Chapter 9
Logical, Ontological and Cognitive Aspects of Object Types and Cross-World Identity with Applications to the Theory of Conceptual Spaces

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Abstract Types are fundamental for conceptual domain modeling and knowledge representation in computer science. Frequently, monadic types used in domain models have as their instances *objects* (*endurants*, *continuants*), i.e., entities persisting in time that experience qualitative changes while keeping their numerical identity. In this paper, I revisit a philosophically and cognitively well-founded theory of object types and propose a system of modal logics with restricted quantification designed to formally characterize the distinctions and constraints proposed by this theory. The formal system proposed also addresses the limitations of classical (unrestricted extensional) modal logics in differentiating between types that represent mere properties (or *attributions*) ascribed to individual objects from types that carry a *principle of identity* for those individuals (the so-called *sortal types*). Finally, I also show here how this proposal can complement the theory of *conceptual spaces* by offering an account for kind-supplied principles of *cross-world identity*. The account addresses an important criticism posed to conceptual spaces in the literature and is in line with a number of empirical results in the literature of cognitive psychology.

9.1 Introduction

Types are fundamental for conceptual domain modeling and knowledge representation in computer science. Frequently, monadic types used in domain models have as their instances *objects*, i.e., entities that persist in time while keeping their identity (as opposed to *events* such as a kiss, a business process or a birthday party). What I term here *object* refers to what is sometimes termed Endurant or Continuant in the literature. Examples of objects include physical and social persisting entities of

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F. Zenker, P. Gärdenfors (eds.), *Applications of Conceptual Spaces*,
Synthese Library 359, DOI 10.1007/978-3-319-15021-5_9

everyday experience such as balls, rocks, planets, cars, students and Queen Beatrix but also fiat objects such as the Dutch part of the North Sea and a non-smoking area of a restaurant.

In this paper, I revisit the philosophically and cognitively well-founded theory of object types first proposed in Guizzardi et al. (2004). The ontological distinctions and postulates proposed by this theory are discussed in the next section. In Sect. 9.3, I present the main contribution of this paper, namely, a system of modal logics with restricted quantification designed to formally characterize the distinctions and constraints proposed by this theory. That section also discusses how the proposed formal system addresses the limitations of classical (unrestricted extensional) modal logics in some fundamental aspects regarding the notions of object persistence and cross-world identity. In Sect. 9.4, the paper elaborates on how this theory can be employed to analyze and address some problems faced by the theory of *conceptual spaces* (Gärdenfors 2000) with respect to the issues of identity and persistence. Section 9.5 briefly discusses related work. Finally, Sect. 9.6 concludes the paper with final considerations.

9.2 Ontological Distinctions Among Object Types

Van Leeuwen (1991) presents an important grammatical difference occurring in natural languages between common nouns (CNs) and arbitrary general terms (adjectives, verbs, mass nouns, etc. . . .). Common nouns have the singular feature that they can be combined with determiners and serve as argument for predication in sentences such as: *(i) (exactly) five mice were in the kitchen last night; (ii) the mouse that ate the cheese has been in turn eaten by the cat.*

In other words, if we have the patterns *(exactly) five X . . .* and *the Y which is Z . . .*, only the substitution of X, Y, Z by CNs will produce sentences that are grammatical. To verify this, we can try substituting the adjective *red* in the sentence (i): *(exactly) five red were in the kitchen last night.* A request to "count the red in this room" cannot receive a definite answer: should a red shirt be counted as one or should the shirt, the two sleeves, and two pockets be counted separately so that we have five reds? The problem here is not that one would not know how to complete the count but that one would not know how to start, since arbitrarily many *subparts of a red thing are still red.*

The explanation for this feature, which is unique of CNs, draws on the function that determinates (demonstratives and quantifiers) play in noun phrases, which is to determine a certain range on individuals. Both reference and quantification requires that the things that are referred or that form the domain of quantification are determinate individuals, i.e., their conditions for individuation and identity must be determinate.

According to van Leeuwen (1991), this syntactic distinction between the two linguistic categories reflects a semantical and ontological one, and so the distinction between the grammatical categories of CNs and arbitrary general terms can be explained in terms of the ontological categories of *sortal* and *characterizing types* (Strawson 1959), which are roughly their ontological counterparts. Whilst the latter supply only a *principle of application* for the individuals they collect, the former supply both a principle of application and a *principle of identity*. A principle of application is one in accordance with which we judge whether a general term applies to a particular (e.g., whether something is a person, a dog, a chair or a student). A principle of identity supports the judgment whether two particulars are the same, i.e., in which circumstances the identity relation holds.

Cognitive psychologist John Macnamara (1986, 1994) has investigated the role of sortal concepts in cognition and provided a comprehensive theory for explaining the infant's process of learning proper and common nouns. He proposed the following example: suppose a little boy (Tom), who is about to learn the meaning of a proper name for his puppy. When presented with the word "Spot", Tom has to decide what it refers to. A demonstrative such as "that" will not suffice to determinate the bearer of the proper name. How to decide that the referent of "that", which changes all its perceptual properties, is still *Spot*? In other words, which changes can Spot suffer and still be the same? As Macnamara (among others) shows, answers to these questions are only possible if *Spot* is taken to be a proper name for an individual, which is an instance of a sortal universal. The principles of identity supplied by the sortals are essential to judge the validity of all identity statements. For example, if for an instance of the sortal *statue* losing one of its pieces will not alter the identity of the object, the same does not hold for an instance of *lump of clay*.

The claim that we can only make identity and quantification statements in relation to a sortal amounts to one of the best-supported hypothesis in the philosophy of language, namely, that the identity of an individual can only be traced in connection with a sortal universal, which provides a *principle of individuation* and *identity* to the particulars it collects (Macnamara 1986, 1994; Gupta 1980; Lowe 1989; van Leeuwen 1991).

As argued by Kripke (1980), a proper name is a rigid designator, i.e. it refers to the same individual in all possible situations, factual or counterfactual. For instance, it refers to the individual Mick Jagger both now (when he is the lead singer of Rolling Stones and 71 years old) and in the past (when he was the boy Mike Philip living in Kent, England). Moreover, it refers to the same individual in counterfactual situations such as the one in which he decided to continue attending the London School of Economics instead of pursuing a musical career. We would like to say that the boy Mike Philip is identical with the man Mick Jagger that he later became. However, as pointed out by Wiggins (2001) and Perry (1970), statements of identity only make sense if both referents are of the same type. Thus, we could not say that a certain boy is the same boy as a certain man since the latter is not a boy (and vice-versa). However, as Putnam put it, when a man *x* points to a boy in a picture and says "I am that boy," the pronoun "I" in question is typed not by man but by a type subsuming both man and boy (namely, person), which embraces *x*'s entire existence (Putnam 1994). A generalization of this idea is thesis D, proposed by David Wiggins

(Wiggins 2001): *if an individual falls under two sortals F and F′ in the course of its history there must be exactly one ultimate sortal G that subsumes both F and F′*.

