Categorical Structures of Self-adaptation in Collective Adaptive Systems

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Abstract. An adaptive system is currently on spot: collective adaptive system (CAS), which is inspired by the socio-technical systems. CASs are characterized by a high degree of adaptation, giving them resilience in the face of perturbations. In CASs, highest degree of adaptation is *self-adaptation*. The overarching goal of CAS is to realize systems that are tightly entangled with humans and social structures. Meeting this grand challenge of CASs requires a fundamental approach to the notion of self-adaptation. To this end, taking advantage of the categorical approach we construct, in this paper, algebraic structures of self-adaptation in CASs.

Keywords: Adaptedness · Categorical approach · Collective adaptive system · Self-adaptation · Self-adaptive trait

1 Introduction

The socio-technical structure of our community increasingly depends on systems, which are built as a collection of varied agents and are tightly coupled with humans and social interrelations. Their agents more and more need to be able to develop, cooperate and work all by themselves as a part of an artificial community. Hence, for such collective adaptive systems (CASs), one of major challenges is how to support self-adaptation in the face of changing interactions [8,9]. In other words, how does a CAS understand relevant interrelations and then self-adapt to become better able to live in its interactions?

Dealing with this grand challenge of CASs requires a well-founded modeling and in-depth analysis on the notion of self-adaptation. With this aim, we construct, in this paper, categorical structures of self-adaptation in CASs where a collective of entities is able to self-adapt its configuration and self-optimize its performance in the face of changing interactions [10].

2 Outline

The paper is a reference material for readers who already have a basic understanding of CAS and are now ready to know the novel approach for formalizing self-adaptation in CAS using categorical language [1–5].

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Formalization is presented in a straightforward fashion by discussing in detail the necessary components and briefly touching on the more advanced components. Several notes explaining how to use the formal aspects, including justifications needed in order to achieve the particular results, are presented.

We attempt to make the presentation as self-contained as possible, although familiarity with the notion of self-adaptation in CAS is assumed. Acquaintance with the algebra and the associated notion of categorical language is useful for recognizing the results, but is almost everywhere not strictly necessary.

The rest of this paper is organized as follows: Sections 3 and 4 present the notions of collective adaptive systems (CASs) and self-adaptation, respectively. In section 5, categorical structures of self-adaptation in CASs are developed in detail. Finally, a short summary is given in section 6.

3 Collective Adaptive Systems (CASs)

We define collective adaptive systems (CASs) as the following among various definitions that have been offered by different researchers:

Definition 1. CASs *are systems that consist of a collective of heterogeneous components, often called agents, that interact and adapt or learn.*

Hence, CASs are characterized by a high degree of adaptation, giving them resilience in the face of perturbations. We see that, in CASs, highest degree of adaptation is *self-adaptation* and we are interested in approaches to this characteristic of CASs.

This definition is concerned with three major factors of CAS:

– *A collective of heterogeneous agents* is large enough to build up systems that are tightly entangled with humans and social structures. Their agents increasingly need to be able to evolve, collaborate and function as a part of an artificial society. More importantly, the agents interact dynamically, and their interactions are either physical or involving the exchange of information.
– *Interactions* are rich, non-linear and primarily, but not exclusively, with immediate neighbors. They can be recurrent, i.e. any interaction can feed back onto itself directly or after a number of intervening stages. CASs are dynamic networks of interactions
– *Self-adaptation* is the self-evolutionary process whereby a CAS becomes better able to live in its interactions.

4 Self-adaptation

An interesting aspect of CASs is that it makes distinction between self-adaptation (i.e. system-driven personalization and modifications) and self-adaptability (i.e. user-driven personalization and modifications). *Self-adaptedness* is the state of being self-adapted, i.e. the degree to which a CAS is able to live and reproduce in a given set of interactions. *Self-adaptive trait* is an aspect of the developmental pattern of the CAS which enables or enhances the probability of that CAS surviving and reproducing.

Formally, let self-* be the set of self-_'s. Each self-_ to be an element in self-* is called a *self-* facet* [6]. That is,

$$\text{self-}* = \{\text{self-\_} \mid \text{self-\_ is a self-* facet}\} \tag{1}$$

Thus, self-adaptation is a facet of self-*, that is, self-adaptation is a member of self-*. In other words, using categorical language, this is written as $1 \xrightarrow{\text{self-adaptation}} \text{self-}*$. CASs are self-adaptive in that the individual and collective behavior mutate and self-organize corresponding to interactions. Self-adaptation indicates that CAS is a mimicry of socio-technical systems.

