Reconstructing Low Degree Triangular Parametric Surfaces Based on Inverse Loop Subdivision

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Abstract. In this paper, we present an efficient local geometric approximate method for reconstruction of a low degree triangular parametric surface using inverse Loop subdivision scheme. Our proposed technique consists of two major steps. First, using the inverse Loop subdivision scheme to simplify a given dense triangular mesh and employing the result coarse mesh as a control mesh of the triangular Bézier surface. Second, fitting this surface locally to the data points of the initial triangular mesh. The obtained parametric surface is approximate to all data points of the given triangular mesh after some steps of local surface fitting without solving a linear system. The reconstructed surface has the degree reduced to at least of a half and the size of control mesh is only equal to a quarter of the given mesh. The accuracy of the reconstructed surface depends on the number of fitting steps k, the number of reversing subdivision times i at each step of surface fitting and the given distance tolerance ε. Through some experimental examples, we also demonstrate the efficiency of our method. Results show that this approach is simple, fast, precise and highly flexible.

Keywords: Triangular Bézier · Parametric surface · Loop subdivision · Reconstructing surface · Surface Fitting

1 Introduction

Modeling method using 3D mesh to describe a surface of an object in the real world is widely used in fields. It consists of small polygonal pieces that can be linked together. These small pieces are often triangles or quadrangles. Beside the 3D meshes, the parametric surfaces have also proven to be the most popular representation method for their specific characteristics including continuousness, smoothness and exact processing [18]. Both of them play an important role for versatile design and research tool in many fields [19], such as Computer Graphic (CG), Computer Aided Geometric Design (CAGD), Reverse Engineering (RE) and Virtual Reality (VR), simulating the surfaces of topography, ect...with applications ranging from simulated surgeries to animated films in the movie industry.

The smooth surface reconstruction from the data points of the given polygon mesh, which surface is recreated in the computer and conserved most of its physical charac-

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teristics, is a difficult and challenging problem because of the following reasons: constructing control polygon meshes, complicated procedures of refinement and reparameterization.

The most of methods interpolate or approximate the parametric surfaces or sudivision surfaces from rectangular meshes. The rectangular surfaces as tensor-product surfaces, are reconstructed by solving linear equation systems, solving a least squares problem and it is difficult to control locally [11,12]. Recently, the iterative geometric fitting methods that do not require the solution of a linear system studied and improved. However, those methods recreated the subdivision surfaces [2,7,14,16] or rectangular parametric surfaces [1, 15, 17] as tensor-product Bézier, tensor-product B-splines by using the initial mesh as the control mesh of fitting surface and so they required that the number of control vertices is equal to the number of data points. The most of the input data is large, so the degree of reconstructed parametric surface is very high.

The Bézier surface is a common form of the paramatric surface and a mathematical description of the surface very used in computer graphics as it is much smoother and compact than mesh. We can distinguish a triangular Bézier surface from a rectangular surface by basing on the basic function over the parameter domain, as known Bernstein polynomial, and the number of control points. Comparing with the rectangular paramatric surfaces, the triangular ones allow to join flexibly and suit arbitrary topological type. The other hand, since the triangular Bernstein basic function is more complex than both the rectangular Bernstein and B-spline basic function, the research of multivariata Bernstein basis over a triangular doman is very significant and still an open question.

Besides, the subdivision surfaces have also become very popular in the computer graphics and geometric modeling domain. They have been applied widely for the representation and manipulation of multiresolution surfaces having arbitrary topological type. There are many subdivision schemes that have been studied and applied such as subdivision schemes of Catmull-Clark, Doo-Sabin, Loop, etc [5]. The Loop subdivision scheme is popularly applied to an arbitrary triangular mesh based on the method called approximate vertex insertion [3]. Inverse subdivision aims at constructing a coarse mesh from a given dense mesh. As the inverse subdivision can be stopped after each step, different multiresolution representation can be obtained. Thus, we want to benefit by the inverse subdivision scheme for the simplification of subdivision meshes.