A proof of thesis D can be found in Guizzardi (2005). Intuitively, one can appreciate that it is not the case that two incompatible principles of identity could apply to the same individual *x*, otherwise *x* will not be a viable entity (determinate particular) (van Leeuwen 1991). For instance, suppose an individual *x* that is an instance of both *statue* and *lump of clay*. Now, the answer to the question whether losing one of its pieces will alter the identity of *x* is indeterminate, since each of the two principles of identity that *x* obeys imply a different answer. As a consequence, we can say that if two sortals F and F′ intersect (i.e., have common individuals in their extension), the principles of identity contained in them must be equivalent. Moreover, F and F′ cannot supply a principle of identity for *x*, since both sortals apply to x only contingently, and a principle of identity must be used to identify *x* in all possible worlds. Therefore, there must be a sortal G that supplies the principle of identity carried by F and F′. The unique ultimate sortal G that supplies the principle of identity for its instances is named a *substance sortal* or a *kind* (Gupta 1980; Guizzardi et al. 2004).

In the example above, person can only be the sortal that supports the proper name Mick Jagger in all possible situations because it applies necessarily to the individual referred to by the proper name, i.e., instances of person cannot cease to be so without ceasing to exist. As a consequence, the extension of a kind is world invariant, i.e., for all x, if x is an instance of a rigid type G then x must be an instance of G in all possible worlds. This meta-property of universals is called *modal constancy* (Gupta 1980) or *rigidity* (Guarino and Welty 2009). Every *kind* G is a rigid universal. Moreover, a kind G can be specialized into other sortals $F_1, \ldots, F_n$ that are themselves rigid. Take for instance the kind person. This kind can be specialized by the sortals male person and female person, which (in the biological sense) are themselves rigid sortals. I name the rigid sortals F_i that specialize a kind (thus inheriting its principle of identity) *subkinds* (Guizzardi et al. 2004).

Examples of *non-rigid* sortals include universals such as *boy* and *adult man* in the example previously discussed, but also *student*, *employee*, *caterpillar* and *butterfly*, *philosopher*, *writer*, *alive* and *deceased*. Actually, these examples of sortals are not only non-rigid, but they are *anti-rigid*. Non-rigidity is the simple logical negation of rigidity, i.e., a type is non-rigid if it does not apply necessarily to at least one of its instances. In contrast, a type is anti-rigid if it does not apply necessarily to all its instances. In other words, if a type F is anti-rigid then for all instances x of F there is a possible world in which x is not an instance of F. Sortals that possibly apply to an individual only during a certain phase of its existence are called *phased-sortals* (Wiggins 2001). As a consequence of thesis D, we have that: *for every phased-sortal PS that applies to an individual, there is a kind (substance sortal) S of which PS is a specialization*.

Although Frege argued at length that *"one cannot count without knowing what to count"* (Frege 1980), in artificial logical languages inspired by him, natural language general terms such as common nouns, adjectives and verbs are treated uniformly as predicates. For instance, if we want to represent the sentence "there are tall men," in the Fregean approach of classical logic we would write $\exists x\ (man(x) \wedge tall(x))$. This reading puts the count noun man (which denotes a sortal) on an equal logical footing with the predicate tall. Moreover, in this formula, the variable x is interpreted as an alleged universal kind *Thing* (or entity). So, the natural language reading of the formula should be "there are things that have the property of being a man and the property of being tall." As argued in Hirsch (1982), concepts such as thing, (entity, element, among others) are *dispersive*, i.e., they cover many concepts with different principles of identity and do not denote sortals. This view is corroborated by many empirical studies in cognitive science (Xu et al. 2004).

The claims presented in this section are represented in a list of psychological claims proposed by Macnamara (1994) and are supported by a number of empirical studies (Xu et al. 2004; Bonatti et al. 2002; Waxman and Markow 1995; Booth and Waxman 2003). For instance, results from Xu et al. (2004) show that between 9 and 12 months of age a sortal-based system of individuation and identity emerges in infants' cognition. As remarked by the authors: "during this period, infants' worldview undergoes fundamental changes: They begin with a world populated with objects . . . By the end of the first year of life, they begin to conceptualize a world populated with sortal-kinds . . . In this new world, objects are thought of not as 'qua object' but rather 'qua dog' or 'qua table'."

9.3 A Logical System with Sortal and Characterizing Types

The formal characterization of the ontological distinctions discussed in the previous section requires some sort of modal treatment. In classical (extensional) modal logics, no distinction is made between different types of types. Types are represented as predicates in the language that divide the world (at each situation) into two classes of elements: those that fall under them and those that do not. This principle determines the extension of each type at each situation. Classical (one-place) predicates, being functions from worlds to sets of individuals, properly represent the *principles of application* that are carried by all types but fail to represent the *principles of identity*, which are unique of sortals. Equivalently, they treat all objects as obeying the same principle of identity.

Suppose that there is an individual person referred to by the proper name *John*. As discussed in the previous section, proper names for objects refer rigidly and, hence, if we say that John weights 80 kg at t_1 but 68 kg at t_2 we are in both cases referring to the same individual, namely the particular John. Now, let x_1 and x_2 be snapshots representing the projection of John at time boundaries t_1 and t_2, respectively. The truth of the statements $overweight(John,t_1)$ and $overweight(John,t_2)$ depends only on whether overweight applies to the states x_1 or x_2, respectively. In other words, the judgment if an individual i is an instance of a characterizing type G (e.g., overweight) in world w depends only whether the principle of application carried by G applies to the state of i in w. Now, how can one determine that, despite of possibly significant dissimilarities, x_1 and x_2 are states of the same particular John?

As previously argued this is done via a principle of cross-world identity and supplied by the substance sortal person, of which John is an instance.

These differences between sortals and characterizing types are made explicit in the formal language L_{sortal} defined in this section in the following way:

- The intension of the proper name John is represented by an *individual concept J*, i.e., a function that maps to a snapshop x_i of John in each possible world w. The notion of individual concepts, first introduced by Leibniz, refers to a singleton property that only holds for one individual;
- Sortal universals, such as person, are represented as *intensional properties*, which are functions from possible worlds to sets of individual concepts. For instance, for the sortal person there is a function ℓ that maps every world w to a set of individual concepts (including J). An individual x is a person in world w iff there is an individual concept $k \in \ell(w)$ such that $k(w) = x$;
- Individual concepts represent the principle of identity supplied by the universal person such that if $J(w) = x_1$ and $J(w') = x_2$ then we say that x_1 in w is the same person as x_2 in w', or in general: for all individuals x,y representing snapshots of an individual C of type T we say that x in w is the same T as y in w' iff C is in the extension of T and $C(w) = x$ and $C(w') = y$;
- Whilst the principle of identity is represented by sortal determined individual concepts that trace individuals from world to world, the principle of application considers individuals only at a specific world. For instance, John is overweight in word w iff *overweight(J(w),w)* is true.

