Chapter 12
Using Mode Shapes for Real Time ODS Animation

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Abstract In a rotating machine, the dominant forces are applied at multiples of the machine running speed, called orders. An ***order-tracked ODS*** is assembled from the peaks at one of the order frequencies in a set of response frequency spectra of a machine. An order-tracked ODS is a convenient way to visualize and monitor the health of the machine.

In this paper, it is shown how ***modes participate*** in an order-tracked ODS of a rotating machine, and how they participate differently at different operating speeds. It is also shown how the ***modal participation*** can be used to ***expand*** an order-tracked ODS so that it is suitable for display on a model of the machine. With an animated ODS display, changing vibration levels and vibration ***hot spots*** can be observed while the machine is running.

Keywords Operating deflection shapes (ODS's) • Experimental modal analysis (EMA) mode shapes • Finite element analysis (FEA) mode shapes • Modal assurance criterion (MAC)

12.1 Introduction

It is well known that most rotating machines will exhibit different vibration levels under different loads and speeds. A convenient way to troubleshoot these problems is to visualize the operating deflection shapes (ODSs) of the machine in ***"real time"***. This is conveniently done by attaching multiple accelerometers to the machine surfaces, and acquiring vibration data from the machine while it is running. In addition to visualizing the deflection of the machine in real time, an ODS can be used to diagnose machine faults by numerically comparing its current values with previously archived values.

All machines and mechanical structures have resonances, also called modes of vibration. Each resonance has a ***unique natural frequency***, ***damping value,*** and ***mode shape***. It is well known that if a resonance is excited, the response of the machine or structure can be excessive. In this sense, modes are referred to a ***"mechanical amplifiers"***. Over time, sustained excessive vibration levels will cause a variety of failures.

When excited, ***modes participate in*** or ***contribute to*** the overall vibration response of a machine. In an ideal sense, a mechanical structure has an infinite number of modes, but in a practical sense only a few modes participate significantly in its response.

In this paper, it is first shown how ***modal participation*** in an ODS is calculated**.** Once the ***modal participation*** is known, mode shapes with many components in them can be used to ***expand*** the ODS, thus providing a more accurate and realistic description of the machine's vibration. When displayed in animation, an expanded ODS is useful for observing overall vibration levels and for quickly spotting ***areas of excessive vibration (hot spots)*** under different operating conditions.

12.1.1 Operating Deflection Shape (ODS)

If two or more sensors (located at different points and/or in different directions) are used to measure the response of a machine while it is running, this data is called an **Operating Deflection Shape (ODS)**. An ODS is a ***vector of complex values***, each component of which has a magnitude & phase, or real & imaginary parts. When an ODS is displayed in animation on a model of the machine, it shows how each test point is deflected with a magnitude & phase ***relative*** to the deflection of all other test points.

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In order to display it in animation, an ODS must have the correct ***relative magnitude & phase*** between all its components. This relative magnitude & phase requirement among its components is the reason why it is called a deflection ***"shape"***.

12.1.2 Ways to Acquire ODS Data

There are two ways to acquire an ODS so that the relative magnitudes & phases of all its components are valid;

1. Acquire ***data simultaneously from all sensors***
   A multi-channel simultaneous acquisition system is required in order to acquire data from all sensors at once. An ODS can be assembled either from the ***same sample*** of data in simultaneously acquired time records, or from the ***same sample*** of data in multiple frequency (Fourier) spectra which were calculated from the acquired time records.
2. Acquire ***data simultaneously from at least two sensors***, with one sensor remaining at the ***same fixed reference sensor*** location throughout the acquisition process.
   In this case, data is acquired sequentially in ***multiple measurement sets***, with each set containing data from ***one or more roving sensors*** and the ***same (fixed) reference sensor***. In order to preserve the correct relative magnitude & phase in all shape components, a ***cross channel*** function must be calculated between all roving responses and the reference response. A Cross spectrum between the roving and reference responses is commonly calculated. Another function called an ODS FRF can be calculated. An ODS FRF is the Auto spectrum of a roving response combined with the phase of the Cross spectrum between the roving and reference responses.

12.2 Order-Tracked ODS

In a rotating machine, excitation forces are caused by unbalances, shaft misalignments, broken or misaligned gear teeth, reciprocating parts, and many other faults. All of these faults involve rotating parts, and therefore create excitation forces that are ***cyclic*** in nature. These forces are functions of the running speed of the machine and are called ***"orders"***. The first order is equal to the running speed, the second order equal to twice the running speed, the third order equal to three times the running speed, and so on. In a rotating machine, the dominant forces are applied only at multiples of the machine running speed. In other words, forces are applied only in ***discrete narrow frequency bands.***

An ODS assembled from the peaks at one of the order frequencies in a set of response frequency spectra is called an ***order-tracked ODS***. The machine speed should remain unchanged during data acquisition, but could change between acquisitions due to varying operating conditions. Therefore, in order to obtain an order-tracked ODS, the machine speed must also be measured (using a tachometer), and order peaks located in the spectra from which an order-tracked ODS can be assembled.

