

Reading Mathematics: More than Words and Clauses; More than Numbers and Symbols on a Page

Mary A. Avalos, Alain Bengochea, and Walter G. Secada

Abstract Reading comprehension research in mathematics has focused primarily on the teaching of generic content area reading strategies (Alvermann D, Moore D, Secondary school reading. In: Barr R, Kamil M, Mosenthal P, Person PD (eds) Handbook of reading research, vol II. Longman, New York, pp 951–983, 1991; Pearson PD, Fielding L, Comprehension instruction. In: Barr R, Kamil M, Mosenthal P, Pearson PD (eds) Handbook of reading research, vol II. Longman, New York, pp 815–860, 1991). In contrast, mathematics education research has focused on ensuring that students understand and can translate the symbols and register of mathematics (Crandall et al., 1989) to and from everyday language to solve problems. Both approaches have been used to support the treatment of mathematics as a fixed body of facts and procedures that are to be acquired by the learner. More recent thinking, however, views school mathematics as a “way of knowing” (National Council of Teachers of Mathematics, Professional standards for teaching mathematics. Author, Reston, 1991, Principles and standards for school mathematics. Author, Reston, 2000; Siegel M, Fonzi J, Read Res Q 30:635, 1995) and incorporates “mathematical texts” as affordances that can support students’

This work was made possible due to a grant by The Institute of Educational Sciences, Award Number R305A100862. The content of this chapter does not necessarily reflect the views or policies of IES or the U.S. Department of Education nor does mention of trade names, commercial products, or organizations imply endorsement by the U.S. Government.

The authors would like to acknowledge the collaborative effort that made this work possible, including the teachers, students, and district administrators who contributed invaluablely to this project, as well as Margarita Zisselsberger, Edwing Medina, Naomi Iuhasz, Kristen Doorn, Robin Shane, and Shantell Saunders.

M.A. Avalos (✉) • W.G. Secada
Department of Teaching and Learning, University of Miami School of Education and Human Development, Coral Gables, FL, USA
e-mail: maavalos@miami.edu; wsecada@miami.edu

A. Bengochea
Crane Center for Early Childhood Research and Policy (CCEC), The Ohio State University, Columbus, OH, USA
e-mail: bengochea.1@osu.edu

development of mathematical literacy (Draper RJ, Siebert D, Rethinking texts, literacies, and literacy across the curriculum. In: Draper RJ, Broomhead P, Jensen AP, Nokes JD, Siebert D (eds) (Re)Imagining content area literacy instruction. Teachers College Press, New York, pp 20–39, 2010; Siegel M, Fonzi J, *Read Res Q* 30:632–673, 1995). From our work as an interdisciplinary team, we argue for an interdisciplinary perspective of reading comprehension as applied to reform-oriented mathematics-teaching practices. We begin by reviewing the literature on adolescents' reading comprehension of mathematics and then present a small study investigating how sixth and seventh grade students approached reading math textbooks. We end by proposing a revised definition of reading comprehension for mathematics grounded in the results of our study. In building on multiple theories we redefine reading comprehension in mathematics using the work of Rosenblatt (*The reader, the text, the poem*. Southern Illinois University Press, Carbondale, 1978, 1982), Kintsch (1988), and Halliday (*Language as a social semiotic*. Edward Arnold, London, 1978) to respectively incorporate the transactional, constructivist, and language-dependent nature of thinking and reasoning necessary to create meaning and successfully comprehend mathematical texts.

Keywords Mathematics • Textbook comprehension • Opportunity to learn

1 Introduction

Mathematics textbooks are densely packed with information, using more concepts per sentence and paragraph as compared to other texts that are used in school (Schell, 1982). To convey meaning, mathematics textbooks use technical vocabulary and symbols specific to math, everyday language in semantically different ways, specific genre structures and language features within written texts, and visual graphic representations (Halliday, 1978; Huang & Normandia, 2008; Schleppegrell, 2007; Spanos, Rhodes, Dale, & Crandall, 1988). These features contribute to the complexity of math textbooks that is common in elementary and middle school (Secada, Zisselsberger, Langer-Osuna, & Avalos, 2011; Zisselsberger, Avalos, & Secada, 2012). Often, students must engage in nonlinear reading patterns as they zigzag among written words, tables, graphics, and symbols to make sense of problems and other information found in the textbooks (Carter & Dean, 2006).

The organization of most U.S. mathematics textbooks follows an *exposition—examples—exercises* model, structuring tasks in a sequence intended to build students' conceptual understandings for each lesson (Love & Pimm, 1996). Expository text introduces a concept, generally within the problem setting; next, worked-out examples are introduced and are used to demonstrate how to apply problem solving methods. The lesson concludes with a set of exercises and problems that provide student practice. Generally, problem difficulty within a lesson or chapter flows from easier tasks towards increasingly complex tasks. Most mathematics textbooks contain sidebars and pictures that are both proximally and distally related to the lesson, along with a mixed review of problems from previous lessons, practice

problems, and vignettes that connect the mathematics lesson to other content areas or cultures; all of this can be confusing for students to navigate (Metsisto, 2005).