In this paper, we propose an efficient local geometric approximate method for reconstruction of a low degree triangular parametric surface using inverse Loop subdivision scheme. Our proposed technique consists of two major steps. First, using the inverse Loop subdivision scheme to simplify the given dense triangular mesh and employing the result coarse mesh as a control mesh of the triangular Bézier surface. Second, fitting this surface locally to data points of the initial triangular mesh. The obtained parametric surface is approximate to all data points of the given triangular mesh after some steps of local surface fitting. In contrast with traditional reconstruction methods, our method does not solve linear systems. Therefore, this approach completely avoids the parametric dependency problem.

The main contribution of our work is that we proposed an efficient method for reconstructing a triangular Bézier surface from data points of the initial triangular mesh. The reconstructed surface has degree reduced to at least of a half and the size of control mesh is only equal to a quarter of the given mesh. The accuracy of the reconstructed surface depends on the number of fitting steps k, the number of reversing subdivision times i at each step of fitting and the given distance tolerance ε.

The remainder of this article is organized as follows. We describe the inverse Loop subdivision scheme and the triangular Bézier surface in Section 2. In Section 3, an efficient approximate method to recreate the parametric surface from the triangular mesh is proposed. And Section 4 presents some experimental results. Finally, some concluding remarks are drawn in the last Section 5.

2 Inverse Loop Subdivision and Triangular Bézier Surface

In order to simplify the given dense triangular mesh by using an inverse subdivision scheme and employing the result coarse mesh as a control mesh of the triangular Bézier parametric surface, we describe the inverse Loop subdivision scheme and the triangular Bezier surface in this section.

2.1 Inverse Loop Subdivision

Loop subdivision [3] is a process that add new vertices and new faces to a coarse arbitrary triangular mesh to create a finer triangular mesh by breaking triangular faces of this coarse mesh into the smaller triangular faces. In each step of Loop subdivision, to add the new vertices and faces in a triangular mesh, each triangular face is divided into four smaller new triangular faces by splitting each of them and connecting new vertices together (Fig 1).

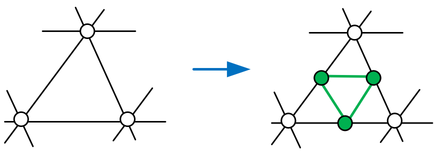


Fig. 1. Each triangular is replaced by the four smaller new triangles

Let us consider an initial coarse triangular mesh $M^0(m)$ with m data points $\{P_j^0 \mid_{j=1...m}\}$. From the initial mesh M^0, by applying the Loop subdivision successively through masks or transformative matrices, a hierarchy of meshes M^1, M^2, M^3, etc is generated then gradually converges to the smooth surface of object. After each step i of Loop subdivision, the vertices of meshes M^i include two types:

- The old vertices of the triangular mesh are modified, which are called vertex-vertices (e.g. vertex p^i of mesh M^i corresponding to vertex p^{i-1} of mesh M^{i-1} in Figure 2).

- The new vertices are inserted into the edges of the triangular mesh, which are called edge -vertices (e.g. vertices $p_j^i\,|_{j=1,2,3...l}$ of mesh M^i corresponding to edges of M^{i-1} in Figure 2).

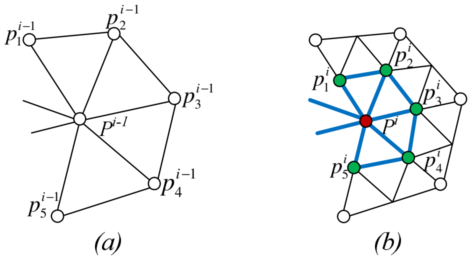


Fig. 2. A segment of a triangular mesh before (a) and after (b) Loop subdivision

The inverse of Loop subdivision aims at constructing a coarser triangular mesh from the given fine triangular mesh. As the inverse Loop subdivision can be stopped after each step, different multiresolution representation can be obtained.

