Educational Paths to Mathematics: Which Paths Forward to What Mathematics?

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Abstract Most people involved in mathematics education agree that it is complex, multi-layered, dynamic, multi- and interdisciplinary. To study and to improve mathematics education on the various levels of its curricula and its practices has been a goal of the Commission Internationale pour l'Etude et l'Amélioration de l'Enseignement des Mathématiques (CIEAEM, the International Commission for Study and Improvement of Teaching Mathematics) since it was created and established in the 1950s. CIEAEM continues to investigate the actual conditions and the possibilities for the development of mathematics education. This introductory chapter provides the rational for the book by looking at historical developments in school mathematics. The structure of the sourcebook is explained at the end of the introduction.

Mathematics education is a multi-facetted endeavour that has been regarded from many theoretical points of view. Some believe that it has to do with the transmission and the acquisition of mathematical knowledge, while others emphasise the learners' mathematical constructions and the teachers' role in providing appropriate mathematical environments. Mathematics education is concerned with the formation of the learners' identities, but also with the institutional (re-)production of a mathematically-educated workforce. It is about the transposition, or recontextualisation, of academically produced mathematics into a mathematics curriculum, and it is also about how people activate and integrate mathematical skills and knowledge in everyday contexts. Mathematics education has been regarded as a process, a product, a discourse, a practice, an activity system, a material reality, a research domain, a field of academic research and an area of study. Most people involved in mathematics education agree that it is complex, multi-layered, dynamic, multi- and interdisciplinary. Some argue that mathematics education is the key to the

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development of individual, national and global prosperity. But, is this all true, and if so, in which sense?

Looking Back: Developments in School Mathematics

Mathematics education is political, as can be seen in disputes over the question: “What is school mathematics?” 300 years ago, this question would have been quite difficult to pose, let alone to answer. The mathematics that was taught varied heavily across countries and the different institutions of learning according to their education and formation purposes. But then, a process of international modernisation started, and, according to Gispert and Schubring (2011), two countries were leading this process: France and Germany.

The Construction of School Mathematics

In France, an important and influential development was realised in the early eighteenth century: The idea of a generalised science education, including mathematics, was propagated. Until then, mathematics as such was taught mostly in military schools and only to a very small extent at the university within philosophical studies. The publication and distribution of textbooks in mathematics promoted the development of the discipline. The authors, who have been teachers in different types of schools, advocated for it, emphasising the moral and cultural value of mathematics.

The development reached a new level when schooling shifted from being a privilege for a social elite to being a part of an overall system of education, organized by state authorities, with formalized teacher education programmes and compulsory for all children. From this moment on, various stakeholders were, and still are, involved in contesting the very nature of school mathematics. As Gispert (2011) claims, these stakeholders can be grouped as follows: experts (mathematicians), field professionals (high school mathematics teachers), political and economic actors. She further argues that these three spheres of actors are associated with three registers of modernity: a mathematical, an educational, and a socio-economic register.

During the nineteenth century, the status and importance of mathematics in secondary education in France remained marginal within the “classical canon” of a humanistic secondary education which, at that time, formed the country’s elite. In contrast to the elite’s education, the school curriculum for the children of the middle classes (called the “upper primary”) included a strong emphasis in science education, in which “practical” mathematics played a significant role. A very important reform took place in 1902. It unified secondary education. It implemented, in parallel with the prestigious traditional pathway focused on teaching classical humanities, a “modern” sector based on language and science education. This sector combined two visions of mathematics, cultural and practical, and incorporated new contents: among others, breaking with Euclidean geometry, and introducing the quasi-experimental study of functions in connection with physics. The reform was mainly driven by mathematicians; high school teachers seized it a little later.

This reform challenged the theoretical and disinterested view of the formation of the elite conveyed by the classical secondary education. After World War I, the reform was accused of being inspired too much by the German model, that had been developed during the nineteenth century in Prussia, and betraying the spirit of the French classical humanities. These developments in Prussia are an interesting historical case in which the controversies about the nature of school mathematics came to a certain end.

