satisfies condition 2 of the definition of d-model, this follows from the second step of the construction of $KB_{\mathcal{P}}$. (3) This follows directly from the assumption for this case. So $\mathcal{I}$ is a d-model for (KB, δ) in this case.

The second case is when there is some $\mathcal{P}_x \in \mathsf{P}^n(\mathsf{Ind}_{KB})$ for which there is a classical model $\mathcal{I}$ for $KB_{\mathcal{P}_x}$ and $\mathcal{P}_x \succ P_1$. Again, there are two possibilities as in case of $\mathcal{P}_1$. Either the first case above holds for $\mathcal{P}_x$, or there is some $\mathcal{P}_y \succ \mathcal{P}_x \in \mathsf{P}^n(\mathsf{Ind}_{KB})$ for which the second case holds. In the latter situation, the argument repeats, eventually giving rise to an ascending chain with respect to the order $\succ$ on $\mathsf{P}^n(\mathsf{Ind}_{KB})$. However, since $\mathsf{P}^n(\mathsf{Ind}_{KB})$ is finite this chain has a maximal element and thus the first case applies. Therefore, there is a d-model for (KB, δ). □

The following theorem is a direct consequence of Lemma 1 and the finiteness of δ.

Theorem 1. *The task of determining d-satisfiability of default-knowledge-bases is decidable.*

It should be noted that in case of Reiter's defaults it is known that for normal default theories an extension always exists, but in the case of free defaults it can be easily seen that there might be some default-knowledge-bases which do not have a d-model. This is not completely satisfactory of course. However, at this stage it is unknown whether a stronger result can be obtained without giving up decidability. Though the notion of d-satisfiability is important for checking that the default-knowledge-base modelled is consistent and can be used for reasoning-based query services, the more interesting problem in the case of default-knowledge-bases is to handle d-entailment inference services. As it can be observed that d-entailment checking is not directly reducible to satisfiability checking of the default-knowledge-base,[6] we define a mechanism of checking d-entailments and show that this is also decidable.

Proposition 2. *Let (KB, δ) be a default-knowledge-base. If $\mathcal{I}$ is a d-model of (KB, δ), then there exists $\mathcal{P} \in \mathsf{P}^n(\mathsf{Ind}_{KB})$ such that $\mathcal{I}$ is a classical model of $KB_{\mathcal{P}}$ and all classical models of $KB_{\mathcal{P}}$ are d-models of (KB, δ).*

Proof. Given $\mathcal{I}$ we construct a $\mathcal{P} \in \mathsf{P}^n(\mathsf{Ind}_{KB})$ as follows: Given a default $C_i \sqsubseteq_d D_i \in \delta$, let X_i be the maximal subset of Ind_{KB} such that $X_i^{\mathcal{I}} \subseteq (C_i \sqcap D_i)^{\mathcal{I}} \cup (\neg C_i)^{\mathcal{I}}$. Given all these $X_i's$, let $\mathcal{P} = \{X_1, \ldots, X_n\}$.

Clearly, $\mathcal{I}$ is then a classical model of $KB_{\mathcal{P}}$.

Furthermore, since $\mathcal{I}$ is a d-model of (KB, δ), there is no $\mathcal{P}_x \in \mathsf{P}^n(\mathsf{Ind}_{KB})$ such that $KB_{\mathcal{P}_x}$ has a classical model and $P_x \succ \mathcal{P}$. By construction of $KB_{\mathcal{P}}$ all classical models of $KB_{\mathcal{P}}$ satisfy the three d-model conditions for (KB, δ) because (1) $KB \subseteq KB_{\mathcal{P}}$, all axioms of KB are satisfied, (2) the second step of the construction of $KB_{\mathcal{P}}$ ensures the second condition of d-model is satisfied. (3) Since $\mathcal{P}$ satisfies the maximality condition, all classical models of $KB_{\mathcal{P}}$ also satisfy the maximality condition of being a d-model ensured by step one of the construction of $KB_{\mathcal{P}}$. $\square$