To reverse Loop subdivision, we have to construct the exact formulas to map the set of vertices of mesh M^i into the set of vertices of mesh M^{i-1}. Assume that the positions of edge-vertices and vertex-vertices in Loop subdivision scheme are correlative with the α and β weights. We have to determine the positions of vertex p^{i-1} from vertices p^i and $p_j^i\,|_{j=1,2,3...l}$. In other words, we must determine the weights η and μ correlative with the α and β weights by using inverse formulas. The expression of the inverse vertex-vertices p^{i-1} based on p^i and the neighbor vertices $p_j^i\,|_{j=1,2,3...l}$ of mesh M^i is determined as follows:

$$p^{i-1} = \mu.p^i + \eta.\sum_{j=1}^{l} p_j^i \tag{1}$$

With $\alpha = \frac{1}{l}\left(\frac{5}{8} - \left(\frac{3}{8} + \frac{1}{4}\cos\left(\frac{2\pi}{l}\right)\right)^2\right)$; $\beta = 1 - l\alpha$

$$\mu = \frac{5}{8\beta - 3} \text{ and } \eta = \frac{\beta - 1}{n\left(\beta - \frac{3}{8}\right)} \tag{2}$$

where l is valence of vertex p^i.

For the boundary vertices, by applying the inverse masks for cubic B-spline of Bartels and Samavati [13], we have the inverse formula:

$$P^{i-1} = -\frac{1}{2}P_1^i + 2P^i - \frac{1}{2}P_2^i \tag{3}$$

2.2 Triangular Bézier Surface

A triangular Bézier parametric surface S over the triangular domain Δ(A,B,C) defined by the following equation [18]:

$$S(u,v,w) = \sum_{i+j+k=n} B^n_{i,j,k}(u,v,w)p_{ijk} \tag{4}$$

where n is the degree of surface, p_{ijk} are the control points of the control polyhedron with $(n+1)(n+2)/2$ points, $u+v+w=1$ and

$$B^n_{i,j,k}(u,v,w) = \frac{n!}{i!\,j!\,k!}u^i v^j w^k \tag{5}$$

is called a Bernstein polynomial over the triangular domain Δ.

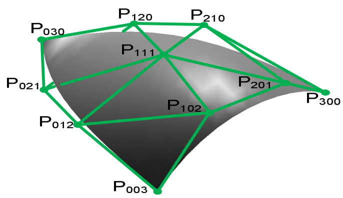


Fig. 3. A triangular Bézier parametric surface for degree 3

A triangular Bézier parametric surface has shape properties as follows: convex hull property, end point interpolation, end point tangency, the boundary curves are Bézier curves formed by the boundary control points and affine invariance. The degree of the triangular Bernstein polynomial function is lower than the rectangular one and the control polyhedron is a triangular mesh, so the triangular Bézier surface allow to perform the surface of 3D objects flexibly.

Let us consider a triangular mesh $M^0(m)$ with m data points, the degree n of the triangular Bezier surface which has $M^0(m)$ as its control mesh is determined as follows:

$$n = \frac{1}{2}(\sqrt{1+8m} - 3) \tag{6}$$

After steps of reversing Loop subdivision i, the degree of triangular Bézier surface will be equal to $n/2^i$. Then, the number of reversing Loop subdivision i must satisfy the following condition:

$$0 < i \le log_2(\frac{1}{2}(\sqrt{1+8m} - 3)) \quad (7)$$

3 Reconstructing the Low Degree Parametric Surface

In this section, we proposed a geometric approximate method to reconstruct the parametric surface *S* from the given triangular mesh $M^0(m)$ with *m* data points sampled from the surface of 3D object.

By using the inverse Loop subdivision scheme to simplify the given dense triangular mesh and employing the vertices of this result coarse mesh as the control vertices of a triangular Bézier surface. After that, this parametric surface is locally translated to gradually converge to an ideal smooth surface that pass thought all data points of the initial triangular mesh. After some steps of local surface fitting, the obtained parametric surface is approximate to all data points of the given triangular mesh. The proposed method is presented by diagram in figure 4.