Jahnke (1986) discusses the origins of the dispute over what school mathematics should be in early nineteenth-century Prussia—a quite decisive moment in the development of mathematics curricula. He shows how, during a short period after a military defeat against the French revolutionary troops in 1806, the debate about the constitution and form of school mathematics affected a radical change in the perception of what the difference between school mathematics and academic mathematics should be. During the dispute it became clear that it would no longer be possible to simply define school mathematics as academic mathematics on a lower level and a lower degree of difficulty. There is more to do than merely to simplify academic mathematical knowledge in order to build up a school mathematics curriculum. Jahnke reconstructs the historical process of the development of what he calls “the ‘deep structure’ of school mathematics” (p. 86), a structure that “has remained essentially the same since the early nineteenth century” (ibid.). He shows how mathematicians were extremely active in promoting the view that school mathematics should be uniform in all schools and free from any practical concerns. However, their insistence was not successful. State administration, particularly school inspectors and headmasters of prestigious secondary (Gymnasium) schools in Berlin, argued that it is not “appropriate to exclude ‘common arithmetic’ from the curriculum” (p. 91) and that “it is necessary to find ways and means of linking ‘common arithmetic’ and higher mathematics, everyday knowledge and scientific knowledge” (ibid.). Ultimately, the structure of the school mathematics curriculum was based on the concept of ‘mathematical operation’ thus founding higher school mathematics on elementary arithmetic: School algebra was constructed as the study of the formal properties of the arithmetical operations; infinitesimal calculus was constructed as formal school algebraic theory. The extension of the number concept, the ‘principle of permanence’, defined the macro-structure of the school mathematics curriculum from the early nineteenth century to the present.

Towards the Problem-Centred Curriculum

Much later, in the mid- to late-twentieth century, new initiatives of curriculum development in mathematics have brought about an orientation towards 'problem-solving' and 'mathematical modelling'. Pólya's conception of mathematics as an essential problem solving activity has often been quoted as the main root for the first of the two orientations (e.g., 1962). It was his idea to convert the ontological concept of mathematics as problem solving into an epistemological programme. The student should learn mathematics in a way that is analogous to the way mathematicians work. Less related to 'pure mathematics' than the first, mathematical modelling is often taken as an idealisation of the developmental activities within applied mathematics. In contrast to the field of applied mathematics, a mathematical modelling approach to school mathematics only rarely aims at the development of new mathematical algorithms and technology that can be used to solve real problems or to engage mathematically in real situations. Instead, as English and Sriraman (2010) adhere to, mathematical modelling can be conceptualised as an advance on existing classroom word problem solving. Arguably, the "problems" in mathematical modelling differ from the "problems" in mathematics as problem solving.

Both orientations only tacitly challenge the macro-structure of the school mathematics curriculum. They are not meant to re-evaluate the importance of mathematical operations and the principle of permanence. The recontextualisation of problem solving and mathematical modelling within the institutional frame of schooling brings about a transformation of ontological statements into didactic principles and pedagogic strategies. By this mechanism, problem solving and mathematical modelling, as didactic principles and pedagogic strategies, appear as official curricular paths to mathematics (cf. Jablonka and Gellert 2012). It is part of the self-concept of the mathematics research community to regard the resulting modifications of curriculum material, classroom activities, attainment descriptions, etc. as topics for empirical research and as impulses for design activities.