12.3 Variable Speed Rotating Machine

Figure 12.1 shows a variable speed rotating machine, instrumented with eight tri-axial accelerometers. An accelerometer is attached to the top of each bearing block, and six accelerometers are attached to the base plate; three on the front edge and three on the back edge. This test setup was used to measure order-tracked ODSs under different machine speeds.

A laser tachometer with its beam pointed at the outer wheel of the machine was used to measure the machine speed, as shown in Fig. 12.1. The outer wheel had reflective tape on it, so the laser measured the once-per-revolution speed of the machine.

A spectrum of the tachometer signal is shown in Fig. 12.2. The running speed of the machine is obtained from the ***lowest frequency peak*** in the tachometer spectrum.

Figure 12.3 contains a model of the machine that was used to display ODSs in animation. Each of the test points is displayed as a cube icon and is numbered.

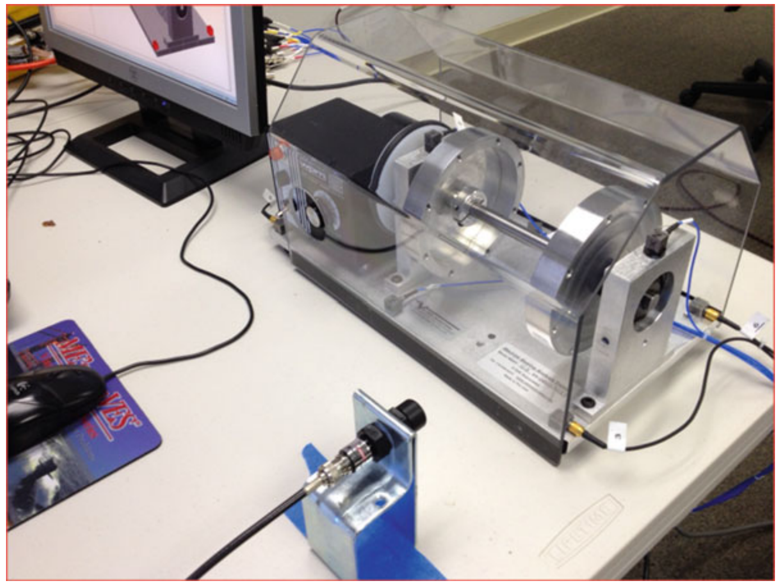


Fig. 12.1 Variable speed rotating machine

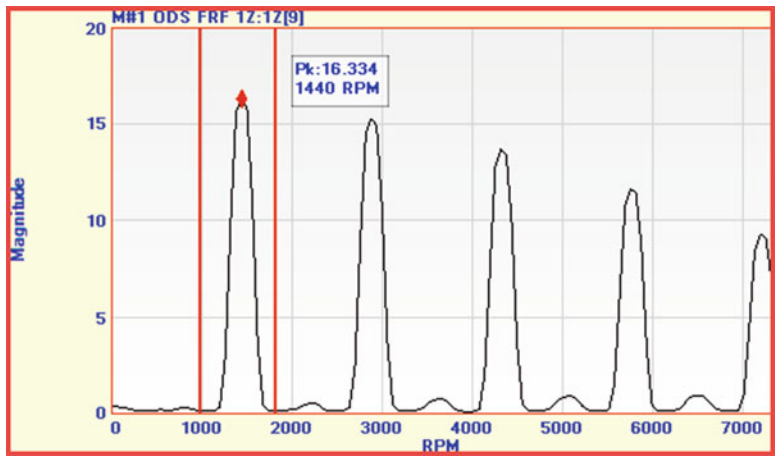


Fig. 12.2 Laser tachometer spectrum

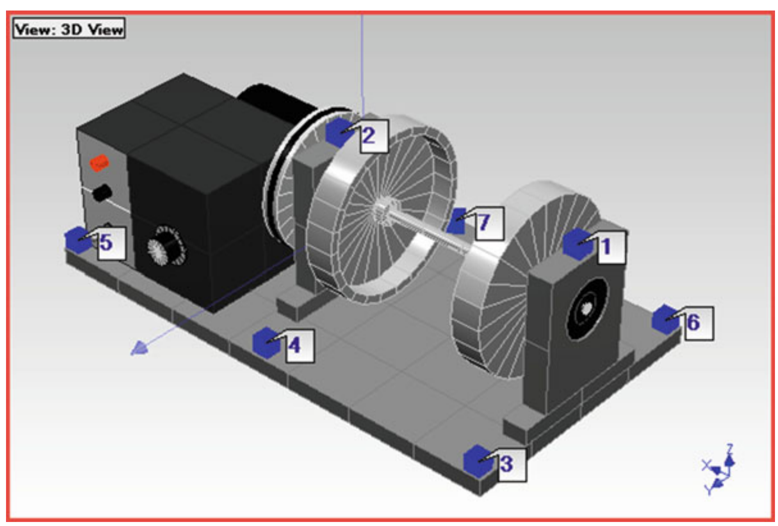


Fig. 12.3 Rotating machine model

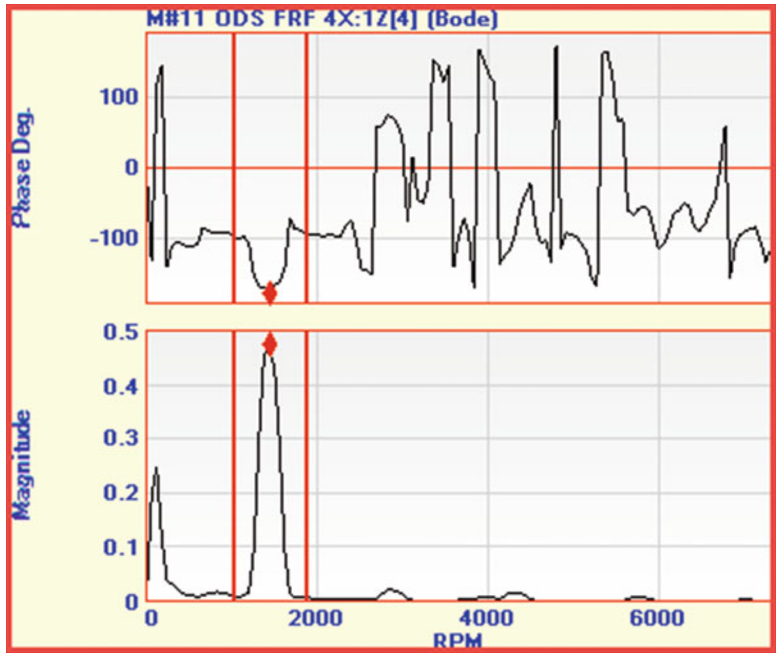


Fig. 12.4 Typical ODS FRF

12.4 Multiple Measurement Sets

A total of 24 ODS FRFs were calculated from data acquired sequentially from the machine in ***eight measurement sets***. Each measurement set was acquired with a 4-channel analyzer that simultaneously acquired four signals of accelerometer data. In each measurement set, three time waveforms were acquired from one of the tri-axial accelerometers, together with a time waveform from the reference accelerometer. Accelerometer 1Z, which measured the Z direction acceleration on the outside bearing block, was used as the reference.

Figure 12.4 shows a typical ODS FRF acquired from the rotating machine. Its magnitude is the Auto spectrum of the acceleration response, and its phase is the phase of the Cross spectrum between the response and the reference response.