Consider the two sets

$$\mathcal{P}_{KB\text{-d}} = \{\mathcal{P} \in \mathsf{P}^n(\mathsf{Ind}_{KB}) \mid KB_{\mathcal{P}} \text{ is classically satisfiable}\}$$
$$\mathcal{P}_{KB\text{-d-model}} = \{\mathcal{P} \in \mathcal{P}_{KB\text{-d}} \mid \mathcal{P} \text{ is maximal w.r.t. } \succ\}$$

and note that they are both computable in finite time. We refer to all the $KB_{\mathcal{P}}$'s generated from all $\mathcal{P} \in \mathcal{P}_{\text{KB-d-model}}$ as *d-model generating knowledge bases.*

Lemma 2. *A DL axiom α is d-entailed by a default-knowledge-base (KB, δ) iff it is classically entailed by every $KB_{\mathcal{P}}$ obtained from KB, δ, and all $\mathcal{P} \in \mathcal{P}_{KB\text{-}d\text{-}model}$.*

Proof. This is a consequence of Proposition 2, since all classical models of each $\{KB_{\mathcal{P}} \mid \mathcal{P} \in \mathcal{P}_{\text{KB-d-model}}\}$ are also the d-models of the knowledge base. $\square$

We assume KB is in a decidable description logic $\mathcal{L}$ that supports nominals and full negation. It is a well known result that all common inference tasks are reducible to a satisfiability check in DLs that support full negation [5]. Furthermore, $KB_{\mathcal{P}}$ is constructed by adding GCIs involving concept expressions using nominals and conjunctions, so we can safely assume that $KB_{\mathcal{P}}$ also falls under the DL $\mathcal{L}$. Hence, all the d-model generating knowledge bases are in $\mathcal{L}$.

Theorem 2. *(Decidability of d-entailment) Let $\mathcal{L}$ be a decidable DL with full negation and nominal support. Then the tasks of subsumption checking, instance checking, and class satisfiability are decidable for default-knowledge-bases with KB in $\mathcal{L}$.*

[6] This is due to non-monotonicity of the logic.

Proof. Given a default knowledge base (KB, δ), then by Lemma 2 the inference tasks can be reduced as follows:

- Subsumption: $C \sqsubseteq D$ is d-entailed by (KB, δ) iff $KB_{\mathcal{P}} \cup \{C \sqcap \neg D\}$ is classically unsatisfiable for all $\mathcal{P} \in \mathcal{P}_{\text{KB-d-model}}$.
- Instance checking: $C(a)$ is d-entailed by (KB, δ) iff $KB_{\mathcal{P}} \cup \{\neg C(a)\}$ is classically unsatisfiable for all $\mathcal{P} \in \mathcal{P}_{\text{KB-d-model}}$.
- Class satisfiability: a class C is satisfiable iff $C \sqsubseteq \bot$ is not d-entailed by (KB, δ).

Consider the task of checking $(KB, \delta) \models_d C \sqsubseteq D$. Then $(KB, \delta) \models_d C \sqsubseteq D$ iff $KB_P \cup \{C \sqcap \neg D\}$ is classically unsatisfiable for all $\mathcal{P} \in \mathcal{P}_{\text{KB-d-model}}$. Since $P_{\text{KB-d-model}}$ is finitely computable and checking classical satisfiability is decidable in $\mathcal{L}$, checking the satisfiability for each $KB_{\mathcal{P}}$ is decidable. Hence, checking $(KB, \delta) \models_d C \sqsubseteq D$ is decidable. Similar arguments hold for the other tasks. $\square$

5 Default Role Inclusion Axioms

So far we have restricted our attention to default concept inclusions. We made this restriction for the purpose of obtaining a clearer presentation of our approach. However, as may be clear by now, we can also carry over our approach to cover default role inclusions, and we discuss this briefly in the following.

We use the notation $R \sqsubseteq_d S$ for free (normal) role defaults. As in the case of default concept inclusion axioms for role defaults, we restrict the exceptions to these defaults to be pairs of named individuals only. The intuitive semantics of $R \sqsubseteq_d S$ is that for every pair (a, b) of named individuals in the knowledge base, if R holds then assume S also holds unless it leads to an inconsistency. For all other pairs of individuals (with at least one unnamed individual), if R holds then S also holds. We extend the definition of default-knowledge-bases and adjust the other definitions in the following.