Fit to and Fit for the Data-Driven Society

A seemingly different kind of curricular renovation occurred during the last decades in numerous countries with the introduction (or expansion) of statistics, of chance and probability in the primary and secondary mathematics curriculum. This mathematical area has been integrated into official curriculum descriptions and attainment standards, thus actually bearing the potential to shift or diversify the macro-structure of the school mathematics curriculum. Why did the shift occur, or: in which way is it a shift of the macro-structure? Note that the introduction of stochastics in mathematics education fits well to the three dimensions of "modernity" defined by Gispert above: mathematical "modernity" as it takes into account recent developments in academic mathematics, pedagogical "modernity" through the use of "real problems" to motivate students and build interdisciplinary links, and the socio-political importance of statistics and data analysis in "modern" societies. Indeed, it can be argued that the focus of the traditional school mathematics curriculum on mathematical operations (during the primary grades) and school algebra (during the secondary grades) is in a critical relation to the concept of the Western democracy or, more precisely: of technocracy. In a technocracy, political decisions based on calculations require a populace that is used to trust the legitimacy of calculations—and not necessarily a populace that is able to critically evaluate the mechanism by which political decisions are made legitimate. A curriculum focussed on mathematical operations and school algebra may perfectly contribute to customise and familiarise the student population with the imposed legitimacy of calculations. In the second half of the twentieth century, partly due to the advancement of computer technology, more and more of the calculations, that inform political decisions and by which political decisions are communicated, became of probabilistic and statistic character. Introducing statistics, chance and probability in the school mathematics curriculum might then be regarded less a challenge to its macro-structure than an attempt to repair the familiarisation with the mathematical operations and representations mentioned in legitimacy claims in political decisions. However, some scepticism seems appropriate here about the way stochastic is introduced in the school mathematics curriculum and the claim that it can offer new paths to mathematics. As Fabre (2010) holds, knowledge is multi-dimensional. These dimensions can be classified as historical, systematic and operational. School curricula mostly emphasise the last one. Consequently, statistics is often reduced to a set of techniques that students need to master, at worst to mathematics-in-contexts that are supposed to motivate students. School statistics, chance and probability seems to be a recontextualisation of stochastics that does not take into account its complex epistemology, in particular the tension between a data analysis approach and a modelling approach. Apparently, there is a constant threat that the new and the different is systematically recontextualised and, thus, subordinated to the traditional foci of the mathematics curriculum. In any case, research in mathematics education is concerned with scrutinising the impact of the curricular change on mathematics classroom practice and beyond.

ICT Challenging the Mathematics Curriculum

The technological development of the last decades is a factor that might alter the forms of mathematics education practices both on the curricular and the non-curricular level. Research in mathematics education has extensively focussed on the curricular potential of ICT, perhaps best illustrated by the attempts to render school geometry more dynamically. Although an initial period in which ICT had been promoted like the silver bullet for a mathematics education for the twenty-first century has faded away, ICT still seems to have the potential of rendering some mathematics classroom traditions obsolete. This is another area for research and design activities—and this volume is further exploring the topic. From a political perspective, we might ask whether the turn to ICT is related to new mechanisms by which political decisions are generated, communicated and made legitimate—and the respective ethics involved—or if the broad availability of ICT is related in some way or another to a form of democracy that considers technocracy a risk. Anyway, even if schools are simply taking up technological standards, the potential of ICT for new educational paths to mathematics seems to be beyond doubt.

Looking Forward: Why and Where?

A profound criticism has arisen recently. What are we actually doing when always looking for new educational paths to mathematics? Are we uncritically bound to the ideology that we have to continuously reform mathematics education, because the whole enterprise of mathematics education is not running as we wish it would? But, could it ever? Can the reasons for mathematics education not being a fully developed success story be suppressed by an improved mathematics education? Or are we to face eternal frustration—education cannot compensate for society (Bernstein 1970)—but cannot stop producing new ideas, new strategies, new theories and paradigms, because … because of what? Because mathematics educators and others have indeed been successful in constituting mathematics education as an ethical system in which mathematical knowledge is "good" (Lundin 2012), "mathematics for all" even a Lacanian "supreme good" (Pais 2012) and, at the same time, establishing the mathematics educators as the key producers of knowledge in mathematics education? This is, of course, a reflection from a cynical point of view. From this perspective, mathematics education appears as an ingenious self-reproducing machinery. However, the metaphor of the machinery is essentially modern and suppresses all personal motives, uninterested commitment and illogical behaviour of those who like to improve mathematics education. There are many mathematics educators who do not wish to make a better world by means of mathematics, many who do not believe more fair and just mathematics education would cause a more fair and just world, many who do not see our economical and ecological problems resolved once the students achieve a better understanding of mathematics. In lieu thereof, many mathematics educators do not stop producing new ideas because they simply seek to make the learning of mathematics, under the conditions of institutionalised schooling, more meaningful to students. As the students of the twenty-first century seem to be different from those of e.g. the nineteenth century, and because the students are exactly the ones who decide about the meaningfulness of mathematics education activities, there indeed is a never-ending necessity to study and improve mathematics education. Stopping this endeavour can hardly be an alternative.