Notice that the units of the ODS FRF are velocity units, indicating that it has been integrated from acceleration to velocity. Notice also that the magnitude peak appears to be somewhat ***"widened"***. This is because the time waveforms were windowed with a ***"flat top"*** window. When a waveform is a ***narrow band***, this window makes the peak values at the order frequencies more accurate [1].

An order-tracked ODS is assembled by taking the peak values of the ODS FRFs at one of the order frequencies. This is done by using a ***"peak cursor"***, that finds the peak value in a small band surrounding each order peak.

Figure 12.5 shows two order-tracked ODSs displayed side by side on the machine model, one at 985 RPM and the other at 2280 RPM. Of course, an ***animated display*** of the ODSs makes it easier to see the difference between them. The deflections at the eight test points are the actual values acquired from the accelerometers. The deflections at all of the other ***un-measured*** points are being ***"geometrically interpolated"*** from the deflections of nearby test points. The MAC value [2] of the two shapes is displayed on the right. Its low value **(0.10)** indicates that the two shapes are very different.

12.5 Mode Shapes of the Machine

It is apparent from the display in Fig. 12.5 that the ODS probably contains participation of both the ***"rigid body"*** and the ***"flexible"*** mode shapes of the base plate and bearing blocks. Since the machine is resting on four rubber mounts (one under each corner), its rigid body modes will participate in its ODS. The machine has ***six rigid body*** mode shapes. These mode shapes describe the ***free-free motion*** of the machine in space, but it will be shown that they also dominate its ODSs since it is resting on ***four soft springs***.

The rigid body and flexible body mode shapes of the machine were obtained from a finite element model of the base plate and bearing blocks. The easiest way to do this is to treat each of the three parts separately.