Definition 7. *Let KB be a knowledge base in a decidable DL and let δ be a set of default axioms of the form $C \sqsubseteq_d D$ or $R \sqsubseteq_d S$, where C, D and R, S are respectively concepts and roles appearing in KB. Then we call (KB, δ) a* default-knowledge-base. *Furthermore:*

- *The definition of $\mathsf{Ind}_{KB}, \mathsf{P}(\mathsf{Ind}_{KB}), \mathsf{P}^n(\mathsf{Ind}_{KB})$ carry over from Definition 3, where n is the number of axioms of the form $C \sqsubseteq_d D$ in δ.*
- *$\mathsf{P}(\mathsf{Ind}_{KB} \times \mathsf{Ind}_{KB})$ denotes the power set of $\mathsf{Ind}_{KB} \times \mathsf{Ind}_{KB}$.*
- *$\mathsf{P}^m(\mathsf{Ind}_{KB} \times \mathsf{Ind}_{KB})$ is the set of m-tuples obtained from the Cartesian product: $\mathsf{P}(\mathsf{Ind}_{KB} \times \mathsf{Ind}_{KB}) \times \ldots_{mtimes} \times \mathsf{P}(\mathsf{Ind}_{KB} \times \mathsf{Ind}_{KB})$, where m is the number of default role axioms in δ.*

For simplicity of presentation we assume that δ is arranged such that all default concept inclusion axioms appear before all default role inclusion axioms. Now, consider the set $\mathcal{D}_{KB} = \mathsf{P}^n(\mathsf{Ind}_{KB}) \times \mathsf{P}^m(\mathsf{Ind}_{KB} \times \mathsf{Ind}_{KB})$ which is a set of tuples, where each tuple is of the form $((X_1, \ldots, X_n), (Y_1, \ldots, Y_m))$ such that $(X_1, \ldots, X_n) \in \mathsf{P}^n(\mathsf{Ind}_{KB})$ and $(Y_1, \ldots, Y_m) \in \mathsf{P}^m(\mathsf{Ind}_{KB} \times \mathsf{Ind}_{KB})$. An interpretation for default-knowledge-bases with default role inclusion axioms should now map δ to a tuple as follows. $\delta^{\mathcal{I}} = (\mathcal{X}^{\mathcal{I}}, \mathcal{Y}^{\mathcal{I}}) \in \mathcal{D}_{KB}$, where $\mathcal{X}^{\mathcal{I}} = (X_1^{\mathcal{I}}, \ldots, X_n^{\mathcal{I}})$ and $\mathcal{Y}^{\mathcal{I}} = (Y_1^{\mathcal{I}}, \ldots, Y_m^{\mathcal{I}})$ such that $X_i^{\mathcal{I}} = (C_i^{\mathcal{I}} \cap D_i^{\mathcal{I}} \cap \Delta_{Ind}^{\mathcal{I}}) \cup ((\neg C)^{\mathcal{I}} \cap \Delta_{Ind}^{\mathcal{I}})$ and $Y_j^{\mathcal{I}} = (R_j^{\mathcal{I}} \cap S_j^{\mathcal{I}} \cap (\Delta_{Ind}^{\mathcal{I}} \times \Delta_{Ind}^{\mathcal{I}})) \cup ((\neg R_j)^{\mathcal{I}} \cap \Delta_{Ind}^{\mathcal{I}} \times \Delta_{Ind}^{\mathcal{I}})$, for all $C_i \sqsubseteq_d D_i \in \delta$ and $R_j \sqsubseteq_d S_j \in \delta$. In other words $X_i^{\mathcal{I}}$ denotes the extension of the named individuals that satisfy the i^{th} default concept axioms and $Y_j^{\mathcal{I}}$ denotes the extension of pairs of named individuals that satisfy the j^{th} default role axiom.

To ensure the maximal application of the default axioms we need the preference relation to be adapted to this setting.