Due to these considerations, in the language L_{sortal} presented below, the primitive elements in the domains of quantification are momentary states of objects, not the objects themselves. Ordinary objects of everyday experience (endurants, continuants) are instead represented by individual concepts. In the sequel, I formally define the syntax and semantics of L_{sortal}.

9.3.1 Syntax of L_{sortal}

Let L_{sortal} be a language of modal logics with identity with a vocabulary $V = (K,B,A,P,T)$ where: (a) T is a set of individual constants; (b) P is a non-empty set of n-ary predicates; (c) A is a set of phased-sortals (anti-rigid sortal types); (d) B is a set of subkinds; (e) K is a non-empty set of kinds (*substance sortal type*); (f) $R = K \cup B$ is named the set of rigid sortal types and the set $C = R \cup A$, the set of sortal types. The alphabet of L_{sortal} contains the traditional operators: $=$ (equality), $\neg$ (negation), $\rightarrow$ (implication), $\forall$(universal quantification), $\Box$ (necessity). The notions of term, sortal and formula are defined as follows:

Definition 1

1. All individual constants and variables are terms;
2. All sortal types belong to the category of sortal types;

3. If s and t are terms, then $s = t$ is an atomic formula;
4. If P is a n-place predicate and $t_1 \ldots t_n$ are terms, then $P(t_1,..,t_n)$ is an atomic formula;
5. If A and B are formulas, then so are $\neg A$, $\Box A$, $(A \rightarrow B)$;
6. If S is a sortal classifier, x is a variable and A is a formula, then $(\forall S,x)A$ is a formula. ■

The symbols $\exists$ (existential quantification), $\wedge$ (conjunction), $\vee$ (disjunction), $\diamond$ (possibility) and $\leftrightarrow$ are defined as usual:

Definition 2

7. $(A \wedge B) =_{def} \neg\ (A \rightarrow \neg B)$;
8. $(A \vee B) =_{def} ((A \rightarrow B) \rightarrow B)$;
9. $(A \leftrightarrow B) =_{def} (A \rightarrow B) \wedge (B \rightarrow A)$;
10. $\diamond A =_{def} \neg\ \neg\ A$
11. $((\exists S, x)\ A =_{def} \neg (\forall S, x) \neg A$
12. $((\exists! S, x)\ A =_{def} (\exists S, y)\ (\forall S, x)\ (A \leftrightarrow (x = y)$ ■

In L_{sortal}, all quantification is restricted by sortals. The quantification restricted in this way makes explicit what is only implicit in standard predicate logics. As previously discussed, suppose we want to state the following proposition: (a) *There are red tasty apples*. In classical predicate logic we would write down a *logical* formula such as (b) $\exists x\ (apple(x) \wedge tasty(x) \wedge red(x))$. In an ontological reading, (b) states that *"there are things which are red, tasty and apple*." The theory presented in the previous section denies that we can conceptually grasp an individual under a general concept such as thing or entity or, what is almost the same, that a logic (or a domain representation language) should presuppose the notion of a *bare particular*. Moreover, it states that only a sortal (e.g., apple) can carry a principle of identity for the individuals it collects, a property that is absent in characterizing types such as red and tasty. For this reason, a logical system, when used to represent a formalization of conceptual models of reality, should not presuppose that the representations of natural general terms such as apple, tasty and red stand in the same logical footing. For this reason, (a) should be represented as $(\exists Apple,x)\ (tasty(x) \wedge red(x))$ in which the sortal binding the variable x is the one responsible for carrying its principle of identity.

In L_{sortal}, sortal classifiers are never used in a predicative position. Therefore, if $S \in C$ is a sortal type, the predicate $s(x)$ (in lowercase) is a meta-linguistic abbreviation according to the following definition.

Definition 3

$s\ (t) =_{def} (\exists S, x)\ \ (x = t)$ ■

According to this definition, the sentence "John is a man" is better rendered as "John is identical to a man". In opposition, in the sentence "John is tall," the copula represents the "is" of predication, which denotes a relation of mere equivalence.

9.3.2 Semantics of L_{sortal}

Definition 4 (Model Structure) A model structure for L_{sortal} is defined as an ordered couple ⟨W, D⟩ where: (i) W is a non-empty set of possible worlds; (ii) L_{sortal} adopts a varying domain frame (Fitting and Mendelsohn 1998) and, thus, instead of a set, D is a function that assigns to each member of W a non-empty set of elements. In order to avoid issues that are not germane to the purposes of this article, we simply assume here a universal accessibility relation between worlds (ibid.). ■

Given a model structure *M* $(= \langle W, D \rangle)$, the intension of an individual constant can be represented by an *individual concept*, i.e., a function *i* that assigns to each world $w \in W$, an individual in *D(w)*. Formally, we have that:

Definition 5 (Individual Concept) Let $M = \langle W, D \rangle$ and $U = \bigcup_{w \in W} D(w)$. An individual concept i in M is function from W into U, such that $i(w) \in D(w)$ in all worlds. For a given model structure M, we define I as a set of individual concepts defined for that structure. ■

The intension of an n-place predicate is defined (as usual) as an n-ary property, i.e., a function that assigns to each world $w \in W$ a set of n-tuples. If a tuple $\langle d_1 \ldots d_n \rangle$ belongs to the representation of a predicate at world *w*, then $d_1 \ldots d_n$ stand in *w* in the relation expressed by the predicate.

Definition 6 (Property) An n-ary property $(n > 0)$ in M is a function P from W into $\wp(D(w))^n$, i.e., if $\langle d_1 \ldots d_n \rangle \in P(w)$, then $d_1 \ldots d_n \in D(w)$. ■

The intension of sortal classifiers is defined such that both the principles of application and identity are represented. This is done by what Gupta (1980) calls *sorts*, i.e., *separated intensional properties*.

Definition 7 (Sort) Let $M = \langle W, D \rangle$ be a model structure. An intensional property in M is a function ℓ from W into the powerset of individual concepts in M (i.e., $\wp(I)$).

An intensional property assigns to each world a set of individual concepts, and it can be used to represent the intension of a sortal type in the following way. Suppose that ℓ represents the intension of the sortal type S and that the individual concept i belongs to ℓ at world w, i.e., $i \in \ell(w)$. Then i(w) is a S in w, and i(w′) is identical to i(w) in w.

Let ℓ be an intensional property in M, and let $L = \bigcup_{w \in W} \ell(w)$.

Now, let i,j be two individual concepts such that $i,j \in L$. We say that the intensional property ℓ is separated iff: if there is a world $w \in W$ such that $i(w) = j(w)$ then, for all $w' \in W$, $i(w') = j(w')$, i.e., $i = j$.