5 Categorical Structures of Self-adaptation

In this section, we construct self-adaptation monoid and then a category of self-adaptation monoids in order to consider the significant properties of the self-adaptation.

5.1 Self-adaptation Monoid

In [7], self-adaptation is specified by the morphism $Self\text{-}A : (CAS \times Inter^{n\in T}) \longrightarrow (CAS \times Inter^{n\in T})$, which defines the set $\{Self\text{-}A_{i\in\mathbb{N}}(CAS \times Inter^{n\in T}, CAS\times\ Inter^{n\in T})\}$ of self-adaptive traits. Let $\mathbf{Self\text{-}A}^{n\in T}$ be the set of such self-adaptive traits, then

$$\mathbf{Self\text{-}A}^{n\in T} = \{Self\text{-}A_{i\in\mathbb{N}}(CAS \times Inter^{n\in T}, CAS \times Inter^{n\in T})\} \tag{2}$$

Note that, in the case, we write $Self\text{-}A^{n\in T}_{i\in\mathbb{N}}$ to stand for $Self\text{-}A_{i\in\mathbb{N}}(CAS \times Inter^{n\in T}, CAS\times\ Inter^{n\in T})$. Thus, we have

$$\mathbf{Self\text{-}A}^{n\in T} = \{Self\text{-}A^{n\in T}_{i\in\mathbb{N}}\} \tag{3}$$

This set with the composition operation ";" satisfies two following properties:

Composition of self-adaptive traits. Let f and g be members of $\mathbf{Self\text{-}A}^{n\in T}$, then the composition of self-adaptive traits $f; g : (CAS \times Inter^{n\in T}) \longrightarrow (CAS \times Inter^{n\in T})$ is as $g : (f : (CAS \times Inter^{n\in T}) \longrightarrow (CAS \times Inter^{n\in T})) \longrightarrow (CAS \times Inter^{n\in T})$. In other words, let $f = Self\text{-}A^{n\in T}_{i\in\mathbb{N}}$ and $g = Self\text{-}A^{n\in T}_{j\in\mathbb{N}}$ then

$$(Self\text{-}A^{n\in T}_{i\in\mathbb{N}}\ ;\ Self\text{-}A^{n\in T}_{j\in\mathbb{N}}) = Self\text{-}A_{j\in\mathbb{N}}(Self\text{-}A^{n\in T}_{i\in\mathbb{N}}, CAS \times Inter^{n\in T}) \tag{4}$$

Identity of self-adaptive traits. There exist identities $1_{n\in T} : (CAS \times Inter^{n\in T}) \longrightarrow (CAS \times Inter^{n\in T})$ of self-adaptive traits in $\mathbf{Self\text{-}A}^{n\in T}$ such that, for every f in $\mathbf{Self\text{-}A}^{n\in T}$, $1_{n\in T}; f = f; 1_{n\in T} = f$ to be held. In other words, this can be specified by

$$\begin{aligned} Self\text{-}A_{i\in\mathbb{N}}^{n\in T} &= Self\text{-}A_{i\in\mathbb{N}}(1_{n\in T}, CAS \times Inter^{n\in T}) \\ &= Self\text{-}A_{i\in\mathbb{N}}(CAS \times Inter^{n\in T}, 1_{n\in T}) \\ &= Self\text{-}A_{i\in\mathbb{N}}(CAS \times Inter^{n\in T}, CAS \times Inter^{n\in T}) \end{aligned} \tag{5}$$

Thus, $\mathbf{Self\text{-}A}^{n\in T}$ with the composition operation ";" is called *self-adaptation monoid*. Moreover, the monoid $\mathbf{Self\text{-}A}^{n\in T}$ is also a monoid category including only one object to be the set $\{Self\text{-}A_{i\in\mathbb{N}}^{n\in T}\}$, each of whose members is a self-adaptive trait, and by the composition operation as a morphism, then the associativity and identity on the morphisms are completely satisfied.