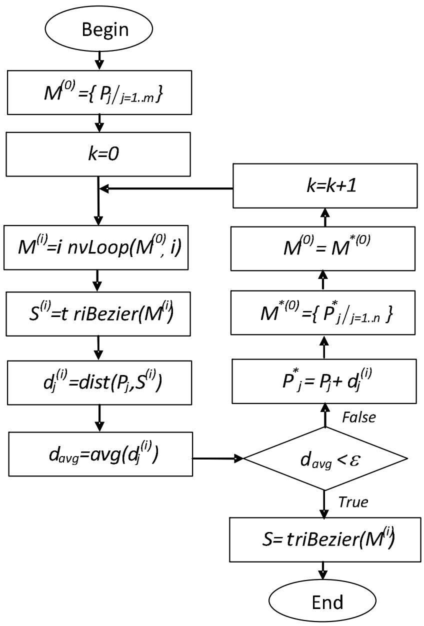


Fig. 4. Approximative surface fitting algorithm

with denoting the expressions as followings:

- $M^{(0)}=\{ P_j|_{j=1..m} \}$ is the initial triangular mesh with P_j are data points sampled from the surface of 3D object.
- $M^{(i)}=invLoop(M^{(0)}, i)$ is the triangular mesh $M^{(0)}$ after *i* steps of reversing of Loop subdivision.

- $S^{(i)}=triBezier(M^{(i)})$ is the triangular Bézier parametric surface with the control mesh $M^{(i)}$.
- $d_j^{(i)}=dist(P_j,S^{(i)})$ is the distance from point P_j of the given mesh $M^{(0)}$ to the parametric surface $S^{(i)}$.
- $d_{avg}^{(i)}=avg(d_j^{(i)})$ is average of the distances $d_j^{(i)}$.
- P^*_j is point P_j after fitting.
- $M^{*(0)}$ is the triangular mesh that is reconstructed by the new points P^*_j.

The quality and accuracy of the reconstructed surface depends on the number of fitting steps k, the number of reversing subdivision times i at each step of fitting and the given distance tolerance ε.

Considering this computation cost is a constant time, the projections of initial mesh points P_j are executed m times, and suppose that the repeat-until iterates for k times. The value k depends on given tolerance ε. Then the approximative surface fitting algorithm has an asymptotic complexity $\theta(m \times k)$.

4 Experimental Results

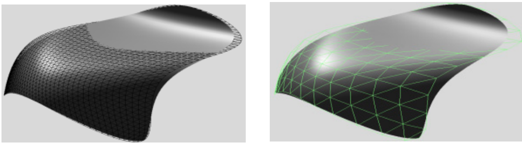
In this section, we presents some experimental results to prove the effective of the proposed method, the influence of the number of fitting steps k, the number of reversing subdivision times i and the given distance tolerance ε on the quality and accuracy of the reconstructed surface.

Given an initial triangular mesh M^0 with 1225 points and 2304 faces, corresponding to a control mesh of triangular Bézier surface with degree 48. The size of the inverse subdivision mesh can be predicted according to selected value of i. After $i=4$ of inverse subdivision steps, we obtain a coarse inverse subdivision mesh used as a control polyhedron of a triangular Bézier surface. These control polyhedrons have 10 points and 9 faces, corresponding to triangular Bézier surface with degree 3. The relative of the number of reversing Loop subdivision times i, the degree of parametric surface and the size of control mesh are presented in Table I. After each step of local fitting, the result surface has the degree decreased at least of a half and the size of control mesh is only equal to a quarter of the given mesh.

Table 1. The degree of parametric surface corresponding to the number of inverse subdivision

i	Degree n	Number of points m	Number of faces
1	24	325	576
2	12	91	144
3	6	28	36
4	3	10	9

Figure 5a uses the initial mesh as the control mesh of parametric surface, and figure 5b uses the control mesh of parametric which is the initial mesh simplified by i=2 steps of reversing subdivision. Result shows that in case of the obtained parametric surfaces both are equivalent together, but the degree of the second surface reduced to 2^i times.



(a).i=0 and n=48 *(b).i=2 and n=12*

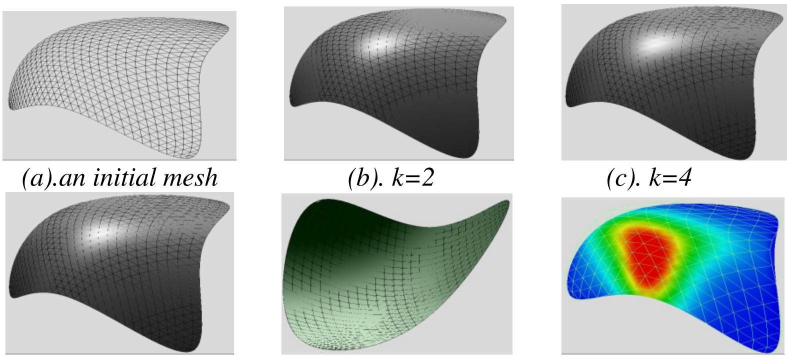
Fig. 5. The result surface after *i* times of inverse subdivision with k=1

In order to prove the influence of the number of fitting steps k and the given distance tolerance ε for the convergence of parametric surface to the initial mesh, we analyse the convergence by the maximum distances d_{max} and average distances d_{avg} between the initial mesh points and the reconstructed approximative surface.