Definition 8. *(Preference relation $>_{KB,\delta}$) Given a knowledge base KB, a set of default axioms δ, and $\mathcal{I}$ and $\mathcal{J}$ be two interpretations of (KB, δ). We say that $\mathcal{I}$ is preferred over $\mathcal{J}$, written $\mathcal{I} >_{KB,\delta} \mathcal{J}$, if*
1. *conditions 1-4 of Definition 4 hold,*
2. *$Y_i^{\mathcal{I}} \supseteq Y_i^{\mathcal{J}}$ for all $1 \leq i \leq m$, where $Y_i^{\mathcal{I}} \in \mathcal{Y}^{\mathcal{I}}$ and $Y_i^{\mathcal{J}} \in \mathcal{Y}^{\mathcal{J}}$,*
3. *$Y_i^{\mathcal{I}} \supset Y_i^{\mathcal{J}}$ for some $1 \leq j \leq m$, where $Y_i^{\mathcal{I}} \in \mathcal{Y}^{\mathcal{I}}$ and $Y_i^{\mathcal{J}} \in \mathcal{Y}^{\mathcal{J}}$.*

where m is the number of role inclusion axioms in δ.

The definition of d-model now carries over from Definition 5, the only difference being that the new definition $>_{KB,\delta}$ of the preference relation is used when default role axioms are also included.

To show the decidability of reasoning with any default-knowledge-base (KB, δ) with role defaults, we assume that KB is in a decidable DL $\mathcal{L}$ which supports nominal concept expression, boolean role constructors, concept products, and the universal role $\mathcal{U}$. In [22], it was shown that expressive DLs can be extended with boolean role constructors for simple roles without compromising on complexity and decidability. For some tuple $\mathcal{P} \equiv ((X_1, \ldots, X_n), (Y_1, \ldots, Y_m)) \in \mathcal{D}_{KB}$, let $KB_{\mathcal{P}}$ be the knowledge base that is obtained by adding the following axioms to KB. For each $C_i \sqsubseteq_d D_i \in \delta$ add the following.

1. $\overline{X_i} \equiv (C_i \sqcap D_i \sqcap \{a_1, \ldots, a_k\}) \sqcup (\neg C_i \sqcap \{a_1, \ldots, a_k\})$, where $\overline{X_i}$ is the nominal expression $\{x_1, \ldots, x_m\}$ containing exactly the named individuals in X_i, and $\{a_1, \ldots, a_k\} = \mathsf{Ind}_{KB}$.
2. $C_i \sqcap \neg\{a_1, \ldots, a_k\} \sqsubseteq D_i$, where $\{a_1, \ldots, a_k\} = \mathsf{Ind}_{KB}$.

And for each $R_j \sqsubseteq_d S_j \in \delta$, add the following.

1. For each $(a, b) \in Y_j$, add the ABox axiom $R_{a,b}(a, b)$ and the axiom
$$\{x\} \sqcap \exists R_{a,b}.\{y\} \sqsubseteq \{a\} \sqcap \exists \mathcal{U}.(\{y\} \sqcap \{b\}),$$
where $R_{a,b}$ is a fresh role name, and $\{x\}$ and $\{y\}$ are so-called *nominal schemas* as introduced in [19]: They are a kind of nominal variables, which can stand for any nominal. In fact, this axiom can easily be cast into a set of axioms not containing nominal schemas, as shown in [19]. The axiom just given enforces that $R_{a,b}^{\mathcal{I}} \cap (\Delta_{Ind}^{\mathcal{I}} \times \Delta_{Ind}^{\mathcal{I}}) = \{(a, b)\}$.
2. $\bigsqcup_{(a,b) \in \mathcal{Y}_j} R_{a,b} \equiv (R_j \sqcap D_j \sqcap \mathcal{U}_g) \sqcup R_j \sqcap \neg\mathcal{U}_g$, where $\mathcal{U}_g \equiv \mathsf{Ind}_{KB} \times \mathsf{Ind}_{KB}$.
3. $R_j \sqcap \neg\mathcal{U}_g \equiv S_j$, where $\mathcal{U}_g = \mathsf{Ind}_{KB} \times \mathsf{Ind}_{KB}$.

The construction just given for role defaults is analogous to the one for class inclusion defaults, with the exception that we do not have a nominal constructor for roles. However, for the specific setting we have here, we can obtain the same result by using the axioms from points 1 and 2 just given.

It should also be noted that the above outlined construction of $KB_{\mathcal{P}}$ can be computed in a finite number of steps.

The remainder of the decidability argument for d-entailment now carries over easily from section 4, and we omit the details. It should be noted that the availability of boolean role constructors is required for our argument, and that corresponding simplicity restrictions may apply, depending on the concrete case.