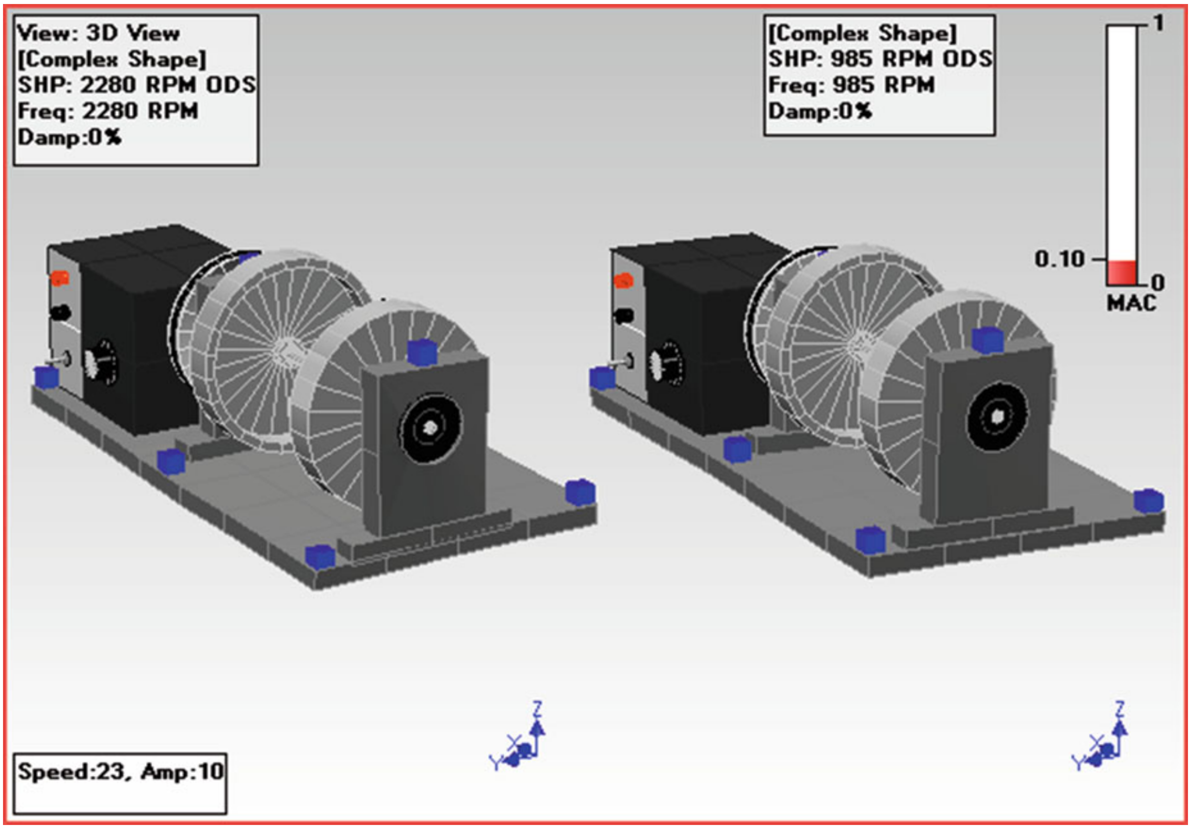


Fig. 12.5 ODSs at 985 and 2280 RPM

FEA models of the base plate and one of the bearing blocks were built first. Then the FEA modes of the base plate and the bearing block were calculated separately.

Figure 12.6 lists the FEA mode shapes of the base plate. It was modeled using 112 FEA brick elements. Figure 12.7 lists the FEA modes of the bearing block. It was modeled using 71 FEA bricks and 6 FEA prisms. Notice that both substructures have 6 rigid body modes.

The Structural Dynamics Modification (SDM) method [3] was used to ***"attach"*** the two bearing blocks to the base plate using stiff springs. This is called ***"substructuring". FEA springs*** were used to model the connections of the blocks to the base plate. Figure 12.8 shows one of the bearing block models connected to the base plate using ***18 very stiff FEA spring***s (with 1E6 lbf/in stiffness). Each spring applies stiffness between a point on a bearing block and a point on the base plate in one (X, Y, or Z) direction.

The mode shapes of the base plate and bearing blocks attached together are listed in Fig. 12.9. Notice that there are still ***six rigid body*** mode shapes, as expected.

12.6 Modal Participation Matrix

The mode shapes of the base plate and bearing blocks will be used to expand the order-tracked ODSs acquired from the eight accelerometers on the rotating machine. As a first step, a ***modal participation matrix*** that contains the contribution of each mode shape to the ODSs is calculated. The ***modal participation matrix***, introduced in a previous paper [4], is defined by Eq. 12.1 below,

$$[U][P] = [V] \tag{12.1}$$

$[V]$ = matrix of ***ODSs*** (n by o)
$[U]$ = matrix of mode shapes (e by m)
$[P]$ = ***modal participation*** matrix (m by o)