In a similar spirit, to study and to improve mathematics education on the various levels of its curricula and its practices has been a goal of the *Commission Internationale pour l'Etude et l'Amélioration de l'Enseignement des Mathématiques*

(CIEAEM, the International Commission for the Study and Improvement of Mathematics Education) since it was created and established in the 1950s. CIEAEM continues to investigate the actual conditions and the possibilities for the development of mathematics education. The commission regularly organises conferences characterised by exchange and discussion of research work and of experiences based on the craft knowledge of teaching at all levels. It fosters the dialogue between researchers and educators in all domains of practice. Whereas the founding members of CIEAEM—mathematicians, mathematics educators, psychologists and philosophers—had focused on interrelating an academic mathematical perspective with a modern pedagogy based on psychological models of cognitive development. They missed recognising the social dimension of mathematics education, this missing dimension has since been largely integrated into the work of the commission—as you will immediately notice when reading this sourcebook.

The Structure of the Sourcebook

The volume begins with three chapters that exemplarily illustrate the obstacles that any attempt to promote mathematics education might face. These obstacles are no minor ones. They are written in the social fabric of mathematics education. The first chapter raises a fundamental concern: Why should the students be interested in learning mathematics at all? Paola Valero argues that there is "a cultural gap between the forms of subjectivity promoted by mathematics as areas of schooling, and the forms of subjectivity experienced by students in their everyday life". In order to counter the referred students' lack of interest in mathematics, it seems necessary not to restrict the remedial activities to the pedagogic and the didactic but to consider the students' identity constructions as closely connected to the cultural politics of schooling. The second chapter investigates how the cultural politics of mathematics education play out in rural areas. The study, reported by Robert Klein, is based on the expectation that these places "would be engaged in meaningful efforts to connect mathematics instruction to local places and communities". Although the study found a variety of support for making local connections between mathematics education and locally relevant issues, it concludes that the support remained mainly on the level of rhetoric. Instead, mathematics education interacts in a rather alienating way on the students' developing identities: mathematics "inhabits nowhere rather than round here". The third chapter exposes a kind of cultural micro-politics of schooling. Christine Knipping, David Reid and Hauke Straehler-Pohl adjust their analytical lenses on the micro-dynamics of the mathematics classroom. By drawing on mathematics instruction practices in 'officially selective' and 'officially inclusive' school systems, they trace how the basic political principle of meritocracy translates into disparity producing interactional mechanisms in the classroom. A classroom culture is generated in which the conflicting nature of the distribution of access to mathematical knowledge, and thus to mathematical identities, is naturalised.

The three chapters constitute the first part of the volume and serve as a horizon for the following seventeen chapters. These seventeen educational paths to mathematics try—not to attack but—to understand, to redefine and to engage with the reported, and other, obstacles. What they essentially do is look for resources.

We grouped the seventeen paths to mathematics in seven parts of two to three chapters each. All parts end with a commentary. The intention behind this second part of the volume is to offer new ideas of educational paths to mathematics. The chapters differ from standard research articles in mathematics education that aim at the development of methodologies or theories. Although many of the chapters are indeed outcomes of systematically controlled research projects (and reference to research papers in this respect is given in the chapters), the focus here is not on the details of data construction and analysis etc., but on new mathematical activities and conceptions enriching the repertoire of educational paths to mathematics.

The volume is remarkably international. Teachers and researchers from 14 countries authored 20 chapters and 7 commentaries. The reader is invited to reflect on the particular effect of presenting avenues to mathematics contrived in diverse national settings in which the praxis of mathematics education might look different compared to what happens in the reader's place.

