k-Distinct Strong Minimum Energy Topology Problem in Wireless Sensor Networks

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Abstract. Given a set of sensors, the strong minimum energy topology (SMET) problem is to assign transmit power to each sensor such that the resulting topology containing only bidirectional links is strongly connected and the total energy of all the nodes is minimized. The **SMET** problem is known to be NP-hard. Currently available sensors in the market support a finite set of transmission ranges. So we consider the k-DISTINCT-SMET problem, where only k transmission power levels are used. We prove that the k-DISTINCT-SMET problem is NP-complete for $k \geq 3$. However, on the positive side, we show that the 2-DISTINCT-SMET problem can be solved in polynomial time. The energy cost of transmitting a bit is higher than the cost of computation, and hence it may be advantageous to organize the sensors into clusters and form a hierarchical structure. This motivated the study of **k-Distinct-rStrong Minimum Energy Hierarchical Topology (k-Distinct-rSMEHT) problem:** Given a sensor network consisting of n sensors, and integers k and r, assign transmit powers to all sensors out of the k distinct power levels such that (i) the graph induced using only the bi-directional links is connected, (ii) at most r sensors are connected to two or more sensors by a bidirectional link and (iii) the sum of the transmit powers of all the sensors is minimum. We Propose a $\frac{r+1}{2}$- approximation algorithm for the k-Distinct-rSMEHT problem for any fixed r and arbitrary k.

Keywords: Wireless Sensor Network, Topology Control Problem, Transmission Power Assignment, Graph Theory, NP-complete, Heuristics.

1 Introduction

A wireless sensor network (WSN) consists of a collection of autonomous devices, each of which consists of a digital circuitry, radio transceiver, transmission amplifier and a small battery. The communication among these nodes is based on radio propagation. Since the battery of each sensor is of limited capacity

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and it is not possible to replace the battery always, energy conservation is a critical issue in order to increase the lifetime of a sensor network. Each sensor node u has an omni-directional antenna, which can transmit signal within a certain specified range $r(u)$. Node u can directly communicate with other nodes located within its range. In general, communication is *multi-hop* in nature, where intermediate nodes are used to relay the transmission until destination is reached.

Given a set of sensors S, and a connectivity constraint Π, the minimum range assignment problem, MIN-RANGE(Π) is to assign transmission range f, to the nodes in S such that given connectivity constraint Π is satisfied and $cost(f)$ is minimum. The connectivity constraint Π could be simple connected, strongly connected or bi connected (see [4]). The symmetric connectivity is the strong form of connectivity among all, as it contains only bidirectional links. The bidirectional/symmetric edges are preferred in wireless sensor networks, because the signal transmitted over a link are to be acknowledged. Bidirectional links also simplify routing protocols. The current MAC layer protocols such as IEEE 802.11 and S-MAC take into account only bidirectional links. The bidirectional range assignment is studied by cheng *et al.* [3] and they named this problem as *Strong Minimum Energy Topology* (SMET) problem. The SMET problem is known to be NP-hard [3].

Currently available sensors in the market support a finite set of transmission ranges. So we consider k-DISTINCT-SMET problem, where only k transmission power levels are used. We prove that the k-DISTINCT-SMET problem is NP-complete for $k \geq 3$. However, on the positive side, we show that the 2-DISTINCT-SMET problem can be solved in polynomial time. The energy cost of transmitting a bit is higher than the cost of computation, and hence it may be advantageous to organize the sensors into clusters and form a hierarchical structure. This motivated the study of **k-Distinct r Strong Minimum Energy Hierarchical Topology (k-Distinct-rSMEHT) problem:** Given a sensor network consisting of n sensors and an integer r, assign transmit powers to all sensors out of the k distinct power levels such that (i) the graph induced using only the bi-directional links is connected, (ii) at most r sensors are connected to two or more sensors by a bidirectional link and (iii) the sum of the transmit powers of all the sensors is minimum. We propose a $\frac{r+1}{2}$- approximation algorithm for the k-Distinct-rSMEHT problem for any fixed r and arbitrary k.

The rest of the paper is organized as follows. In Section 2, we present a summary of related work. In Section 3, we define the SMET problem and the special cases of the SMET problem. In Section 4, we propose an efficient algorithm for the 2-DISTINCT-SMET problem. In Section 5, we present the k-Distinct-rSMEHT problem for arbitrary r and k and propose a $(\frac{r+1}{2})$-approximation algorithm for the k-Distinct-rSMEHT problem for fixed r and arbitrary k. Finally, Section 6 concludes the paper.