Finally, a sort in a model structure M is an intensional property that is separated. ■

The requirement of separation proposed in Gupta (1980) states, for example, that if two individual concepts for person, say 007 and James Bond, apply to the same object in a world *w* then they apply necessarily to the same object. This prevents unlawful conceptualizations in which a substantial individual splits or in which two individuals can become one while maintaining the same identity.

Given a sort ℓ in *M*, we designate by $\ell[w]$ the set of objects that fall under ℓ in *w*. Formally,

Definition 8 $\ell[\mathrm{w}] = \{\mathrm{d}: \mathrm{d} \in \mathrm{D(w)}$ and there is an individual concept $\mathrm{i} \in \ell(\mathrm{w})$ such that $\mathrm{i(w)} = \mathrm{d}\}$. ■

Moreover, we define the set of objects in *w* that are *possibly* ℓ, i.e.,

Definition 9 $\ell\,|[\mathrm{w}]| = \{\mathrm{d}: \mathrm{d} \in \mathrm{D(w)}$ and there is an individual concept $\mathrm{i} \in \ell(\mathrm{w}')$ such that $\mathrm{i}(\mathrm{w}') = \mathrm{d}\}$. ■

We now are able to define the notion of *counterpart* relative to a sort ℓ.

Definition 10 (Counterpart) We say that d in world w is the same ℓ as d′ in w′ iff there is an individual concept i that belongs to ℓ at some world (i.e., there is a w′′ such that $\mathrm{i} \in \ell(\mathrm{w}'')$) and $\mathrm{i(w)} = \mathrm{d}$ and $\mathrm{i}(\mathrm{w}') = \mathrm{d}'$. The ℓ counterpart in w′ of the individual d in w is the unique individual d′ such that d′ in world w′ is the same ℓ as d in w. ■

Finally, we are then able to define a model for L_{sortal}:

Definition 11 (Model) A model in $\mathrm{L}_{\mathrm{sortal}}$ can be defined as a triple $\langle \mathrm{W}, \mathrm{D}, \delta \rangle$ such that:

1. 1.$\langle \mathrm{W}, \mathrm{D} \rangle$ is a model structure for $\mathrm{L}_{\mathrm{sortal}}$;
2. δ is an interpretation function assigning values to the non-logical constants of the language such that: it assigns an individual concept to each individual constant c $\in$ T of $\mathrm{L}_{\mathrm{sortal}}$; an n-ary property to each n-place predicate p $\in$ P of $\mathrm{L}_{\mathrm{sortal}}$; a sort to each sortal type S $\in$ C of $\mathrm{L}_{\mathrm{sortal}}$.

   The interpretation function δ must also satisfy the following constraints:
3. If S $\in$ R then the sort ℓ assigned to S by δ must be such that: for all w, w′ $\in$ W, $\ell(\mathrm{w}) = \ell(\mathrm{w}')$, i.e., all rigid sortals are world invariant (modally constant);
4. Let $\mathrm{S} \in (\mathrm{B} \cup \mathrm{A})$ be a subkind or an anti-rigid sortal type. Then, there is a kind S′ $\in$ K such that, for all $\mathrm{w} \in \mathrm{W}$, $\delta(\mathrm{S})(\mathrm{w}) \subseteq \delta(\mathrm{S}')(\mathrm{w})$;
5. Let S, S′$\in$ K be two kinds and let ℓ and ℓ' be the two sorts assigned to S and S′ by δ, respectively. Then we have that: there is a $\mathrm{w} \in \mathrm{W}$ such that $\ell(\mathrm{w}) \cap \ell'(\mathrm{w}) \neq \varnothing$ iff $\ell = \ell'$, i.e., sorts representing kinds do not intersect unless they are identical. In other words, this restriction states that individuals belong to one single substance sortal, i.e., they obey one single principle of identity;
6. Let S $\in$ A be a phased (anti-rigid) sortal type. The sort ℓ assigned to S by δ must be such that: for all w $\in$ W, and for all individual concepts $\mathrm{i} \in \ell(\mathrm{w})$, there is a world w′ $\in$ W such that $\mathrm{i} \notin \ell(\mathrm{w}')$;

7. Let S, S′∈ K be two kinds and let ℓ and ℓ' be the two sorts assigned to S and S′ by δ, respectively. Then we have that: there is a w ∈ W such that $\ell[w] \cap \ell'[w] \neq \varnothing$ iff $\ell = \ell'$. Differently from (5) above, this restriction has it that individual states of objects can only be referred to by individual concepts of the same kind. ■

We are now able to define an assignment for L_{sortal}:

Definition 12 (Assignment) An assignment for L_{sortal} relative to a model ⟨W, D, δ⟩ is a function that assigns to each variable of L_{sortal} an ordered pair ⟨ℓ,d ⟩, where ℓ is a sort relative to the modal structure ⟨W,D⟩ and d ∈ $U = \bigcup_{w\in W} D(w)$.

If a is an L_{sortal} assignment then $a_o(x)$ is the object assigned to variable x by a and $a_S(x)$ is the sortal to which x is bound. Moreover, it is always the case that $a_o(x) \in a_S(x)|[w]|$ for all variables. ■

Definition 13 An assignment a′ for L_{sortal} is an ℓ variant of a at x in w iff: a′ is just like a except perhaps at x (abbreviated as $a' \sim_X a$),

1. $a'_s(x) = \ell$,
2. $a'_o(x) \in \ell[w]$. ■

Definition 14 The w′ variant of an assignment a relative to w (abbreviated as f(w′,a,w)) is the unique assignment a′ that meets the following conditions:

(i) $a'_S(x) = a_S(x)$ at all variables x,
(ii) $a'_o(x)$ in w′ is the $a_S(x)$ counterpart of $a_o(x)$ in w relative,

at all variables x. ■

Definition 15 (Truth-Theoretical Semantics) Finally, let α be an expression in L_{sortal}, and let the semantic value of α at world w in model M relative to assignment *a* be the value of the valuation function $\boldsymbol{\nu}^w_{M,a}$.

With these definitions, we can define the semantics of L_{sortal} as follows:

(a) If α is an individual constant or a sortal type, then $\boldsymbol{\nu}^w_{M,a}(\alpha) = \delta(\alpha)(w)$.
(b) If α is variable, then $\boldsymbol{\nu}^w_{M,a}(\alpha) = a_o(\alpha)$
(c) If α is an atomic formula $t_1 = t_2$, then $\boldsymbol{\nu}^w_{M,a}(\alpha) = T$ if $\boldsymbol{\nu}^w_{M,a}(t_1) = \boldsymbol{\nu}^w_{M,a}(t_2)$. Otherwise $\boldsymbol{\nu}^w_{M,a}(\alpha) = F$.
(d) If α is an atomic formula $P(t_1 \ldots t_n)$, then $\boldsymbol{\nu}^w_{M,a}(\alpha) = T$ if $\langle \boldsymbol{\nu}^w_{M,a}(t_1) \ldots \boldsymbol{\nu}^w_{M,a}(t_n) \rangle \in \delta(P)(w)$. Otherwise $\boldsymbol{\nu}^w_{M,a}(\alpha) = F$.
(e) If α is the formula ¬A, then $\boldsymbol{\nu}^w_{M,a}(\alpha) = T$ if $\boldsymbol{\nu}^w_{M,a}(A) = F$. Otherwise $\boldsymbol{\nu}^w_{M,a}(\alpha) = F$.
(f) If α is the formula (A → B), then $\boldsymbol{\nu}^w_{M,a}(\alpha) = T$ if $\boldsymbol{\nu}^w_{M,a}(A) = F$ or $\boldsymbol{\nu}^w_{M,a}(B) = T$. Otherwise $\boldsymbol{\nu}^w_{M,a}(\alpha) = F$.
(g) If α is the formula (∀S,x)A, then $\boldsymbol{\nu}^w_{M,a}(\alpha) = T$ if $\boldsymbol{\nu}^w_{M,a'}(A) = T$ for all assignments a′ which are δ(S) variants of a at x in w. Otherwise $\boldsymbol{\nu}^w_{M,a}(\alpha) = F$.
(h) If α is the formula □A, then $\boldsymbol{\nu}^w_{M,a}(\alpha) = T$ if $\boldsymbol{\nu}^{w'}_{M,f(w',a,w)}(A) = T$ for all w′∈ W. Otherwise $\boldsymbol{\nu}^w_{M,a}(\alpha) = F$. ■

9.3.3 Discussion

The language L_{sortal} has been proposed based on the first of four systems introduced by Anil Gupta in his *Logic of Common Nouns* (Gupta 1980). Gupta, however, does not elaborate on different types of sortals. Consequently, restrictions (3) to (7) on δ in definition 11 are simply not defined in his system. Restriction (7), in particular, would have to be rejected by Gupta, as a consequence of his contingent (or relative) view of identity. Note that restriction (7) implies (5) but not vice-versa.

It is widely accepted that any relation of identity must comply with Leibniz's law: if two individuals are identical then they are necessarily identical (van Leeuwen 1991). Relativists, however, adopt the thesis that it is possible for two individuals to be identical in one circumstance but different in another. A familiar example, cited by Gupta, is that of a statue and a lump of clay. The argument proceeds as follows: Suppose that in world *w* we have a statue *st* of the Dalai Lama which is identical to the lump of clay *loc* that this statue is made of. In *w*, *st* and *loc* have exactly the same properties (e.g., same shape, weight, color, temperature, etc.). Suppose now that in world w', a piece (e.g., the hand) is subtracted from *st*. If the subtracted piece is an inessential part of a statue then the statue st' that we have in w' is identical to *st*. In contrast, the lump of clay *loc'* which st' is made of is different from *loc*. In summary, we have in w' the same statue as in *w* but a different lump of clay. In Gupta's system, without restriction (7), we have it that for two individual concepts *i* and *j* such that $i(w) = j(w)$, it remains possible a world w' such that $i(w') \neq j(w')$. In other words, the formula (α) $(\exists \text{Statue,x}\ (\text{x} = \text{dl}) \wedge \exists \text{LoC,y}\ (\text{y} = \text{dl}) \wedge \diamond(x \neq y))$ is satisfiable.

I reject this line of reasoning for two reasons. Firstly, I support the view that Leibniz's rule must hold for a relation to be considered a relation of identity, otherwise, any equivalence relation such as *being an instance of the same class* would have to be considered a relation of identity. Secondly, if Gupta's primitive elements are thought of as momentary states, then (α) does not in fact qualify as a statement of relative identity. It actually expresses that two objects can coincide (i.e., share the same state) in a world *w* but not in a different world w' (van Leeuwen 1991). Notice, however, that if restriction (7) is assumed, formula (α) is no longer satisfiable.

Proof (a) if (x = dl) is true then there is an individual concept st of statue that refers in the actual world w to the same entity d as dl; (b) if (y = dl) is true then there is an individual concept loc of LoC that refers in the actual world w to the same entity d as dl; (c) by transitivity of equality, st and loc refer to the same d in world w and, consequently, d is then both of the kind Statue and of the kind LoC in w; (e) due to (7), the intensions of Statue and LoC are identical; (f) finally, due to separation, st and loc must coincide in every world. □

Now we are in a position to choose between two alternatives related to the interpretation of momentary states. The first is to assert restriction (7) and take a *multiplicationist* (Guizzardi 2005) stance such that *st* and *loc* do not actually share the same state in *w* in the strong sense. Rather, I consider the states *st(w)* and *loc(w)* to be numerically different albeit instantiating the same types (properties).

A second stance is to assume that two continuants (endurants) can indeed share a state in the numerical sense. If we accept this, a simple way of modifying L_{sortal} to account for coincidence as manifested in Gupta's system consists in:

(a) removing the constraint (7) in definition 11;
(b) including the operator $\approx$ for coincidence, with the following semantics: If α is an atomic formula $t_1 \approx t_2$, then $\boldsymbol{\nu}^{w}_{M,a}(\alpha) = T$ if $\boldsymbol{\nu}^{w}_{M,a}(t_1) = \boldsymbol{\nu}^{w}_{M,a}(t_2)$. Otherwise $\boldsymbol{\nu}^{w}_{M,a}(\alpha) = F$;
(c) defining the identity relation between individual constants as $(t_1 = t_2) =_{def} \Box(t_1 \approx t_2)$, i.e., two continuants are identical if they coincide in every possible world.

A version of L_{sortal} which takes the *multiplicationist* stance can serve in support of two goals: defining the semantics of object-oriented and database languages in computer science, and to circumvent some of the limitations in representing modal (temporal) information in terminological languages such as OWL (Web Ontology Language).[1] In the sequel, I will briefly present an example of the first goal. For an example of the latter, I refer readers to Zamborlini and Guizzardi (2010). In the next section, I employ the proposed framework to address issues of cross-world identity and dynamic classification in conceptual spaces.

For instance, in the Unified Modeling Language (UML),[2] a *de facto* standard for conceptual modeling in computer science, types are represented in so-called class diagrams.[3] In contrast, the instances of these types are represented in object diagrams. See Figs. 9.1 and 9.2 below. In Fig. 9.1, we have a representation of the type person characterized by the properties *name*, *social security number*, *age* and *height*, as well as the type car characterized by the properties *chassis number*, *color*, *kilometer count* and *manufacturing date*. Moreover, the diagram represents a relational property *owns*, defined between instances of person and instances of car together with some integrity constraints on this relational property (while people can own *zero-to-many* cars, we assume that a car must be owned by exactly one person). In Fig. 9.2, we have a representation of an instance of the type person (John) and two instances of the type car (car_1 and car_2), as well as representation of two instances of the relational property *own*. Notice that individuals that appear in an UML object diagram (such as is given in Fig. 9.2) are not endurants. They are not persons like you and me, or cars like mine or yours. These are snapshot entities, momentary states of endurants. However, the instances of a UML class diagram (Fig. 9.1) are not snapshot entities; instead they are so-called *oid (object identifiers)*. Although

[1] www.w3.org/2004/OWL/

[2] http://www.uml.org/

[3] What are termed classes in UML are akin to what I name types here, not to the well-known set-theoretical notion of classes. In other words, classes in UML are intensional not extensional entities.