'Working with adults' is the heading of the second part. Gail FitzSimons reflects on her experience with pharmaceutical operators engaged in vocational education courses on 'Calculations' and 'Basic Computer Skills'. She shows that, if the workers' knowledge and experience, their artefacts and practices, their voices and stories are taken into account and incorporated in the course material, then a path to the up to now unthinkable is offered. In contrast, Vera Helena Giusti de Souza, Rosana Nogueira de Lima, Tânia Maria Mendonça Campos and Leonardo Gerardini face a situation of young adults returning to school in order to catch up on their school-leaving certificate. As these learners do not dispose of a shared work experience, the authors design a mathematical modelling activity with bank loan systems that might be important for the young adults in their near future. In his commentary on both chapters, Javier Díez-Palomar emphasises the importance of using, and further developing, strategies to bridge the gap between academic knowledge and common sense based on experience in order to generate more democratic mathematical activities.

'Working with pre-schoolers' is the focus of the third part. Anna Chronaki, Georgia Moutzouri and Kostas Magos open this part with an outdoor activity for Roma and non-Roma children. They designed 'Number in Cultures' as a counter event in which the correspondence amongst number words and symbols was explored in three languages: Greek, Romany, and Arabic. Their study concludes that such a counter event can open a space for marginalised children, mathematical knowledge and silenced identities. The following two chapters are about the issue of fairness and fair sharing in children aged 3–6. Zoi Nikiforidou and Jenny Pange discuss how logico-mathematical activities in pre-schoolers' classrooms may contribute to the children's developing understanding of fairness as an intersection of their cognitive, social and moral development. Julie Cwikla and Jennifer Vonk investigate if, and how, fair sharing tasks facilitate pre-schoolers' access to fractional concepts. They find evidence that children can comprehend or acquire fractional concepts before whole numbers are consolidated. In their commentary on the three chapters, Michaela Kaslová and Sixto Romero Sánchez expose the historical background to recent work with pre-schoolers. Particular attention is paid to the influence of Comenius' principles and to the developments in many European countries at the turn from the nineteenth to the twentieth century.

'Taking spaces and modalities into account' is the heading of the fourth part in which different 'spaces' and their relation to 'participation' are conceptualised. Susan Gerofsky commences with examples of digital mathematical performances, which may pave ways to complement, or antagonise, the "disembodied, antiperformative traditions in school mathematics pedagogy". An expansion of 'liminal spaces', i.e. "play in the spaces of paradox and contradiction, ambiguity and transition", may lead to deeper levels of mathematical understanding. Liminal performative spaces may offer ample opportunities for students' participation. Luciana Bazzini and Cristina Sabena use a teaching experiment to illustrate how space for interaction is often filled with gestures and narration. They expose the multimodal nature of teaching and learning of mathematics. Awareness of the multimodality of "the 'semiotic game' between the teacher and students" proves to be important for the teacher in order to understand, and facilitate, the students' participation in classroom interaction. Eleni Gana, Charoula Stathopoulou and Petros Chaviaris focus on the space as the material space, in which a classroom teacher and her/his students are spatially related. The authors argue from a social semiotic perspective that the teachers' use of classroom space is involved in enabling "students' social experience in the specific teaching and learning environment". Classroom space is considered as one of the teacher's resources for the delineation of semantically coherent learning activities. In his spirited commentary to the three chapters, Peter Appelbaum distinguishes the part as a revolutionary moment in mathematics education. The attentiveness towards performance, towards gestures and narration, and towards spaces allow for new ways "of collaboration, experimentation, aesthetic participation, and playful creation of new worlds to be in".

'Criticising public discourse' is the core of the mathematical activities depicted in Part V. Lluís Albarracín and Núria Gorgorió analyse a teaching experiment in which the school mathematical topic of inconceivable magnitude estimation is related to a critical understanding of media reports about political events. In the teaching experiment, the students disclose the political bias of numbers devised by political parties and other stakeholders, and published by the media. Their critical competence is fostered through their mathematical investigations. Dimitris Chassapis and Eleni Giannakopoulou draw on the role of public media in the legitimation of the recent austerity policies in Greece. They show how mathematical concepts are used in the media to convey policies and political views. From a critical mathematics education perspective, mathematics being used as a discursive instrument within the 'apparatus of truth' can be regarded as the linchpin of the school mathematics curriculum. In her commentary on both chapters, Charoula Stathopoulou puts emphasis on the educational potential of public discourse as a focal point for a critical mathematics curriculum by which the relationship of mathematics with issues of social justice, with manipulation of the public sphere, and with critical citizenship can be made explicit.