SHP: Base plate modes

Select Shape	Frequency (or Time)	Units	Damping (%)
1	0	Hz	0
2	0	Hz	0
3	0	Hz	0
4	0	Hz	0
5	0	Hz	0
6	0	Hz	0
7	439.76	Hz	0
8	500.96	Hz	0
9	1103.2	Hz	0
10	1198.4	Hz	0
11	1532.4	Hz	0
12	1826.2	Hz	0
13	1909.5	Hz	0
14	2322.1	Hz	0
15	2575.5	Hz	0
16	2988.2	Hz	0
17	3434.3	Hz	0
18	3914.9	Hz	0

DOFs	Measurement Type	Units	Shape 1	
			Magnitude	Phase
3X	UMM Mode Shape	in/lbf-sec	1.2648	0
3Y	UMM Mode Shape	in/lbf-sec	0.83429	0
3Z	UMM Mode Shape	in/lbf-sec	11.456	0
4X	UMM Mode Shape	in/lbf-sec	1.2648	0
4Y	UMM Mode Shape	in/lbf-sec	0.034838	0
4Z	UMM Mode Shape	in/lbf-sec	2.1085	180
5X	UMM Mode Shape	in/lbf-sec	1.2648	0
5Y	UMM Mode Shape	in/lbf-sec	0.76462	180
5Z	UMM Mode Shape	in/lbf-sec	15.673	180
6X	UMM Mode Shape	in/lbf-sec	0.41308	0
6Y	UMM Mode Shape	in/lbf-sec	0.83429	0
6Z	UMM Mode Shape	in/lbf-sec	14.96	0
7X	UMM Mode Shape	in/lbf-sec	0.41308	0
7Y	UMM Mode Shape	in/lbf-sec	0.034838	0
7Z	UMM Mode Shape	in/lbf-sec	1.3953	0
8X	UMM Mode Shape	in/lbf-sec	0.41308	0
8Y	UMM Mode Shape	in/lbf-sec	0.76462	180

Fig. 12.6 Mode shapes of the base plate

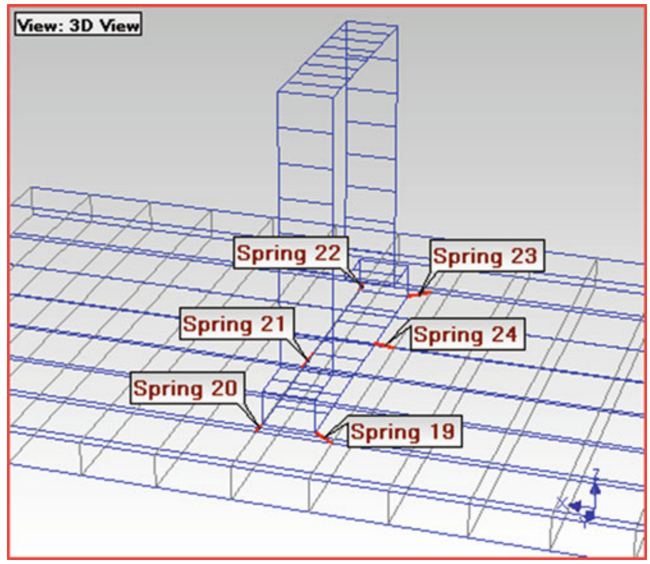
SHP: Inner Bearing Block Modes

Select Shape	Frequency (or Time)	Units	Damping (%)
1	0	Hz	0
2	0	Hz	0
3	0	Hz	0
4	0	Hz	0
5	0	Hz	0
6	0	Hz	0
7	4737.8	Hz	0
8	5901.5	Hz	0
9	8676.5	Hz	0
10	10242	Hz	0
11	11649	Hz	0
12	12608	Hz	0
13	12868	Hz	0
14	14105	Hz	0
15	14986	Hz	0
16	16394	Hz	0
17	17019	Hz	0
18	18256	Hz	0

DOFs	Measurement Type	Units	Shape 1	
			Magnitude	Phase
2X	UMM Mode Shape	in/lbf-sec	1.2257	0
2Y	UMM Mode Shape	in/lbf-sec	6.1956	0
2Z	UMM Mode Shape	in/lbf-sec	1.6901	0
1000X	UMM Mode Shape	in/lbf-sec	9.9352	0
1000Y	UMM Mode Shape	in/lbf-sec	6.1956	0
1000Z	UMM Mode Shape	in/lbf-sec	3.8279	0
1001X	UMM Mode Shape	in/lbf-sec	10.508	0
1001Y	UMM Mode Shape	in/lbf-sec	11.896	0
1001Z	UMM Mode Shape	in/lbf-sec	0.44761	180
1002X	UMM Mode Shape	in/lbf-sec	7.4838	180
1002Y	UMM Mode Shape	in/lbf-sec	6.1956	0
1002Z	UMM Mode Shape	in/lbf-sec	0.44762	180
1003X	UMM Mode Shape	in/lbf-sec	27.927	0
1003Y	UMM Mode Shape	in/lbf-sec	11.896	0
1003Z	UMM Mode Shape	in/lbf-sec	3.8278	0
1004X	UMM Mode Shape	in/lbf-sec	21.462	0
1004Y	UMM Mode Shape	in/lbf-sec	12.573	0

