Chapter 28
Simplifying Transformations for Nonlinear Systems: Part I, An Optimisation-Based Variant of Normal Form Analysis

N. Dervilis, K. Worden, D.J. Wagg, and S.A. Neild

Abstract This paper introduces the idea of a 'simplifying transformation' for nonlinear structural dynamic systems. The idea simply stated; is to bring under one heading, those transformations which 'simplify' structural dynamic systems or responses in some sense. The equations of motion may be cast in a simpler form or decoupled (and in this sense, nonlinear modal analysis is encompassed) or the responses may be modified in order to isolate and remove certain components. It is the latter sense of simplification which is considered in this paper. One can regard normal form analysis in a way as the removal of superharmonic content from nonlinear system response. In the current paper, this problem is cast in an optimisation form and the differential evolution algorithm is used.

Keywords Nonlinearity • Differential evolution • Optimisation • Simplifying transformation • Superharmonics

28.1 Introduction

The machine learning methods that are introduced in this paper via an optimisation algorithm that aims to address the problem of validity relating to the modal analysis of nonlinear structures. Modal analysis is an important tool in structural dynamics as it is used to understand the dynamical characteristics of the structure. Many methods have been proposed in recent years regarding nonlinear analysis, such as nonlinear normal modes or the method of normal forms [1–10].

The motivation for this paper comes from the problem of how to transform second-order equations of motion into a simpler form, as vibration problems are naturally expressed via such differential equations. The method of normal forms can be used as an analytical tool when vibration problems are surrounded by nonlinearities in order to recast the equations of motion in a simpler form.

One can regard normal form analysis in a way as the removal of some superharmonic content from nonlinear system response. In the current work, this problem is cast in an optimisation form and the differential evolution algorithm is used. The advantage of this data-based approach introduced here is that complicated algebraic analysis is not needed and the details of the equations of motion are not required.

The layout of the paper is as follows. Section 28.2 covers the main features of the optimisation algorithm and the proposed transformation, while Sect. 28.3 gives an example of nonlinear analysis based on the technique that is mentioned in Sect. 28.2. The paper finishes with some overall conclusion and future work.

28.2 Simplifying Transformation Strategy and Differential Evolution

In vibration analysis, the relationship between system resonances and external forcing is particularly important especially when nonlinearities are present. Using the approach that is proposed here, leads to a simpler form when simulating resonant behaviour by removing the dominant superharmonic.

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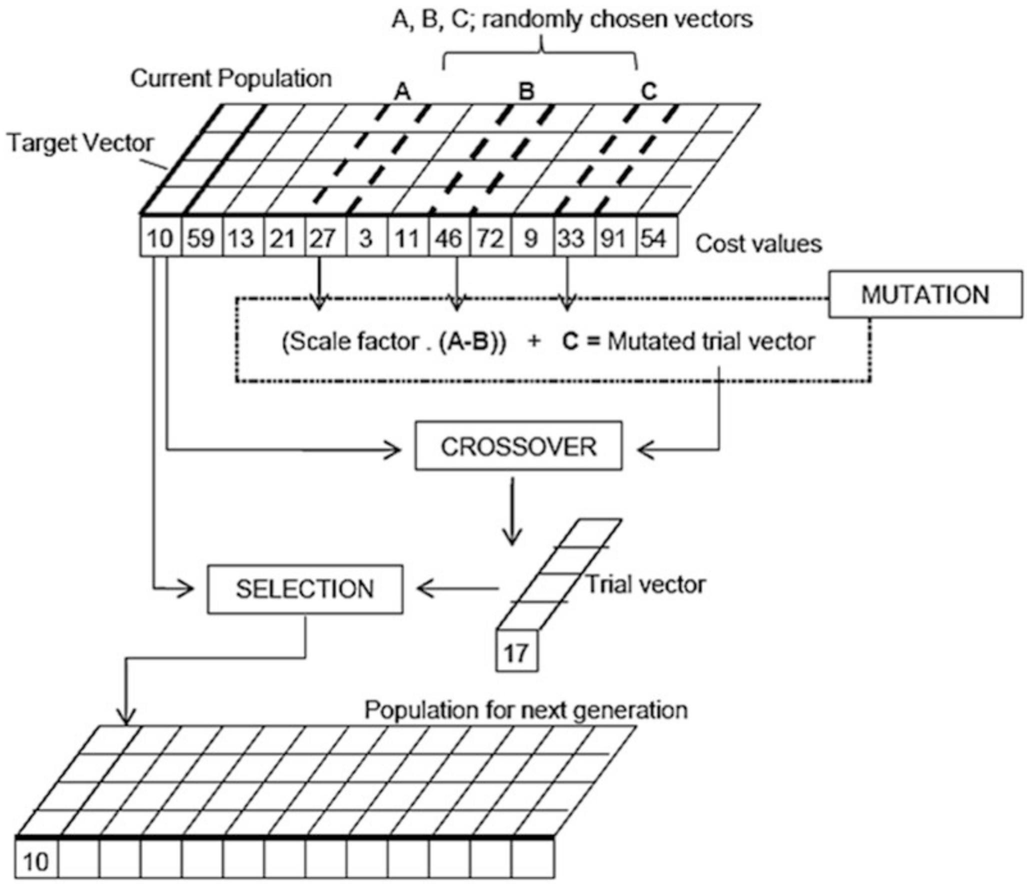