6 Application of Defaults in Ontology Alignment

Variety and semantic heterogeneity are at the very core of many fields like the Semantic Web, Big Data etc. To give a concrete example, many interesting scientific and societal questions cannot be answered from within one domain alone but span across disciplines. Studying the impact of climate change, for instance, requires to consider data and models from climatology, economics, biology, ecology, geography, and the medical science. While all these disciplines share an overlapping set of terms, the meanings of these terms clearly differ between them. A street, for instance, is a *connection* between A and B from the view point of transportation science, and, at the same time, a disruptive *separation* which cuts through habitats from the view point of ecology. Even within single domains, the used terminology differs over time and space [16]. The idea that this variety should be 'resolved' is naive at best. The defaults extension proposed in this work can thus support more robust ontology alignments that respect variety and still allow us to share and integrate the heterogeneous data. In the following we give some concrete examples that cannot be sufficiently addressed with existing ontology alignment frameworks, but would benefit from the proposed extension.

Consider the axioms in Figure 2. The ontology fragment consisting of (17) to (21) reflects a certain perspective on canals valid in a transportation application. In contrast, the axioms (22) to (24) reflect a different but equally valid perspective from an agricultural perspective. Typically, ontology alignment systems would default to a syntactic matching of shared primitives such as *AgriculturalField* or *IrrigationCanal*. However, applied to these two ontology fragments this would yield a logical inconsistency in which some waterbodies would have to be land masses at the same time. Using our proposed free defaults, only certain canals from a would qualify as canals in b, avoiding the inconsistencies.

While in the above example the inconsistency was largely caused by the cardinality restrictions, other cases involve concrete domains. For instance, each US state (and the same argument can be made between counties as well) has its own legally binding definition of what distinguishes a town from a city. Thus, to query the Linked Data cloud for towns it is required to take these local definitions into account. Otherwise one would, among hundreds of thousands of small municipalities, also retrieve Los Angeles, CA or Stuttgart, Germany.[7]

[7] In case of DBpedia via `dbpedia:Stuttgart rdf:type dbpedia-owl:Town`.

$$a\text{:}\mathit{flowsInto} \sqsubseteq a\text{:}\mathit{IsConnected} \tag{17}$$
$$a\text{:}\mathit{IrrigationCanal} \sqsubseteq a\text{:}\mathit{Canal} \tag{18}$$
$$\exists a\text{:}\mathit{flowsInto}.a\text{:}\mathit{AgriculturalField} \sqsubseteq a\text{:}\mathit{IrrigationCanal} \tag{19}$$
$$a\text{:}\mathit{Waterbody} \sqcap a\text{:}\mathit{Land} \sqsubseteq \bot \tag{20}$$
$$a\text{:}\mathit{AgriculturalField} \sqsubseteq a\text{:}\mathit{Land} \tag{21}$$
$$b\text{:}\mathit{flowsInto} \sqsubseteq b\text{:}\mathit{IsConnected} \tag{22}$$
$$b\text{:}\mathit{Canal} \sqsubseteq (\geq 2\ b\text{:}\mathit{IsConnected}.b\text{:}\mathit{Waterbody}) \tag{23}$$
$$\begin{aligned} b\text{:}\mathit{IrrigationCanal} \equiv{} & (=1\ b\text{:}\mathit{isConnected}.b\text{:}\mathit{Waterbody}) \\ & \sqcap (=1\ b\text{:}\mathit{flowsInto}.b\text{:}\mathit{AgriculturalField}) \end{aligned} \tag{24}$$

Fig. 2. Fragments of two ontologies, (17)-(21), respectively (22)-(24), to be aligned

In several cases these state-specific distinctions solely depend on the population count and, thus, could be handled using existing alignment systems. However, in other cases they are driven by administrative divisions of geographic space, are based on historical reasons, or other properties. As argued before, our free defaults can handle these cases.

Let us now return to the example from section 1 and discuss it in more technical depth. We showed that an alignment using axiom (14) leads to inconsistency. Now consider instead using the approach of free defaults, by replacing axiom (14) with $b\text{:}\mathit{hasSpouse} \sqsubseteq_d a\text{:}\mathit{hasSpouse}$. As per our semantics the pair $(b\text{:}\mathit{mike}, b\text{:}\mathit{david})$ will act as an exception to the default role inclusion that we just added and $a\text{:}\mathit{hasSpouse}(b\text{:}\mathit{mike}, b\text{:}\mathit{david})$ will not hold anymore. On the other hand if we also add the axiom $a\text{:}\mathit{hasSpouse} \sqsubseteq_d b\text{:}\mathit{hasSpouse}$ then $b\text{:}\mathit{hasSpouse}(a\text{:}\mathit{john}, a\text{:}\mathit{mary})$ will also hold.