Fig. 12.7 Mode shapes of a bearing block

Fig. 12.8 Springs connecting base plate and bearing blocks



SHP: Unmodified Modes

Select Shape	Frequency (or Time)	Units	Damping (%)
1	0	Hz	0
2	0	Hz	0
3	0	Hz	0
4	0	Hz	0
5	0	Hz	0
6	0	Hz	0
7	439.76	Hz	0
8	500.96	Hz	0
9	1103.2	Hz	0
10	1198.4	Hz	0
11	1532.4	Hz	0
12	1826.2	Hz	0
13	1909.5	Hz	0
14	2322.1	Hz	0
15	2575.5	Hz	0
16	2988.2	Hz	0
17	3434.3	Hz	0
18	3914.9	Hz	0

DOFs	Measurement Type	Units	Shape 1	
			Magnitude	Phase
3X	UMM Mode Shape	in/lbf-sec	1.2648	0
3Y	UMM Mode Shape	in/lbf-sec	0.83429	0
3Z	UMM Mode Shape	in/lbf-sec	11.456	0
4X	UMM Mode Shape	in/lbf-sec	1.2648	0
4Y	UMM Mode Shape	in/lbf-sec	0.034838	0
4Z	UMM Mode Shape	in/lbf-sec	2.1085	180
5X	UMM Mode Shape	in/lbf-sec	1.2648	0
5Y	UMM Mode Shape	in/lbf-sec	0.76462	180
5Z	UMM Mode Shape	in/lbf-sec	15.673	180
6X	UMM Mode Shape	in/lbf-sec	0.41308	0
6Y	UMM Mode Shape	in/lbf-sec	0.83429	0
6Z	UMM Mode Shape	in/lbf-sec	14.96	0
7X	UMM Mode Shape	in/lbf-sec	0.41308	0
7Y	UMM Mode Shape	in/lbf-sec	0.034838	0
7Z	UMM Mode Shape	in/lbf-sec	1.3953	0
8X	UMM Mode Shape	in/lbf-sec	0.41308	0
8Y	UMM Mode Shape	in/lbf-sec	0.76462	180

Fig. 12.9 FEA mode shapes of combined substructures

Writing out Eq. 12.1 for ***matching*** shape components,

$$\underset{\text{(n by m)}}{\begin{bmatrix} u_{1,1} & \cdots & u_{1,m} \\ \vdots & \ddots & \vdots \\ u_{n,1} & \cdots & u_{n,m} \end{bmatrix}} \underset{\text{(m by o)}}{\begin{bmatrix} p_{1,1} & \cdots & p_{1,o} \\ \vdots & \ddots & \vdots \\ p_{m,1} & \cdots & p_{m,o} \end{bmatrix}} = \underset{\text{(n by o)}}{\begin{bmatrix} v_{1,1} & \cdots & v_{1,o} \\ \vdots & \ddots & \vdots \\ v_{n,1} & \cdots & v_{n,o} \end{bmatrix}}$$

n = number of ***matching*** mode shape & ODS components
m = number of mode shapes
o = number of ODSs
e = number of mode shape components

Each column of the ***modal participation matrix*** contains ***complex*** valued scale factors that define the contribution of each mode shape to each ODS. The ***modal participation matrix*** is calculated as a least squared solution to Eq. 12.1,

$$[\mathrm{P}] = \left[[\mathrm{U}]^{\mathrm{h}}[\mathrm{U}]\right]^{-1}[\mathrm{U}]^{\mathrm{h}}[\mathrm{V}] \tag{12.2}$$

h – denotes the *transposed conjugate* matrix
−1 – denotes the *inverse* matrix

12.7 ODS Expansion

Once the modal participation matrix has been calculated, the ODS expansion is done by post-multiplying the mode shape matrix by the modal participation matrix,

$$\left[\overline{\overline{V}}\right] = [U]\,[P] \tag{12.3}$$

$\left[\overline{\overline{V}}\right]$ = expanded ODS matrix

12.8 Modal Participation at Two Speeds

ODS data was acquired from the rotating machine in Fig. 12.1 at two different speeds. The modal participation of the first 10 FEA modes in the 985 RPM ODS was calculated using Eq. 12.2, and is shown in Fig. 12.10.

The modal participation shows that the ***first three rigid body modes*** are the ***dominant contributors*** to the 985 RPM ODS. These modes are being excited and are contributing the most to the ODS at this speed. At this speed, the machine is simply ***"bouncing"*** on its rubber mounts.