2 Previous and Related Work

A WSN is modeled as a complete weighted undirected graph $G = (V, E, c)$, where $c : E \to \mathbb{R}$ is the cost function. Given a spanning sub graph H of G, the cost of H is defined as $C(H) = \sum_{e \in E(H)} c(e)$. For a vertex $u \in V$, the power of u is the maximum cost of an edge in H, incident on u, i.e $P_H(u) = \max_{uv \in E(H)} C(uv)$. The power of a graph is the sum of powers of its nodes. We are interested in the minimum energy spanning subgraph problem where the subgraph is a spanning tree. Minimum energy spanning tree problem is NP-hard for the sensors on a 2-dimensional plane (see [3]).

The minimum energy symmetric connectivity problem is studied in [3]. For more details about topology control in WSN, we refer to [8].

Prim-incremental heuristic for the SMET problem was proposed by [3]. Valley-free heuristic for the SMET problem was proposed by [1]. Panda and Shetty proposed Kruskal-incremental based heuristic algorithm [5] and local search based heuristic algorithm [6] for the SMET problem. A 2-hop strong minimum energy rooted topology (2h-SMERT) and its algorithmic aspects are presented in [7].

3 Special Cases of Strong Minimum Energy Topology Problem

Definition 1 (SMET). *Given a set of sensors in the plane, compute the transmit power of each sensor such that there exists at least one bidirectional path between any pair of sensors and the sum of transmit powers of all the sensors is minimized.*

Let $T = (V, E')$ be a spanning tree of a weighted graph $G = (V, E,)$ having cost function w. Let $P_T(v) = \max\{w(uv) | uv \in E(G)\}$ and $P(T) = \sum_{v \in V} P_T(v)$. The SMET problem now reduces to the problem of finding a spanning tree T of G such that $P(T)$ is minimum. Formally, we define the SMET problem as follows:

Problem: SMET

Instance: (K_n, w, M), where K_n is a complete graph with n nodes, $w : E(K_n) \to \mathbb{R}^+$ is the weight function, and M is a positive real value.

Question: Does there exist a spanning tree T of K_n such that $P(T) \leq M$?

As the status of the SMET problem is known to be NP-hard [3], we examine the computational complexity of the following special cases of the SMET problem.

1 k-Distinct-SMET Problem: The SMET problem with the restriction that the admissible range of a node is one of the k-distinct power threshold values. This special case is important as the currently available sensors in the market supports a discrete set of power ranges.

2 *k-Distinct r Strong Minimum Energy Hierarchical Topology (k-Distinct-rSMEHT) problem:* Given a sensor network consisting of n sensors and integers k and r, assign transmit powers to all sensors out of the k distinct power

levels such that (i) the graph induced using only the bi-directional links is connected, (ii) at most r sensors are connected to two or more sensors by a bidirectional link and (iii) the sum of the transmit powers of all the sensors is minimum.

4 k-Distinct-SMET Problem

In practice, it is usually impossible to assign arbitrary power levels to the transmitters of a radio network. Instead one can only choose from a constant number of pre-set power levels corresponding to a constant number of ranges [2]. In this context, we study the decision version of the k-DISTINCT-SMET problem which is formally defined below:
Problem: k-DISTINCT-SMET-D
Instance: (K_n, w, M), where K_n is the complete graph with n nodes, $w : E(K_n) \to \{c_1, c_2, \ldots, c_k\}$ is a weight function, and M is a positive real number.
Question: Does there exist a spanning tree T of K_n such that $P(T) \leq M$?

Theorem 1. *The* k-DISTINCT-SMET-D *problem is* NP*-complete.*

Proof. The proof is omitted due to space constraint.

4.1 The 2-Distinct-SMET Problem

Consider the case where only two power threshold levels, say, high and low are used. We call this problem 2-DISTINCT-SMET problem. Power assignment in radio networks with two power levels is studied by Carmi and Katz [2].

Let c_1 and c_2 be the two distinct edge costs assigned to K_n. Let $E(c_i) = \{e | w(e) = c_i\}$, $1 \leq i \leq 2$.

Theorem 2. *An optimal solution for* 2-DISTINCT-SMET *problem can be found in polynomial time.*

Proof. The proof is omitted due to space constraint.