To see this formally, consider all the axioms of figure 1 except (14) to be KB and let $\delta \equiv \{(a\text{:}\mathit{hasSpouse} \sqsubseteq_d b\text{:}\mathit{hasSpouse}), (b\text{:}\mathit{hasSpouse} \sqsubseteq_d a\text{:}\mathit{hasSpouse})\}$ and consider an interpretation $\mathcal{I}$ such that $(a\text{:}\mathit{hasSpouse})^{\mathcal{I}} = \{(a\text{:}\mathit{john}^{\mathcal{I}}, a\text{:}\mathit{mary}^{\mathcal{I}})\}$ and $(b\text{:}\mathit{hasSpouse})^{\mathcal{I}} = \{(b\text{:}\mathit{mike}^{\mathcal{I}}, b\text{:}\mathit{david}^{\mathcal{I}}), (a\text{:}\mathit{john}^{\mathcal{I}}, a\text{:}\mathit{mary}^{\mathcal{I}})\}$. Note that forcing $(b\text{:}\mathit{mike}^{\mathcal{I}}, b\text{:}\mathit{david}^{\mathcal{I}})$ in the extension of $a\text{:}\mathit{hasSpouse}$ will result in an inconsistency because of the reasons mentioned in section 1. On the other hand, if we consider an interpretation $\mathcal{J}$ such that $(a\text{:}\mathit{hasSpouse})^{\mathcal{J}} = \{(a\text{:}\mathit{john}^{\mathcal{J}}, a\text{:}\mathit{mary}^{\mathcal{J}})\}$ and $(b\text{:}\mathit{hasSpouse})^{\mathcal{J}} = \{(b\text{:}\mathit{mike}^{\mathcal{J}}, b\text{:}\mathit{david}^{\mathcal{J}})\}$, then clearly $\mathcal{I} >_{\mathit{KB},\delta} \mathcal{J}$, because the extension of b:hasSpouse in $\mathcal{I}$ is greater than that of $\mathcal{J}$. So, $\mathcal{I}$ is preferred over $\mathcal{J}$. In fact $\mathcal{I}$ is also a d-model of this default-knowledge-base.

The above example shows that in case of ontology alignments, using default constructs to map terms could help us avoid potentially inconsistent merged knowledge bases due to subtle semantic differences in the different ontologies.

As final example, we want to discuss the implications our approach has with respect to the use and abuse of owl:sameAs in linked data [12],[8] where this Web ontology language (OWL) construct is used extensively to provide links between

[8] Note owl:sameAs is OWL representation of individual equality in DLs

different datasets. However, owl:sameAs is, semantically, a strong equality which equates entities, and this strong (specification-compliant) interpretation easily leads to complications. For instance, consider two linked datasets a and b where a contains the axioms: *a:airport*(*a:kennedy*) and *a:airport* $\sqsubseteq$ *a:place*, and b contains axioms *b:president*(*b:kennedy*) and *b:president* $\sqsubseteq$ *b:person* plus the disjointness axiom *b:person* $\sqcap$ *a:place* $\sqsubseteq \bot$.

Now, if some text-based co-reference resolution system identifies *a:kennedy* and *b:kennedy* as the same object, then it will result in a link such as *owl:sameAs* (*a:kennedy*, *b:kennedy*). Obviously, this yields to an inconsistency because of the disjointness axiom. However, if we use defaults this could be expressed as {*a:kennedy*} $\sqsubseteq_d$ {*b:kennedy*}, which essentially is another way of saying that *a:kennedy* is identical to *b:kennedy unless* it causes an inconsistency. While [12] argues for a set of variants of equality with differing semantic strength, automatic identification of the exact variant of equality to be used is yet another matter, and presumably rather difficult to accomplish. So for automated co-reference resolution, we would argue that the use of free defaults, which semantically recover from erroneous guesses by the alignment system, are a much more suitable solution.