Fig. 28.1 Schematic of a typical differential evolution algorithm [14]

The transformation is as simple as a polynomial expansion. Adopting a machine learning approach, one can learn the undetermined coefficients from measured data, freeing the system from the dominant superharmonic.

A different approach to both analytic solutions and pattern recognition methods is to treat the removal of superharmonic content from nonlinear system response in terms of an optimisation problem, where the aim is to choose parameters such that the new signal is free from the first superharmonic that appears at three times the natural frequency (for a symmetric nonlinearity). For this purpose, a nonlinear optimisation algorithm based on differential evolution will be used here. For the purposes of this paper a brief description of differential evolution is given and readers are referred to [11–13] for more details. A section will follow with results using the technique on data simulated to represent the theoretical situation that was discussed.

Differential evolution, introduced by Storn and Price [12], is an evolutionary algorithm in the same sense as a genetic algorithm that begins with an initial population of trial solutions to a problem and via successive cycles of mutation, crossover and selection computes an optimal set of solutions. These trial solution are subject to a suitable objective function, in respect to the given problem. In turn, for the current analysis the trial solutions are a vector of parameter guesses that satisfy the condition that the new signal is a simplified transformation as described before.

The optimisation algorithm is summarised in Fig. 28.1. The routine, although seeming complicated, follows a smooth but powerful procedure. An initial population of parameter vectors are randomly generated. Then to each parameter vector of this initial population, the objective function specifies a cost value and a new generation of solutions is born from this initial population. A target vector is chosen from the initial population and then a trial vector is created by ‘mutation’. Mutation takes two random parameter vectors A and B according to Fig. 28.1 from the population and subtracts one from the other by multiplying it by some constant or scaling factor and finally adds it to a third randomly chosen parameter vector (C) from the initial population. The trial vector can be considered as the ‘child’ of the target vector and the mutated vector.

After this chain of actions a new parameter vector is born between mutated trial vector and the target vector and the procedure is called ‘crossover’. A predefined hyperparameter determines if the trial vector takes a parameter value from the target vector or the mutated vector. This new vector will then be selected for the next generation if its cost value is lower than that of the target vector. If not, the target vector is forwarded to the next generation population. This procedure is repeated several times and as the process evolves through a chain of generations, parameter vectors with low cost values will be constructed.

In this analysis a slight variation of DE was used called self-adaptive differential evolution (SADE) [11, 13]. This algorithm evolves in a similar fashion to the DE procedure except that it also learns which values of scaling factor and hyperparameter are most likely to result in a trial vector that will be forwarded to the next generation, giving faster convergence compared to DE algorithm. The main advantage of using this evolutionary population based tool in respect to the parameter estimation problem is that the algorithm will potentially converge on the global minimum cost value rather than a local minimum.

For the purposes of the nonlinear simplifying transformation, a suitable objective function must be chosen on the basis of isolating and remove certain components (superharmonics).

28.3 An Example

The system of interest will be a nonlinear one-DOF lumped parameter system (see Fig. 28.2). Data were simulated using a fixed-step fourth-order Runge-Kutta algorithm with harmonic excitation and the associated displacement was extracted. The model parameters adopted were: $m = 1$, $c_1 = 0.001$, $k_1 = 50$, $k_{nl} = 10^3$. The nonlinearity that is assumed is cubic.

Figure 28.3 shows the resulting power spectral density (PSD) for the simulated physical variable. The method that is used in order to calculate the power spectral densities (PSDs) which follow is the Welch method based on time averaging over short, modified periodograms which could decolour the effect of different random excitation inputs [15]. The signals are split into sections and the periodograms of each section are averaged. Through the Welch method these data sections are overlapped and a window, such as the Hanning window is applied in order to filter each section. The overlapping of the signal sections is usually either 50 % (as in this paper) or 75 %.