The modal participation of the first 10 FEA modes in the 2280 RPM ODS is shown in Fig. 12.11. The participation of modes 2, 5, and 6 shows that they are ***dominating*** the 2280 ODS. At this higher speed, the machine is ***"rocking"*** on its rubber mounts with more deflection at the outer bearing location.

Notice also that the participation scale factors of the dominate modes at both speeds have ***significant real and imaginary parts***. This is because the ODSs are complex valued and the FEA mode shapes are real valued, also called ***normal*** modes. In order to match the normal mode shapes to complex ODSs, the complex modal participation factors, when multiplied by the normal mode shapes, create expanded ODSs that are complex valued.

12.9 ODS Expansion

Finally, Eq. 12.3 was used to expand the 985 and 2280 RPM ODSs. The MAC values for each expanded and un-expanded ODS pair are shown in Fig. 12.12. The MAC value for the **acquired and expanded 985 RPM ODSs** is **0.87**, indicating a strong correlation between this experimental and its expanded ODS. The MAC value between the **experimental and expanded 2280 RPM ODSs** is **0.94**, indicating an even stronger correlation between this experimental and its expanded ODS.

The two off-diagonal MAC values are between the **expanded 985 RPM ODS** and the **experimental 2280 RPM ODS**, and between the **expanded 2280 RPM ODS** and the **experimental 985 RPM ODS**. Both of these values are ***very low*** (**<0.10**), again indicating that the ODSs of the machine at these two speeds are quite different from one another.

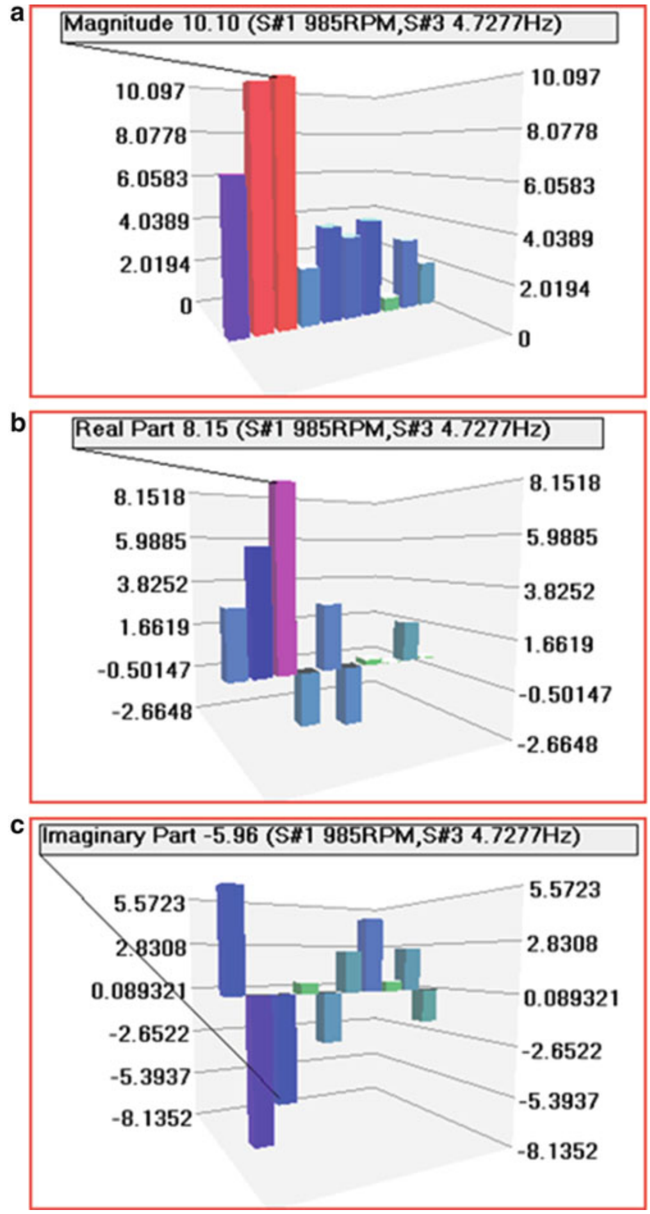


Fig. 12.10 (**a**) Magnitude of modal participation at 985 RPM. (**b**) Real part of modal participation at 985 RPM. (**c**) Imaginary part of modal participation at 985 RPM

12.10 Conclusions

A linear relationship between matrices of mode shapes and matrices of ODSs was presented in a previous paper [4], and was used here to calculate a ***modal participation matrix***. Equation 12.1 merely expresses the well known ***superposition property*** of mode shapes. That is, the overall vibration of a machine or structure contained in its ODS, can be represented as a ***linear combination*** of its mode shapes.

In a wide variety of practical cases, modes can been used to characterize and further understand resonant vibration. Whenever the dynamics of a structure can be represented by its modes, this ***superposition*** property can be utilized, and the participation of each mode in the ODS can be calculated.