7 Conclusion

In this paper, we have provided a new semantics for embedding defaults into description logics, and have shown that reasoning tasks are decidable in our setting. Both the decidable logic from [2] and our work are variants of Reiter's defaults [21]. But the approach in [2] is very restricted and arguably violates some key intuitions. Our proposal provides an improvement, mainly because with our free defaults, the application of defaults is not limited to named individuals. However, we impose that exceptions to the default rules only occur in the named individuals of the knowledge base. Also, our approach to the semantics is model-theoretic whereas most of the previous work on defaults has been mainly based on fixed point semantics [6,7]. We have furthermore given a thorough motivation of the usefulness of free defaults in the context of ontology alignments. Through the examples in section 6, it is shown that the new semantics that we have introduced in this paper is useful when dealing with the integration of heterogeneous ontologies. We believe that our work provides a foundation for a new and more powerful ontology alignment language.

Whether defaults over DLs are decidable when we allow exceptions to also occur over unnamed individuals, is still an open question and we intend to investigate this in the future. Future work also includes smart algorithmization and implementation of d-entailment tasks mentioned in this paper. A naive algorithm can easily be developed by searching for maximal d-model generating tuples from $\mathsf{P}^n(\mathsf{Ind}_{KB})$, i.e. by searching for all maximal $\mathcal{P}$s in $\mathsf{P}^n(\mathsf{Ind}_{KB})$ for which $KB_{\mathcal{P}}$ has a classical model and then using the process outlined in Theorem 2. Although this reasoning procedure appears to be decidable it is very expensive and thus not feasible for practical use. However, the algorithmization could be made smarter by using some optimization techniques. For instance, $\mathsf{P}^n(\mathsf{Ind}_{KB})$ could be represented as an ordered set of tuples where each tuple is a collection of comparable Ps sorted by the $\succ$ relation. The algorithm would then look for maximally satisfying Ps for each tuple by performing a binary search on every tuple. This should significantly improve the performance of the naive approach since the number of steps to find all suitable $\mathcal{P}$s has been reduced by a large factor. These and other optimizations will be central to our investigation of algorithmizations.

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References

1. Aguirre, J.L., Eckert, K., Euzenat, J., Ferrara, A., van Hage, W.R., Hollink, L., Meilicke, C., Nikolov, A., Ritze, D., Scharffe, F., Shvaiko, P., Sváb-Zamazal, O., dos Santos, C.T., Jiménez-Ruiz, E., Grau, B.C., Zapilko, B.: Results of the ontology alignment evaluation initiative 2012. In: Shvaiko, P., Euzenat, J., Kementsietsidis, A., Mao, M., Noy, N.F., Stuckenschmidt, H. (eds.) Proceedings of the 7th International Workshop on Ontology Matching, Boston, MA, USA, 11 November 2012, CEUR Workshop Proceedings, vol. 946. CEUR-WS.org (2012)
2. Baader, F., Hollunder, B.: Embedding defaults into terminological knowledge representation formalisms. J. Autom. Reasoning **14**(1), 149–180 (1995)
3. Bonatti, P.A., Faella, M., Sauro, L.: Defeasible inclusions in low-complexity DLs. Artificial Intelligence (JAIR) **42**, 719–764 (2011)
4. Bonatti, P.A., Lutz, C., Wolter, F.: The Complexity of Circumscription in Description Logic. Journal of Artificial Intelligence Research **35**, 717–773 (2009)
5. Buchheit, M., Donini, F.M., Schaerf, A.: Decidable reasoning in terminological knowledge representation systems. In: Proceedings of the 13th International Joint Conference on Artifical Intelligence, IJCAI 1993, vol. 1, pp. 704–709. Morgan Kaufmann Publishers Inc., San Francisco (1993)
6. Dao-Tran, M., Eiter, T., Krennwallner, T.: Realizing default logic over description logic knowledge bases. In: Sossai, C., Chemello, G. (eds.) ECSQARU 2009. LNCS, vol. 5590, pp. 602–613. Springer, Heidelberg (2009)
7. Eiter, T., Lukasiewicz, T., Schindlauer, R., Tompits, H.: Combining answer set-programming with description logics for the semantic web. In: Proc. of the 9th Int. Conf. on the Principles of Knowledge Representation and Reasoning (KR 2004). AAAI Press (2004)
8. Gelfond, M., Lifschitz, V.: The stable model semantics for logic programming. In: Kowalski, R.A., Bowen, K.A. (eds.) Proceedings of the 5th International Conference and Symposium on Logic Programming, pp. 1070–1080. MIT Press (1988)
9. Giordano, L., Gliozzi, V., Olivetti, N., Pozzato, G.L.: A non-monotonic description logic for reasoning about typicality. Artificial Intelligence **195**, 165–202 (2013)
10. Gottlob, G.: Complexity results for nonmonotonic logics. J. Log. Comput. **2**(3), 397–425 (1992)