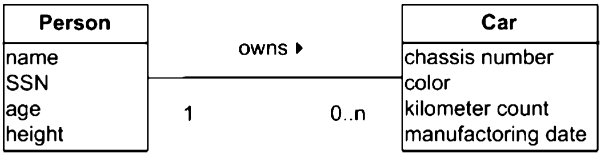

Fig. 9.1 Representation of a Conceptual Schema at the type level in the UML modeling language in the so-called class diagrams

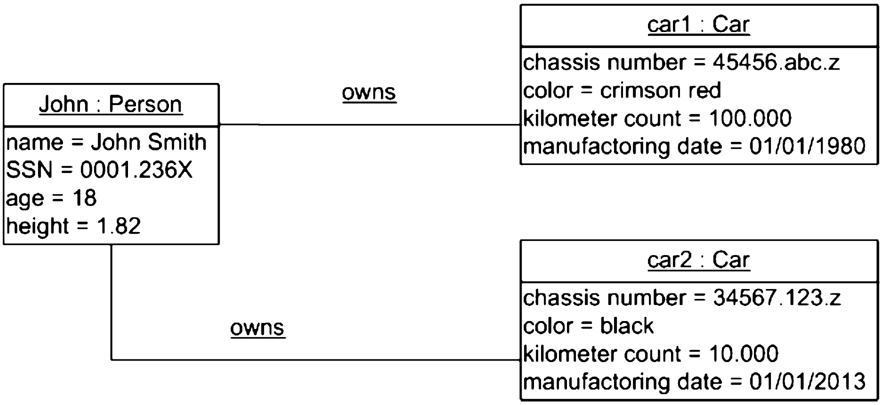

Fig. 9.2 Representation of a Conceptual Schema at the instance level in the UML modeling language in the so-called instance diagrams

this is not made explicit in the definition of the UML standard, an *oid* such as *John* (or car_1 and car_2) is supposed to connect the various snapshot entities (representing momentary states of John) that appear in different UML instance diagrams (hence, the identifier John of type person – symbolized as John:Person - in the header in Fig. 9.2). In summary, *oids* can be interpreted in L_{sortal} as individual concepts; entities in an object diagram can be interpreted instead as momentary states of objects in the sense discussed in Sects. 9.2 and 9.3. It is important to highlight that, in UML, snapshot entities are connected to exactly one *oid*. So, even if two snapshot entities in an instance diagram have the exact same value for all its properties, they still represent two numerically different individuals. Finally, although UML does not make a distinction between sortals and characterizing types, this distinction is available in an evolution of UML for the purpose of conceptual modeling called OntoUML (Guizzardi 2005). In OntoUML, *oids* are defined by classes representing kinds (substance sortals) in the model.

9.4 Cross-World Identity and Classification in Conceptual Spaces

9.4.1 Conceptual Spaces

A proposal to model the relation between the properties and concepts (types) classifying an individual and their representation in human cognitive structures is presented in the theory of *conceptual spaces* developed by the Swedish philosopher and cognitive scientist Peter Gärdenfors (Gärdenfors 2000). The theory is based on the notion of *quality dimension*. The idea is that several perceivable or conceivable properties are associated to quality dimension in human cognition. For example, height and mass are associated with one-dimensional structures featuring a zero point (i.e., isomorphic to the half-line of nonnegative numbers). Other properties such as color and taste are represented by several dimensions. For instance, taste can be represented as a tetrahedron space comprising the dimensions saline, sweet, bitter and sour, and color can be represented in terms of the dimensions hue (a polar dimension), saturation and brightness (two linear dimensions). An illustration of a *color domain* is depicted in Fig. 9.3 below.

According to Gärdenfors, some quality dimensions (especially those related to perceptual qualities) seem to be innate or developed very early in life. For instance, the sensory moments of color and pitch are strongly connected with the neurophysiology of their perception. Other dimensions are introduced by science or human conventions. For example, the representation of Newton's distinction between mass and weight is not given by the senses but has to be learned by adopting the conceptual space of Newtonian mechanics.

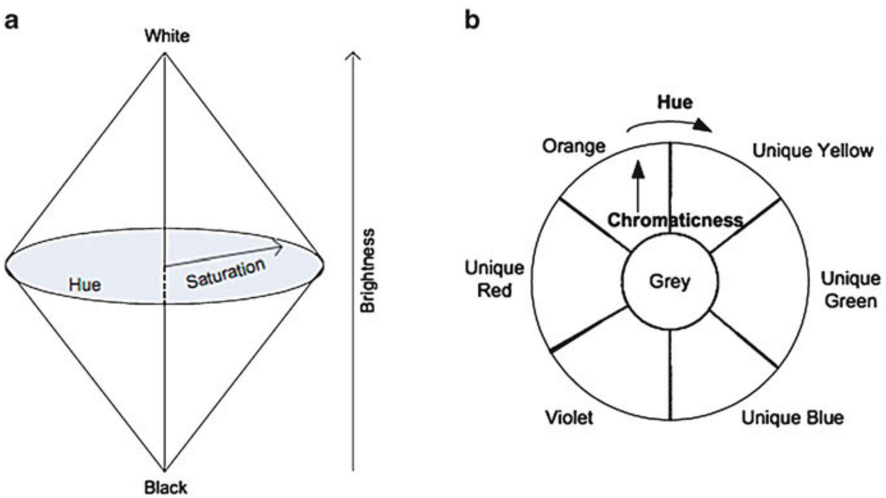

Fig. 9.3 Representations of a color spindle (quality domain for color)

Zenker and Gärdenfors (2015), in this volume, distinguish between *integral* and *separable* quality dimensions: "Dimensions are said to be integral if, to describe an object fully, one cannot assign it a value on one dimension without giving a value on the other. For example, an object cannot be given a hue without giving it a brightness value. Or the pitch of a sound always goes along with its loudness . . . Dimensions that are not integral are said to be separable, as for example the size and hue dimensions". They then define a *quality domain* as "a set of integral dimensions that are separable from all other dimensions". Finally, a *conceptual space* is defined as "collection of one or more domains" (Gärdenfors 2000, p. 26).

Gärdenfors emphasizes that the notion of *conceptual space* should be understood literally, i.e., quality dimensions, quality domains and conceptual spaces are endowed with certain geometrical structures (topological or ordering structures) that constrain the relations between its constituting dimensions. In particular, Gärdenfors uses the notion of a convex region in a metric space to define what he calls a *quality region*. For instance, the different regions in the color circle of Fig. 9.3b define quality regions in that domain. According to him, only attributes representing genuinely substantial properties will form quality regions in a conceptual space. This allows for a geometrical grounding of the difference between what David Lewis (1986) called *natural attributions*, as opposed to *abundant attributions*. In the conceptual space model, natural attributions (e.g., red, person, car) will form convex regions, but abundant attributions will not (e.g., *not-red*, *being-a-car-or-an-apple*).