For all the graphs the vertical axis is the **PSD of displacement** and the **frequency is in Hz**.

As can be seen, the third order superharmonic (which as expected appeared at three times the natural frequency) is the dominant trend in the signal. Following this evidence a simplifying transformation of a polynomial expansion is adopted in the form of:

$$z(i) = a + by(i) + cy(i)^3 + dy(i)^5 \tag{28.1}$$

where $\{z\}$ is the transformed signal, $\{y\}$ is the initial signal and $[a, b, c, d]$ the undetermined parameters of the polynomial expansion.

The task of the optimisation algorithm is to determine the polynomial coefficients and at the same time transform the new signal by zeroing the first dominant superharmonic that appears at three times from the natural frequency.

As can be seen in Fig. 28.4 the method that is introduced successfully removes the superharmonic. However, as can be noted in Fig. 28.5 if the frequency spectrum is widened then the second superharmonic (fifth order) is rising in magnitude to counter balance the reduction of the third order superharmonic. This problem will be investigated in the second part of this paper.

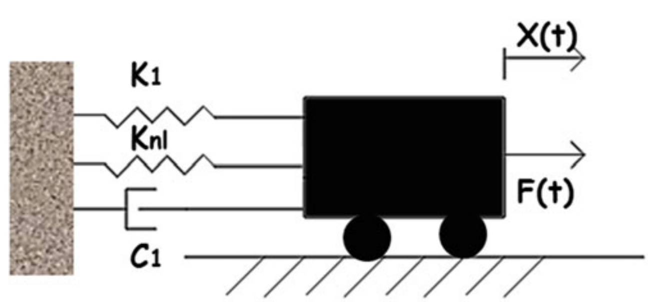

Fig. 28.2 Nonlinear one-DOF lumped parameter system

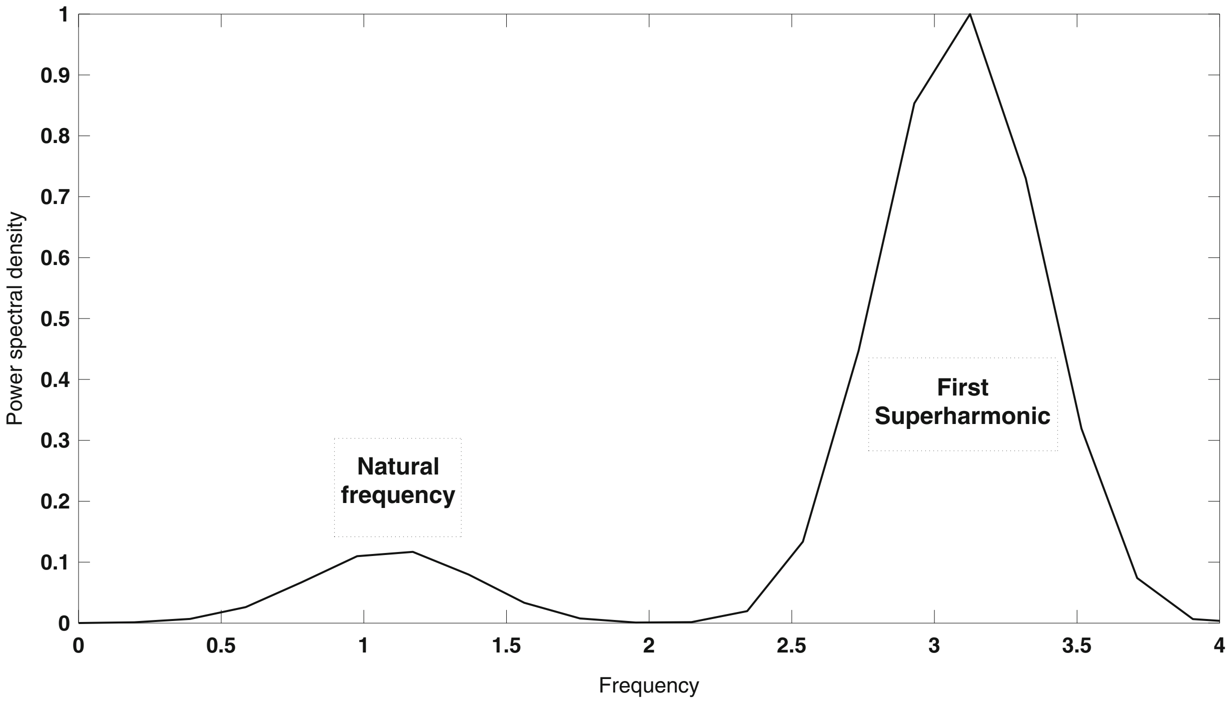

Fig. 28.3 PSD for physical variable

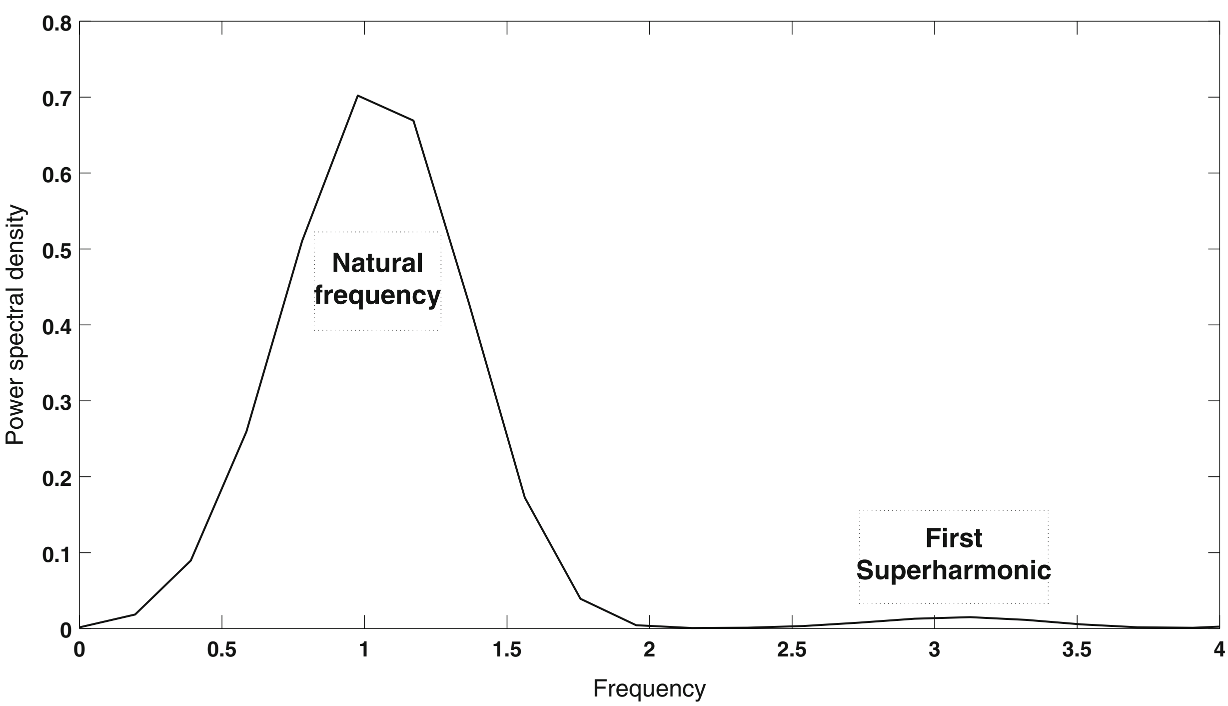

Fig. 28.4 PSD for transformed variable

28.4 Conclusion

The purpose of this paper is to highlight the key utility of some machine learning methods, not only for dynamic analysis of structure but as well as a method of simplification for nonlinear mechanical systems. The main benefit of the approach taken here is that complicated algebraic analysis is not necessary. Furthermore, the physical equations of the system are