11. Grimm, S., Hitzler, P.: Semantic Matchmaking of Web Resources with Local Closed-World Reasoning. International Journal of Electronic Commerce **12**(2), 89–126 (2007)
12. Halpin, H., Hayes, P.J., McCusker, J.P., McGuinness, D.L., Thompson, H.S.: When owl:sameAs isn't the same: an analysis of identity in linked data. In: Patel-Schneider, P.F., Pan, Y., Hitzler, P., Mika, P., Zhang, L., Pan, J.Z., Horrocks, I., Glimm, B. (eds.) ISWC 2010, Part I. LNCS, vol. 6496, pp. 305–320. Springer, Heidelberg (2010)
13. Hitzler, P., Krötzsch, M., Parsia, B., Patel-Schneider, P.F., Rudolph, S. (eds.): OWL 2 Web Ontology Language: Primer. W3C Recommendation 27 October 2009 (2009). http://www.w3.org/TR/owl2-primer/
14. Hitzler, P., Krötzsch, M., Rudolph, S.: Foundations of Semantic Web Technologies. Chapman & Hall/CRC, Boca Raton (2009)
15. Janowicz, K.: The role of space and time for knowledge organization on the Semantic Web. Semantic Web **1**(1–2), 25–32 (2010)
16. Janowicz, K., Hitzler, P.: The digital earth as knowledge engine. Semantic Web **3**(3), 213–221 (2012)
17. Knorr, M., Alferes, J., Hitzler, P.: Local Closed-World Reasoning with Description Logics under the Well-founded Semantics. Artificial Intelligence **175**(9–10), 1528–1554 (2011)
18. Knorr, M., Hitzler, P., Maier, F.: Reconciling OWL and non-monotonic rules for the Semantic Web. In: Raedt, L.D., et al. (eds.) ECAI 2012–20th European Conference on Artificial Intelligence, Montpellier, France, 27–31 August 2012, pp. 474–479. IOS Press, Amsterdam (2012)
19. Krötzsch, M., Maier, F., Krisnadhi, A.A., Hitzler, P.: A better uncle for OWL: nominal schemas for integrating rules and ontologies. In: Sadagopan, S., et al. (eds.) Proceedings of the 20th International World Wide Web Conference, WWW 2011, Hyderabad, India, 2011, pp. 645–654. ACM, New York (2011)
20. Leone, N., Faber, W.: The DLV project: a tour from theory and research to applications and market. In: Garcia de la Banda, M., Pontelli, E. (eds.) ICLP 2008. LNCS, vol. 5366, pp. 53–68. Springer, Heidelberg (2008)
21. Reiter, R.: A logic for default reasoning. Artif. Intell. **13**(1–2), 81–132 (1980)
22. Rudolph, S., Krötzsch, M., Hitzler, P.: Cheap Boolean role constructors for description logics. In: Hölldobler, S., Lutz, C., Wansing, H. (eds.) JELIA 2008. LNCS (LNAI), vol. 5293, pp. 362–374. Springer, Heidelberg (2008)
23. Sengupta, K., Krisnadhi, A.A., Hitzler, P.: Local closed world semantics: grounded circumscription for OWL. In: Aroyo, L., Welty, C., Alani, H., Taylor, J., Bernstein, A., Kagal, L., Noy, N., Blomqvist, E. (eds.) ISWC 2011, Part I. LNCS, vol. 7031, pp. 617–632. Springer, Heidelberg (2011)
24. Shvaiko, P., Euzenat, J.: Ontology matching: state of the art and future challenges. IEEE Trans. Knowl. Data Eng. **25**(1), 158–176 (2013)