'Organising dialogue and enquiry' is the aim of the chapters in Part VI. Ana Serradó, Yuly Vanegas and Joaquin Giménez analyse an example of blended learning in which the students exchange strategies to solve open-ended tasks in on-line forums. They show how the distance produced by the internet can be both beneficial and obstructive to processes in which the students negotiate meaning. The role of the teacher in blended learning settings is particularly highlighted. Luís Menezes, Ana Paula Canavarro and Hélia Oliveira follow a teacher and her fourth-graders through collective mathematical discussions and syntheses of emerging mathematical ideas. They document the teacher's intentions and actions in order to understand educational practice in inquiry-based mathematics classrooms better. In a teaching experiment with eleventh-graders titled "Is our world Euclidean?", Panayota Kotarinou and Charoula Stathopoulou engage the students in discussions about axiomatic definition in Euclidean and non-Euclidean geometries. Their pedagogic technique is 'drama in education'. They show how this technique offers a viable way to foster students' active participation in mathematical enquiry and critical thinking. In their commentary on the three chapters, Lambrecht Spijkerboer and Leonor Santos distinguish between surface and deep approaches for learning and ask whether drama, open-ended tasks and collective mathematical discussions might contribute to deep learning, to dealing with differences in the classroom and to the formation of democratic citizens.

'Providing information technology' is the focus of the seventh part. In the first chapter of this part, Maria Elisabete Brisola Brito Pardo and Nielce Meneguelo Lobo da Costa analyse the challenge to teachers of the introduction of laptop computers in the mathematics classroom. They document that, although the teachers see the potential that technology offers for exploration and articulation with other areas of knowledge, they still find it difficult to deviate from a traditional teaching pattern in which the teacher explains and the students practice. In the second chapter of the part, Gilles Aldon argues for the development of a sufficiently complex theoretical framework necessary to understand better the dynamics and the complexity of using computer technology as a standard in the mathematics classroom. From his perspective, the new standard that technology offers to the teaching and learning of mathematics is crucially related to the ways in which teachers and students document their mathematical activities. Such a new documentary system is related to the processes of memorisation, of organisation of ideas, of creativity and of communication and has thus the potential to re-orientate the dynamics of knowledge construction. In his commentary on both chapters, Fernando Hitt points to the well-documented low impact that the development of ICT has until now on classroom practice. He stresses that empirical research is urgently needed for a systematic and substantial integration of technology into classroom practice.

'Transcending boundaries' completes the educational paths to mathematics. As in the other parts, the two chapters invite the reader to think about the teaching and learning of mathematics beyond the usual patterns of transmission and acquisition of knowledge in school. Javier Díez-Palomar opens the part with a call for family involvement in order to increase democratic participation. On one hand, "parental involvement is recognised as a crucial outside-school aspect in children's mathematics achievement". On the other hand, a broader involvement of the public in the mathematics education enterprise seems to go hand in hand with more participative and democratic practices. In the final chapter, Peter Appelbaum puts forward the idea of "service-learning as teacher education". In the form of 'intergenerational math circles' of secondary students, mathematics teachers and future teachers, the participants experienced radically different forms of teaching and learning. Their experience encourages reflection on "dominant educational structures grounded in competitive individualism". For the future teachers, a redefinition of 'teaching' and 'learning' relates to the possibility of active invention of pedagogical practices, "rather than being a passive implementer of prepackaged curriculum." In their commentary on both chapters, Fragiskos Kalavasis and Corneille Kazadi present a model of the complex structures in education, exemplified by considering parental involvement and service learning. They call for a "new epistemology […], which valorizes the particular and the involvement of all partners in mathematics education."

The end of the volume provides information about the topics of past conferences of CIEAEM.

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