This modal participation matrix shows which modes are ***dominating*** an ODS, and therefore which modes are being excited the most under a specific operating condition.

A key advantage of this technique is that **only mode shapes** are required to calculate the modal participation and expand an ODS for display in animation. Modal frequency and damping are not required.

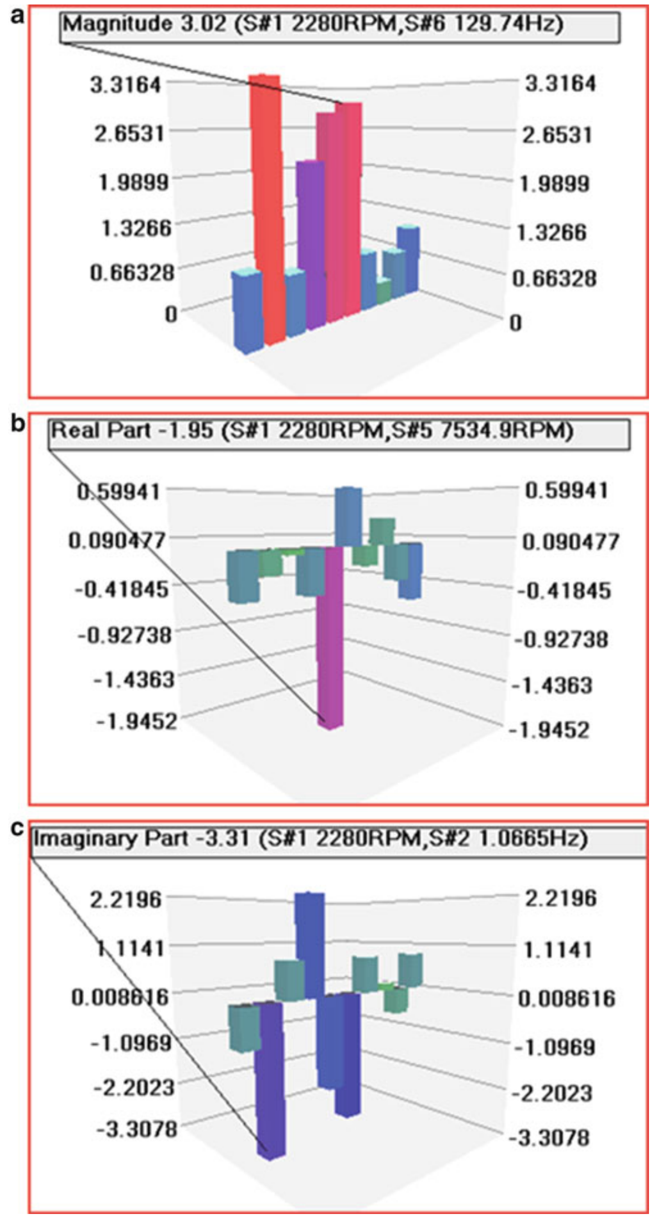


Fig. 12.11 (**a**) Magnitude of modal participation at 2280 RPM. (**b**) Real part of modal participation at 2280 RPM. (**c**) Imaginary part of modal participation at 2280 RPM

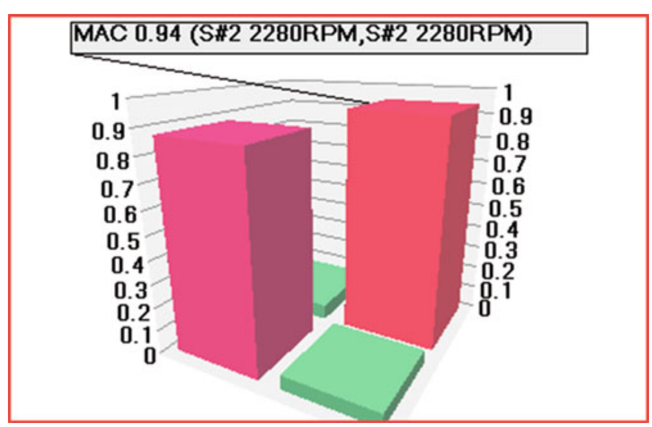


Fig. 12.12 MAC – expanded vs. un-expanded ODSs

In order to construct an analytical modal mode, it was also shown how substructuring and the SDM method can be used to ***"tie together"*** several parts of a machine using stiff FEA springs. SDM calculates the overall mode shapes using the mode shapes of each part, and these mode shapes are easy to obtain by modeling each part in a free-free condition.

ODSs of most rotating machines are ***complex*** valued because they are caused by ***cyclic*** forces within the machine. What makes this technique very appealing is that a set of FEA ***normal*** mode shapes can be ***"curve fit"*** to complex ODS data, resulting in ***complex*** modal participation factors. These participation factors can then be used to expand the ODS, thus providing a realistic animated display of machine deflections on a 3D model of the machine.

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