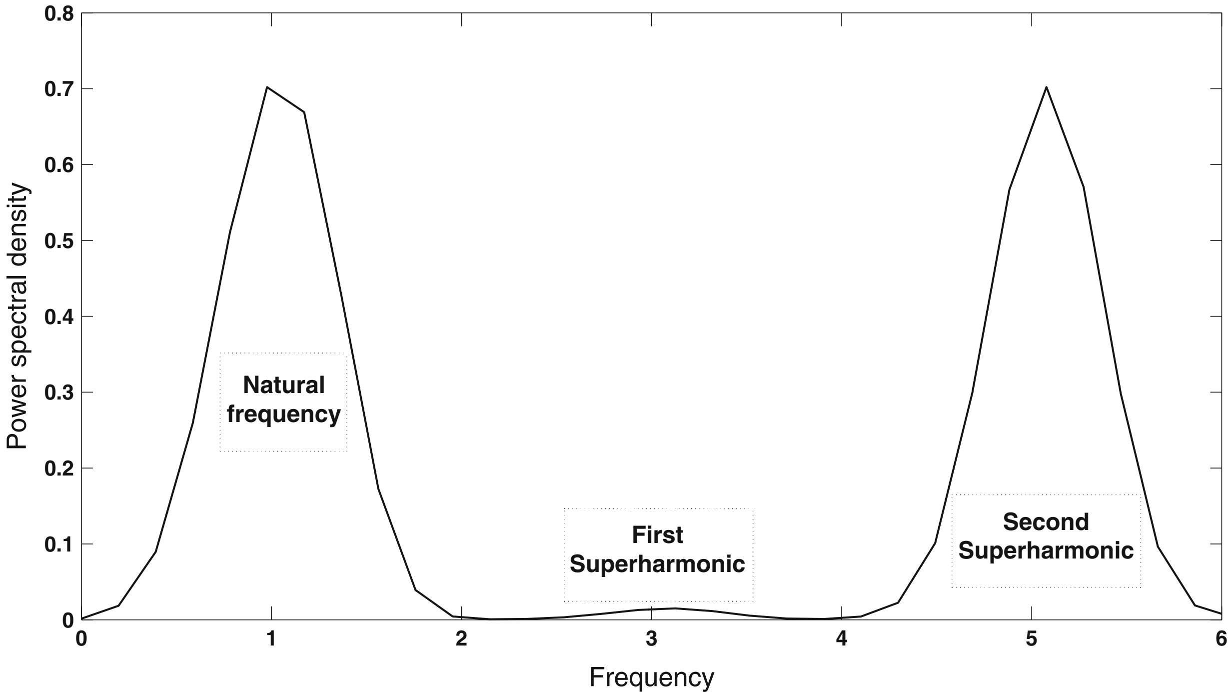

Fig. 28.5 PSD for transformed variable including second superharmonic (fifth order)

not needed. As a result, this machine learning approach is suited to experimental investigation of nonlinear systems using only the measured output responses. A further work in the second part of this paper investigates the appearance of higher order superharmonics.

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References

1. Kerschen G, Golinval J-C, Vakakis AF, Bergman LA (2005) The method of proper orthogonal decomposition for dynamical characterization and order reduction of mechanical systems: an overview. Nonlinear Dyn 41(1–3):147–169
2. Vakakis AF (1997) Non-linear normal modes (nnms) and their applications in vibration theory: an overview. Mech Syst Signal Process 11(1):3–22
3. Worden K, Tomlinson GR (2000) Nonlinearity in structural dynamics: detection, identification and modelling. CRC Press, Boca Raton
4. Worden K, Tomlinson GR (2001) Nonlinearity in experimental modal analysis. Philos Trans R Soc Lond Ser A Math Phys Eng Sci 359(1778):113–130
5. Worden K, Green PL (2014) A machine learning approach to nonlinear modal analysis. In: Catbas FN (ed) Dynamics of civil structures, vol 4. Springer, New York, pp 521–528
6. Rosenberg RM (1962) The normal modes of nonlinear n-degree-of-freedom systems. J Appl Mech 29(1):7–14
7. Shaw SW, Pierre C (1993) Normal modes for non-linear vibratory systems. J Sound Vib 164(1):85–124
8. Neild SA, Wagg DJ (2011) Applying the method of normal forms to second-order nonlinear vibration problems. Proc R Soc A Math Phys Eng Sci 467(2128):1141–1163
9. Poncelet F, Kerschen G, Golinval J-C, Verhelst D (2007) Output-only modal analysis using blind source separation techniques. Mech Syst Signal Process 21(6):2335–2358
10. Dervilis N, Wagg DJ, Green PL, Worden K (2014) Nonlinear modal analysis using pattern recognition. In: Proceedings of ISMA2014, pp 3017–3027
11. Worden K, Manson G, Sohn H, Farrar CR (2005) Extreme value statistics from differential evolution for damage detection. In: Proceedings of the 23rd international modal analysis conference (IMAC XXIII), Orlando, pp 2009–3
12. Storn R, Price K (1997) Differential evolution—a simple and efficient heuristic for global optimization over continuous spaces. J Global Optim 11(4):341–359

13. Qin AK, Suganthan PN (2005) Self-adaptive differential evolution algorithm for numerical optimization. In: The 2005 IEEE congress on evolutionary computation, vol 2. IEEE, pp 1785–1791
14. Cross E (2012) On structural health monitoring in changing environmental and operational conditions. Ph.D. thesis
15. Welch P (1967) The use of fast fourier transform for the estimation of power spectra: a method based on time averaging over short, modified periodograms. IEEE Trans Audio Electroacoust 15(2):70–73