Finally, Gärdenfors makes the following distinction between what he calls *concepts* and *properties* (Gärdenfors 2000): "Properties . . . form as special case of concepts. I define this distinction by saying that a property is based on single domain, while a concept may be based on several domains." In other words, properties define regions completely contained in a quality domain while concepts define regions that cross over multiple quality domains.

9.4.2 Individuation, Identity and Dynamic Classification in Conceptual Space

Regarding the notions of principle of application and principle of identity discussed throughout this article, a number of remarks can be made regarding the conceptual spaces model.

9.4.2.1 Principles of Application in Conceptual Spaces

In the conceptual space model, an individual is identified by a point in a conceptual space. An object (continuant, endurant) like you and me, my car, your house, Susan's cat, the planet Mars, and the Monalisa are identified by a vector in a multi-dimensional space (a hyperspace) so that each component (coordinate) of a vector represents a value of a property on a given quality dimension (e.g., my height, the color of my car, the price of my house). In fact, Gärdenfors admits to the Leibnizian principle of identity for all individuals, i.e., two individuals are the same (in a numerical sense) iff they are represented by the same point in a conceptual space. Provided that individuals are points in a conceptual space, the *principle of application* of a given type can be represented by the geometrical notion of spatial containment in a given region. For instance, we know that my car is red because the coordinates that represent the color of my car (vector) lie within the red region in the color space.

9.4.2.2 Comparing the Sortal and Characterizing Types and the Concepts vs. Properties Distinctions

Gärdenfors' distinction between properties and concepts does not correspond to that between sortal and characterizing types discussed in Sect. 9.2. For once, there are characterizing types that will correspond to regions crossing multiple quality domains. Examples include the types *physical object* (as a supertype of houses, cars, persons), *insurable items* (as a supertype of persons, houses, cars, works of art, buildings). Moreover, the points (individuals) in a region defined by a property (e.g., red) in the sense of Gärdenfors are exemplars not of objects but of what is termed a *quality value* (i.e., the super-determinate value of a quality, a trope, a property instance, an abstract particular) (Guizzardi 2005). If instead, one is willing to conceptualize the object type red (whose instances would include a red car, a red flag, a red apple, a red building), then the corresponding region crosses multiple quality domains (e.g., my red car will be fully conceptualized on many dimensions that are separable). We conclude that, in its present state, the theory of conceptual spaces does not make the distinction between sortal and characterizing types, i.e., a distinction between types that merely offer a principle of application to its instances and types that also offer a principle of identity. As previously mentioned, the theory can adequately represent the former (e.g., my car is an instance of the object type red if it lies within the cross-domain Red region) but not the latter. Given the purposes of this paper, from now on, I focus on regions of conceptual spaces associated to object types. I assume that ordinary objects (in the sense investigated here) will always be associated to *concept regions*, i.e., to regions crossing multiple domains.

9.4.2.3 Limitations of Conceptual Spaces Regarding Cross-World Identity and Dynamic Classification

A region in a conceptual space representing a type such as dog must represent not only the current dogs that exist now but, as put by Gauker (2007), they must *"comprise all and only dogs, since the concept dog correctly applies to each and every dog (that ever has been or ever will be) and to nothing else."* However, as Gauker notes, the regions defined in a conceptual space are static (fixed). In fact, if an individual is represented by a point in a conceptual space, and one adopts a Leibnizian principle of identity (as Gärdenfors does), then an entity cannot suffer any change without ceasing to be the same. As a consequence, as a representation for types (concepts), regions in a conceptual space seem to be only able to represent *rigid types* and immutable individuals. Gauker makes a similar point arguing that *similarity spaces theories* of concepts, in general, and the theory of conceptual spaces in particular, cannot support the structure of judgments (Gauker 2007).

To illustrate this point, suppose a situation in which an individual, John, is a college student. According to the conceptual space theory, this is represented by having a point *x* represent John in the college student region of a conceptual space. Now, suppose that John ceases to be a college student. John must now be represented by a new point *y* outside the college student region in that conceptual space. Notice that by definition the two points x and y (the two vectors containing different component values) are different. Quoting Gauker, the following question arises: *"in what sense the earlier point in college student region represents the same thing as the later point outside the college student region?"* How can we say that these two points represent the same object?

Gauker's example in fact is slightly different. He exposes a situation in which someone initially *judges* John to be a college student (so her belief that John is a college student is represented by a point in the college student region representing John) but later learns that John is in fact not one (so her belief that John is not a college student is represented by a point outside the college student region representing John). Although Gauker's example pertains to belief revision (learning about individuals), for the sake of my argument, its point is exactly the same. After all, someone's mistaken belief about John can be thought of as a conception of John in a counterfactual situation, i.e., one in which the very same individual has *some* properties different from those he now has (like the counterfactual situation where Mick Jagger never quits the London School of Economics and never leads the Rolling Stones). Furthermore, in order to learn things about John, one must recognize or conceptualize the same individual in different (counterfactual) situations as the very same individual. After all, it is not the case that all properties of an individual are manifested in each of encounters with them (Macnamara 1986).

9.4.2.4 Kind-Dependent Identity

In our working example, merely including an extra time dimension to the space of persons (and, hence, to that of college students) will not suffice to address the above issue. Suppose we were to add an extra coordinate to all vectors representing individuals in the person space (and all its sub-regions including that of college student). John being a college student at time t_1 would then be represented by the point x ($\langle x_1 \ldots x_n, t_1 \rangle$), having values for a number of coordinates (including those referring to properties of students); John not being a student at time t_2 would then be represented by the point y ($\langle y_1 \ldots y_n, t_2 \rangle$), having values for a number of coordinates *but not those referring to properties of students*. Notice that our original question still persists: how can we judge that x and y are the same individual in two different situations? Summing up all these different points (among possibly others) and deciding that John actually represents a sequence of time-indexed vectors does not offer any explanatory power. After all, the problem is exactly one of deciding which points should be part of this sequence of vectors. In other words, what kind of changes can an individual suffer and still be the same individual! There must be something that remains the same in all points representing the same individual. Or, using the terminology of similarity spaces, there must be a set of non-zero values for all points representing John (perhaps some of these values are even immutable across these points). *The specific set of these values depends on what kind of entity is being represented by these points*, i.e., it is because John is a person that all points representing John must have values representing properties that must be present for instances of the concept person, regardless if he is a college student in a particular situation, or not.