5.2 A Category of Self-adaptation Monoids

By the self-adaptation monoids $\mathbf{Self\text{-}A}^{i\in T}$, we can construct $\mathbf{Cat}(\mathbf{Self\text{-}A})$ to be a category of self-adaptation monoids. In fact, $\mathbf{Cat}(\mathbf{Self\text{-}A})$ is constructed as follows:

• *Objects*: $Obj(\mathbf{Cat}(\mathbf{Self\text{-}A}))$ is the set of self-adaptation monoids $\mathbf{Self\text{-}A}^{i\in T}$. That is,

$$Obj(\mathbf{Cat}(\mathbf{Self\text{-}A})) = \{\mathbf{Self\text{-}A}^{i\in T}\} \tag{6}$$

• *Morphisms*: Associated with each object $\mathbf{Self\text{-}A}^{i\in T}$ in $Obj(\mathbf{Cat}(\mathbf{Self\text{-}A}))$, we define a morphism $\mathbf{Self\text{-}A}^{i\in T} \xrightarrow{1_{\mathbf{Self\text{-}A}^{i\in T}}} \mathbf{Self\text{-}A}^{i\in T}$, the identity morphism on $\mathbf{Self\text{-}A}^{i\in T}$ such that

$$\mathbf{Self\text{-}A}^{i\in T} \xrightarrow{1_{\mathbf{Self\text{-}A}^{i\in T}} \stackrel{def}{=} 1_{i\in T}} \mathbf{Self\text{-}A}^{i\in T} \tag{7}$$

or

$$\{Self\text{-}A_{k\in\mathbb{N}}^{i\in T}\} \xrightarrow{1_{\mathbf{Self\text{-}A}^{i\in T}} \stackrel{def}{=} 1_{i\in T}} \{Self\text{-}A_{k\in\mathbb{N}}^{i\in T}\} \tag{8}$$

and to each pair of morphisms $\mathbf{Self\text{-}A}^{i\in T} \xrightarrow{f} \mathbf{Self\text{-}A}^{j\in T}$ and $\mathbf{Self\text{-}A}^{j\in T} \xrightarrow{g} \mathbf{Self\text{-}A}^{j\in T}$ such that

$$\mathbf{Self\text{-}A}^{i\in T} \xrightarrow{f \stackrel{def}{=} 1_{i\in T} \times Inter^{j-i}} \mathbf{Self\text{-}A}^{j\in T} \tag{9}$$

and

$$\mathbf{Self\text{-}A}^{j\in T} \xrightarrow{g \stackrel{def}{=} 1_{j\in T} \times Inter^{k-j}} \mathbf{Self\text{-}A}^{k\in T} \tag{10}$$

there is an associated morphism $\mathbf{Self\text{-}A}^{i\in T} \xrightarrow{f;g} \mathbf{Self\text{-}A}^{k\in T}$, the composition of f with g, such that

$$\mathbf{Self\text{-}A}^{i\in T} \xrightarrow{f;g = 1_{i\in T} \times Inter^{k-i}} \mathbf{Self\text{-}A}^{k\in T} \tag{11}$$

For every object in $Obj(\mathbf{Cat}(\mathbf{Self\text{-}A}))$ and the morphisms

$$\mathbf{Self\text{-}A}^{i\in T} \xrightarrow{f \overset{def}{=} 1_{i\in T}\times Inter^{j-i}} \mathbf{Self\text{-}A}^{j\in T} \tag{12}$$

$$\mathbf{Self\text{-}A}^{j\in T} \xrightarrow{g \overset{def}{=} 1_{j\in T}\times Inter^{k-j}} \mathbf{Self\text{-}A}^{k\in T} \tag{13}$$

and

$$\mathbf{Self\text{-}A}^{k\in T} \xrightarrow{h \overset{def}{=} 1_{k\in T}\times Inter^{m-k}} \mathbf{Self\text{-}A}^{m\in T} \tag{14}$$

in $Arc(\mathbf{Cat}(\mathbf{Self\text{-}A}))$, the following equations hold:

Associativity: $(f;g);h = f;(g;h) = 1_{i\in T} \times Inter^{m-i}$
Identity: $1_{\mathbf{Self\text{-}A}^{i\in T}};f = f = f;1_{\mathbf{Self\text{-}A}^{j\in T}}$