5 k-Distinct-rSMEHT Problem

Most wireless networks are structured hierarchically, where some nodes play the role of cluster heads and others as ordinary nodes. So we a study a topology control problem where there are k distinct power levels and at most r sensors are connected to two or more sensors by a bidirectional link and the total power of the network is minimized. The k-Distinct-r SMEHT Problem is formally defined as follows.
Problem: k-DISTINCT-r SMEHT-D
Instance: (K_n, w, r, M), where K_n is a complete graph with n nodes, w :

$E(K_n) \to \{c_1, c_2, \ldots, c_k\}$ is a weight function, r and M are positive real numbers.
Question: Does there exist a spanning tree T of K_n such that $P(T) \leq M$ and T has at most r non-pendent vertices?

Theorem 3. *The k-*DISTINCT-r SMEHT-D *problem is* NP*-complete.*

Proof. The proof is omitted due to space constraint.

5.1 Approximation Algorithm for the k-Distinct-rSMEHT Problem for Fixed r

Given a weighted complete graph (K_n, w) and a fixed positive integer constant r, we propose a $\frac{r+1}{2}$-approximation algorithm for the k-Distinct-rSMEHT problem. In order to explain our algorithm we define, the extension of a tree as follows:

Definition 2 (Extension of a Tree). *Let $T^l = (V_l, E^l)$ be any tree such that $V_l \subset V(K_n)$. Let $V_l = \{v_1, v_2, \ldots, v_l\}$. Let $X_1 = \{x \mid x \in V \setminus \{v_1, v_2, \ldots, v_l\}$ and $w(xv_1) \leq w(xv_j)$ $2 \leq j \leq l\}$. Let $X_i = \{x \mid x \in V \setminus \{v_1 \ldots v_l\} \cup (\cup_{j=1}^{i-1} X_j)$ and $w(xv_i) \leq w(xv_j)$ $1 \leq j \leq l, j \neq i\}$, for $i > 1$. Let $E_i = \{v_i x \mid x \in X_i\}, 1 \leq i \leq l$. The* extension *of T^l, denoted as $Ex(T^l)$, is defined by $Ex(T^l) = (V, Ex(E^l))$, where $Ex(E^l) = E^l \cup (\cup_{i=1}^{l} E_i)$.*

Let T^l be tree of K_n. If T^l is not a spanning tree, then extension of tree T^l is used to construct a spanning tree T of the complete graph K_n.

Lemma 1. *Let T be a minimum energy tree of an instance (K_n, w, r) of the k-Distinct-rSMEHT problem. Let $v_1, v_2, \ldots, v_l, l \leq r$, be the non-pendant vertices of T and $v_{l+1}, v_{l+2}, \ldots, v_n$ be the pendant vertices of T. Let $T' = T[\{v_1, v_2, \ldots, v_l\}]$. Let T'' be the extension of T'. Then, $P(T'') \leq \frac{k+1}{2} P(T)$.*

Proof. The proof is omitted due to space constraint.

Algorithm 1. Algorithm k-Distinct-rSMEHT

Input: (K_n, w, r), where K_n is a complete graph having cost function $w : E(K_n) \to \{c_1, c_2, \ldots, c_k\}$, and r is a fixed integer constant.
Output: A spanning tree T with at most r non-pendant vertices.

```
begin
    Let T be any arbitrary spanning tree of K_n having exactly r non-pendant
    vertices.
    for each subset V_r of V(K_n) of r elements do
        for each spanning trees T_r of K_n[V_r] do
            if P(Ex(T_r)) ≤ P(T)) then
                T = Ex(T_r);
            end
        end
    end
    output(T);
end
```

Note that $Ex(T^l)$ is a spanning tree of K_n and can be computed in $O(nl)$ time. Let $K_n[V_r]$ denote the subgraph of K_n induced by vertices in $V_r \subset V$. We now propose an algorithm to construct a minimum energy spanning tree T of K_n with at most r non-pendent vertices.

Theorem 4. *The algorithm* k-Distinct-rSMEHT *is a* $\frac{r+1}{2}$*-approximation algorithm.*

Proof. The proof is omitted due to space constraint.

6 Conclusion

In this paper we study the k-Distinct-SMET problem and the k-Distinct-rSMEHT problem. We proved that the k-Distinct-SMET problem is NP-complete for $k > 2$. We proposed a polynomial time solution for the k-Distinct-SMET problem, for $k = 2$. We proposed a $\frac{r+1}{2}$- approximation algorithm for the k-Distinct-rSMEHT problem for any fixed r and arbitrary k.

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