As defended here, to decide which points constitute the sequence of points representing an individual in a time-indexed conceptual space, we need the support of a kind K. This kind K will supply a principle of cross-world identity which reports on the properties that must be present in all instances of K (i.e., which dimensions must have non-zero values for points in a given region) and the property values that must remain the same for an entity to remain the same K (i.e., which coordinates must be present for points in a conceptual space to represent the same instance of K).

9.4.2.5 Relating L_{sortal} and Conceptual Spaces

As previously discussed, the conception of object types as regions in a conceptual space can adequately represent the *principle of application* of a given type. However, one should notice that the points in these regions should not be interpreted as objects (continuants, endurants) but as *momentary states of objects*, i.e., the sort of individuals pertaining to the domain of quantification of characterizing types as discussed in Sects. 9.2 and 9.3 (i.e., members of the set $U = \bigcup_{w \in W} D(w)$).

In other words, points in a cross-domain region of a conceptual space corresponding to an object concept should be interpreted as *qualitative characterizations of states of objects* falling under that concept. In particular, unary properties standing for characterizing types in L_{sortal} should correspond to cross-domain regions (object concept regions) in a conceptual space.

My example in the previous section is about time. However, to address objections such as Gauker's, we should take a more general view on an indexing dimension. In other words, the points on such a dimension and the structure of that dimension should correspond exactly to worlds and their accessibility relations, respectively, as discussed in Sect. 9.3. However, for the sake of maintaining generality over the possible interpretations of worlds, I will not assume here that world-structures are additional dimensions on conceptual spaces. Instead, they will be defined as part of an additional structure used for the representation of sortal concepts in the sequel. An additional reason for not including world structures as dimensions in our conceptual spaces is the idea that points, which represent momentary states of entities independently of a world structure are sufficient for applying a principle of application.

I hold that enduring objects of everyday experience cannot be directly represented by standard conceptual spaces. In other words, the instances of sortal types like person, organization, country, car, president, child, planet or statue cannot be directly mapped to points in a conceptual space. To represent such sortal types, we must define associate structures that define suitable projections into conceptual spaces. These structures associate to each sortal type a *sort* ℓ (i.e., a *separated intensional property*) whose extension contains *individual concepts*. Individual concepts can be thought of as projections into a conceptual space defining a suitably constrained *set of points* that represent counterparts in different worlds of the same ordinary object. Sorts, in turn, are sets of individual concepts and, hence, can be thought as projections into conceptual spaces that define regions containing suitably constrained *sets of sets of points*, representing states of ordinary objects of the same sortal type.

So, whilst a point x in a conceptual space can be directly judged to be a *red individual,* an *electrically charged entity* or a *physical object*, that point can only be judged to be a state of person in world w if x belongs to $\ell[w]$. Moreover, whilst regions associated to characterizing types can be defined as similarity regions based uniquely on the similarity of basic points, regions associated to sortal types are *projected* into conceptual spaces by the principle of identity carried by that type. To put it in another way, the latter type of similarity regions are defined in terms of sets of points selected by individual concepts, not in terms of basic points.

In summary, instances of sortals types should be represented by *individual concepts* representing a principle of identity, supplied by the kind they instantiate, which can trace the identity of the same individual by referring to (qualitatively distinct) states in different worlds (represented by points in a conceptual space). These individual concepts are supplied by kinds (substance sortals). However, they can also be dynamically classified possibly under a number of anti-rigid types representing contingent (accidental) properties that can inhere in these individuals. So, returning to our working example, the same individual person, John, can fall in the extension of the type student in a number of situations (in which the states of John will be represented by points in the student region of the person space), and it can fall outside this extension in a number of other situations (in which the states of John will be represented by points outside the student region in the person space). Nonetheless, it is the very same individual, John, that maintains its numerical identity regardless of these contingent (de)classifications as a student.

Finally, given the non-multiplicationalist stance adopted here, the same point in a conceptual space can belong to regions associated to different object types (concepts), i.e., the same point can represent states of individuals falling under different concepts. Moreover, on this stance, two individuals that share a qualitatively indistinguishable state in a given world do have the same state in a numerical sense.

In other words, if the statue *st* and the lump of clay *loc* coincide in world *w* (i.e., $st(w) \approx loc(w)$) then they refer, in that world, to the very same point in a conceptual space.

9.5 Related Work

Modal notions such as were discussed in this paper have been employed by Guarino and Welty (2009) in a number of publications as a way to formally characterize the ontological distinctions comprising the OntoClean evaluation approach for taxonomic structures. OntoClean clearly distinguishes sortal and characterizing types according to their ontological status. However, in the formalizations of that approach, a classical system of modal logics is employed where the focus is on distinguishing between properties w.r.t. to their modal meta-properties (e.g., rigidity versus non-rigidity). As a consequence, these formalizations fail to capture a fundamental distinction between sortal and characterizing types and the unique role of the former category in providing a principle for trans-world identity for objects.

The idea of representing objects of ordinary experience by individual concepts is similar to the solution adopted in the GFO foundational ontology (Heller et al. 2004) in which individual concepts for objects are called *abstract substances* or *persistents*. The notion of a momentary state of objects adopted here is similar to that of *presentials* there. As demonstrated by Heller and Herre, a language such like the one proposed here can play an important role in relating *endurantistic (3D)* and *perdurantistic (4D)* views of entities (i.e., views of entities as space-extended objects with those of entities as spatiotemporal processes). However, in contrast to our approach, GFO does not elaborate on different categories of types (*viz.* kinds, subkinds, phased-sortals and characterizing types). Consequently, no connection between types and identity is developed, and the approach makes no distinction between types that aggregate essential properties (rigid types) and those that aggregate merely contingent ones. Accounting for such distinctions is fundamental not only from a theoretical point of view but also for a number of applications in computer science (Guizzardi 2005). Furthermore, as discussed in Sect. 9.4, this distinction also plays an important role in addressing a criticism targeted at conceptual spaces by Gaulker (2007).

9.6 Summary

I presented a system of modal logic with sortal restricted quantification to suitably capture the intended semantics of a philosophically and cognitively well-founded theory of object types. The proposed logical system formally characterizes the distinction between sortals and general property types where the former exclusively supplies a principle of persistence and cross-world identity to its instances. As a result, we can address the limitations of classical (unrestricted extensional) modal logics which reduce ontologically very different categories to the same logical footing, and advance proposals such as in Gupta (1980) by: (i) refining the notion of sortal types, considering the distinction between substance, rigid and phased-sortals; and (ii) proposing a system that avoids reducing the relation of identity to a mere relation of equivalence. Finally, I also showed how this proposal can complement the theory of conceptual spaces by offering an account for kind-supplied principles of cross-world identity. The proposal is in line with a number of empirical results in cognitive psychology and that can addresses an important criticism of the conceptual spaces model regarding object identity. As I demonstrate here, without addressing issues related to cross-world object identity, the conceptual spaces model is not properly equipped for serving as a general model for cognitive semantics, as it is not properly equipped for defining the semantics of linguistic entities as fundamental as proper names.

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