Table II shows the distance values d_{max} and d_{avg} depend to the number of fitting steps k. As expected, the higher the number k of surface fitting step, the smaller distance values. This is said that the result surface quickly converge to the parametric surface which is interpolate to the data points after several steps of local geometric fitting.

Table 2. The distance values depend to the number of fitting steps k

k	d_{max}	d_{avg}
1	0.01076160	0.003063370
2	0.00669807	0.001669370
3	0.00490922	0.001163310
4	0.00396409	0.000929139
5	0.00300018	0.000704332



(a).an initial mesh *(b). k=2* *(c). k=4*

(d). k=6 (outside) *(e). k=6 (inside)* *(f).result surfaces*

Fig. 6. The initial triangular mesh and the obtained approximate parametric surface after k steps of local geometric fitting

Figure 6a illustrates an initial triangular mesh M^0 with 561 points and 1024 faces, corresponding to a control mesh of triangular Bézier surface with degree n=32. After k=2,4,6 steps of local fitting, the triangular Bézier surface quickly converge to the given mesh (Fig 6b,c,d). Figures 6d and 6e show that the reconstructed surfaces pass through most data points of the given mesh. It is a triangular Bézier surface which has the degree n=16 and the control mesh with 153 points và 256 faces (Fig 6f).

Figure 7 compare the quality of the obtained surface (Fig 7a,c) with the smooth surface interpolated to all data points of the origin mesh (Fig 7b,d) by texture mapping technique (Fig 7a,b) and reflection lines technique (Fig 7c,d) after k=4 steps of local fitting. The result shows that, the reconstructed surface has good quality, approximate the smooth surface interpolated to all data points of the origin mesh.

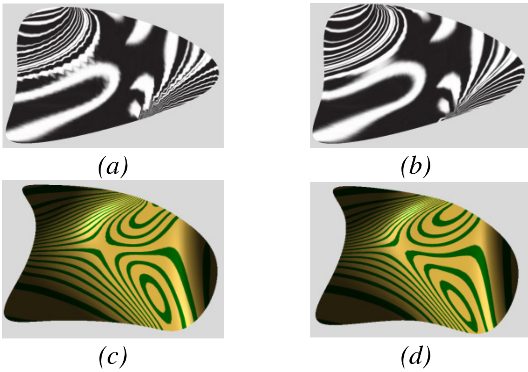


Fig. 7. The obtained surface after k =4 (a,c) and the smooth surface interpolated to all data points of the origin mesh (b,d).

5 Conclusions

In this paper, base on a local geometric approximate fitting method and the inverse Loop subdivision scheme, we proposed an effective technique for reconstructing of low degree triangular Bézier surfaces. The experimental results show that the obtained surface has good quality, approximate to the smooth surface interpolated to all data points of the origin mesh and can be considered as a compression surface. Our proposed technique has some of advantages as follows:

- Avoiding the peak point problem of the control mesh reconstruction by solving a linear system and least-square fitting method, the recontructed surface still pass through most data points of the initial triangular mesh after several steps of local surface fitting.
- Controlling the surface fitting visually and exactly by approximate geometric technique.
- The obtained parametric surface has the degree reduced to at least of a half and the size of control mesh is only equal to a quarter comparing to methods which use the given mesh as a control mesh of the reconstructed surface.
- The proposed technique apply to triangular meshes, so it take over advantage of triangular meshes and parametric surfaces on triangular doman.

With the inverse Loop subdivision as the control mesh of the parametric surface on triangular doman, our method is promising in areas such as mesh compression, surface editing and manipulation and versatile design, ect.

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