(i.e., $1_{i\in T};1_{i\in T} \times Inter^{j-i} = 1_{i\in T} \times Inter^{j-i} = 1_{i\in T} \times Inter^{j-i};1_{j\in T}$)

As a result, the above-mentioned monoid morphisms can be diagrammatically drawn such as

$$\mathbf{Self\text{-}A}^{i\in T} \xrightarrow{1_{i\in T}\times Inter^{\pm k}} \mathbf{Self\text{-}A}_{i\pm k\in T} \tag{15}$$

or

$$\{Self\text{-}A^{i\in T}_{l\in\mathbb{N}}\} \xrightarrow{1_{i\in T}\times Inter^{\pm k}} \{Self\text{-}A^{i\pm k\in T}_{l\in\mathbb{N}}\} \tag{16}$$

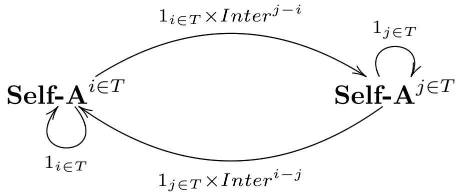
These are all the basic ingredients we need to have the category $\mathbf{Cat}(\mathbf{Self\text{-}A})$. Let us see a general definition of category in [7] for reference.

5.3 Some Properties of Category *Cat(Self-A)*

By the construction of category $\mathbf{Cat}(\mathbf{Self\text{-}A})$), some emerging significant properties are presented in this subsection.

Property 1. *All monoid morphisms of* $\mathbf{Cat}(\mathbf{Self\text{-}A})$ *is monoid isomorphisms*

Proof: This result immediately stems from diagram (15). In fact, for every pair of monoid morphisms in $Arc(\mathbf{Cat}(\mathbf{Self\text{-}A}))$ between $\mathbf{Self\text{-}A}^{i\in T}$ and $\mathbf{Self\text{-}A}^{j\in T}$, we always have the following diagram:



(17)

These monoid morphisms satisfy an isomorphic relationship. Q.E.D.

Property 2. *Isomorphisms between any pair of monoids in* **Cat**(**Self-A**) *are ever isomorphisms between the pair of* CASs*.*

Proof: This comes from the fact that each object of category **Cat**(**Self-A**) is just a CAS. Q.E.D.

From the above-mentioned justification of **Cat**(**Self-A**), we are able to derive $\mathbf{Self\text{-}A}^{i\in T}$. Derivation of every $\mathbf{Self\text{-}A}^{i\in T}$ is simplified by the following facts:

Property 3. *There exists always a self-adaptation monoid* **Self-A***, as simply as it can, in* **Cat**(**Self-A**) *constructed. Hence, it is available to start with.*

Proof: It emerges that

$$\begin{aligned}\mathbf{Self\text{-}A} &= \{Self\text{-}A_{i\in\mathbb{N}}(CAS \times Inter^0, CAS \times Inter^0)\} \\ &= \{Self\text{-}A_{i\in\mathbb{N}}(CAS, CAS)\}\end{aligned} \tag{18}$$

thus

$$1 \xrightarrow{\mathbf{Self\text{-}A}} Obj(\mathbf{Cat}(\mathbf{Self\text{-}A})) \tag{19}$$

Q.E.D.

Property 4. *Given* **Self-A***, we can compute* $\mathbf{Self\text{-}A}^{i\in T}$*.*

Proof: We evaluate self-adaptation monoid $\mathbf{Self\text{-}A}^{i\in T}$ such that

$$1 \xrightarrow{\mathbf{Self\text{-}A}^{i\in T}} Obj(\mathbf{Cat}(\mathbf{Self\text{-}A})) \tag{20}$$

based on the facts that

$$\left(\begin{array}{c} 1 \xrightarrow{\mathbf{Self\text{-}A}} Obj(\mathbf{Cat}(\mathbf{Self\text{-}A})) \\ \text{and} \\ \mathbf{Self\text{-}A} \xrightarrow{1_0 \times Inter^i} \mathbf{Self\text{-}A}^{i\in T} \end{array}\right) \tag{21}$$

Note that $\mathbf{Self\text{-}A} \xrightarrow{1_0} \mathbf{Self\text{-}A}$ Q.E.D.

Property 5. *Given* $\mathbf{Self\text{-}A}^{i\in T}$*, we can compute* $\mathbf{Self\text{-}A}^{j\in T}$ *for every* $j \neq i$*.*

Proof: Self-adaptation monoid $\mathbf{Self\text{-}A}^{j\in T}$ is evaluated such that

$$1 \xrightarrow{\mathbf{Self\text{-}A}^{j\in T}} Obj(\mathbf{Cat}(\mathbf{Self\text{-}A})) \tag{22}$$

based on the facts that

$$\left(\begin{array}{c} 1 \xrightarrow{\mathbf{Self\text{-}A}^{i\in T}} Obj(\mathbf{Cat}(\mathbf{Self\text{-}A})) \\ \text{and} \\ \mathbf{Self\text{-}A}^{i\in T} \xrightarrow{1_{i\in T} \times Inter^{j-i}} \mathbf{Self\text{-}A}^{j\in T} \end{array}\right) \tag{23}$$

Q.E.D.

From the construction of **Cat**(**Self-A**), we see that every $\mathbf{Self\text{-}A}^{i \in T}$ can be formed in the unifying way based on properties 3, 4 and 5. As a result, we gain a substantial procedure of construction at a high abstract level without any excessive inclination towards a specific implementation detail. This is quite helpful when we want to justify whether or not some certain properties of the construction are true. In fact, we can prove

Property 6. *Every monoid* $\mathbf{Self\text{-}A}^{i \in T}$ *can be constructed by any other monoid in* **Cat**(**Self-A**)

Proof: Applying properties 3, 4 and 5 to construct every monoid $\mathbf{Self\text{-}A}^{i \in T}$ from another monoid in **Cat**(**Self-A**) Q.E.D.
This is certainly a property we expect of any construction procedure.

Property 7. **Cat**(**Self-A**) *is a complete graph*

Proof: In fact, this is a consequence stemming from property 6 Q.E.D.
This is indeed a property of our abstract construction mechanism.

6 Conclusions

In this paper, we have investigated algebraic structures of self-adaptation in CASs based on categorical approach.

We have started with investigating self-adaptation in CASs. Then, $\mathbf{Self\text{-}A}^{i \in T}$ has been constructed as a self-adaptation monoid. By the self-adaptation monoids, we have formed **Cat**(**Self-A**) as a category of the self-adaptation monoids for discovering the significant properties of the self-adaptation.

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References

1. Adamek, J., Herrlich, H., Strecker, G.: Abstract and Concrete Categories. John Wiley and Sons (1990)
2. Asperti, A., Longo, G.: Categories, Types and Structures. M.I.T Press (1991)
3. Bergman, G.M.: An Invitation to General Algebra and Universal Constructions. Henry Helson, 15 the Crescent, Berkeley CA 94708, US (1998)
4. Lawvere, F.W., Schanuel, S.H.: Conceptual Mathematics: A First Introduction to Categories, 1st edn. Cambridge University Press (1997)
5. Levine, M.: Categorical algebra. In: Benkart, G., Ratiu, T.S., Masur, H.A., Renardy, M. (eds.) Mixed Motives. Mathematical Surveys and Monographs, chapter I, II, II of Part II, vol. 57, pp. 373–499. American Mathematical Society, USA (1998)

[1] Nguyen Tat Thanh University, Vietnam

6. Vinh, P.C.: Autonomic computing and networking. In: Chapter Formal Aspects of Self-* in Autonomic Networked Computing Systems, pp. 381–410. Springer (2009)
7. Vinh, P.C.: Self-Adaptation in Collective Adaptive Systems. Mobile Networks and Applications (2014, to appear)
8. Vinh, P.C., Alagar, V., Vassev, E., Khare, A. (eds.): ICCASA 2013. LNICST, vol. 128. Springer, Heidelberg (2014)
9. Vinh, P.C., Hung, N.M., Tung, N.T., Suzuki, J. (eds.): ICCASA 2012. LNICST, vol. 109. Springer, Heidelberg (2013)
10. Vinh, P.C., Tung, N.T.: Coalgebraic Aspects of Context-Awareness. Mobile Networks and Applications, August 2012. doi:10.1007/s11036-012-0404-0