Mathematics textbooks usually include short, seemingly disconnected snippets of text on each page that are meant to provide students multiple examples and practice problems within different contexts, making the math textbook look, feel, and read differently than disciplines using more expository-like text structures (Barton & Heidema, 2000; Buehl, 2011). What is more, an analysis of mathematics problems used for the Third International Mathematics and Science Study (TIMSS) shows there is a gradual increase in the use of more expository-type mathematics texts as students advance through the grade levels (e.g., proofs; Valverde, Bianchi, Wolfe, Schmidt, & Houang, 2002). Thus, as math content becomes more abstract in middle and high school, math problems and text increase in density. The need to explore how students approach reading their math texts is important, especially in light of the fact that all students are expected to meet the new and demanding expectations of the Common Core State Standards (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010) mathematical practices.

Österholm (2006, 2008) compared high school and college students' reading of mathematics and expository texts. Two groups of students with equivalent mathematical knowledge read (a) a math text with symbols or (b) an expository text without symbols describing the same construct. Expository math texts without symbols were approached and read differently than mathematics texts with symbols that denote relationships, constitute processes, and represent numerical values. Students who read the text with symbols scored lower on a comprehension post-test. Moore (1994) found that college students had difficulty with the mathematical language of notation, arguing that students' inability to restate definitions and concepts in their own words demonstrated limited comprehension. Not only do these studies indicate the need for secondary students to read mathematical texts differently than other content area texts, but they also point to the need for teachers to scaffold reading instruction and to support students when they are reading mathematical texts. This instructional imperative would seem to be particularly important for socially and culturally diverse learners who may not be familiar with problem contexts that are more typical of mainstream student experiences (Jackson, Garrison, Wilson, Gibbons, & Shahan, 2013) or who may be more accustomed to different problem solving practices (Hoffert, 2009). Additionally, learning "the language of math" may be akin to learning an additional language in and of itself (Barbu & Beal, 2010; Kersaint, Thompson, & Petkova, 2009; Moschkovich, 2010; Wright & Li, 2008).

To better support students' reading across content areas, literacy experts have advocated the use of content area literacy instruction, which has been successful in assisting students' understandings of complex texts (Alvermann & Moore, 1991; Conley, 2008; Ogle, 2009; Pearson & Fielding, 1991; Van Garderen, 2004). Earlier work in content area literacy instruction typically involved the application of generic reading strategies to content texts. Hence, resistance to content area literacy instruction by secondary math teachers was not uncommon because it was

seen as disconnected from the domain knowledge that *mathematics* teachers were responsible to teach, and it was not seen as an essential component for understanding the domain (Draper, 2002; Olson & Truxaw, 2009; Siebert & Draper, 2008). This is particularly true for mathematics teachers who are primarily concerned with students' ability to translate symbols and the register of mathematics (Spanos et al., 1988) to and from everyday language for solving and describing problems (Siegel, Borasi, & Smith, 1989). Another issue with applying generic reading strategies to math texts is that this approach generally treats mathematics as a body of facts and procedures that are to be acquired by the learner rather than treating mathematics as a "way of knowing" (Siegel & Fonzi, 1995, p. 635), of which the former is the antithesis of mathematics reform efforts (National Council of Teachers of Mathematics [NCTM], 1991, 2000).

Content area reading instruction has evolved to promote multi-literacies beyond reading comprehension, thus moving instruction from the application of generic comprehension strategies to the promotion of discipline-specific knowledge and content-specific ways of knowing (Moje, 2008; Shanahan & Shanahan, 2012). A literacies-based approach to mathematics instruction focuses on the "performance of literacy" (Lerman, 2007, p. 755) which includes students' discourse practices and thinking processes, along with their reading, writing, and visual comprehension of mathematics texts so as to more closely reflect how knowledge is created, shared, and evaluated by experts in the field. Thus, making sense of texts shifts from the application of generic strategies to the use and construction of disciplinary-specific practices (Shanahan, Shanahan, & Misischia, 2011). As Heller and Greenleaf (2007) point out, reading in different subject areas requires varying approaches to reading the text that include the knowledge and reasoning processes found in the particular subject.

The discipline of mathematics values literacies that promote reasoning for quantitative situations, problem solving, creating and testing conjectures, communication and evaluation of mathematics solutions, and modeling/application to real life contexts (Siebert & Hendrickson, 2010). In reform-oriented mathematics instruction (NCTM, 1991, 2000), teachers become facilitators of student learning rather than practitioners of a 'pedagogy of telling' (Sizer, 1984, p. 109). The CCSS (2010) mathematical practices promote students' perseverance in problem solving, critical thinking, and sense-making of texts, among other disciplinary-like practices. With this expansion of literacies, the notion of *text* has expanded beyond words on a page to include multiple modes of viewing, organizing, and reading to make sense of information (Buehl, 2011; Draper, Broomhead, Jensen, Nokes, & Siebert, 2010; Langer, 2011; McKenna & Robinson, 1990; Parris, Fisher, & Headley, 2009). Mathematical texts include tables, graphs, diagrams, models, or graphics with data or information related to a problem, equations, proofs, written descriptions of a problem or solution, calculator readouts, and verbal mathematics discussions (Siebert & Hendrickson, 2010). Proficiency with this expanded notion of text is necessary for students to become active and engaged participants of math-practice communities, as called for by the CCSS. While the NCTM standards documents (1991, 2000) emphasize the importance of reading, writing, and communicating

in reform-oriented mathematics classrooms, specific strategies for helping students acquire mathematical literacies are not included or suggested for teachers to emulate or adapt (Draper & Siebert, 2004). This is also the case for the CCSS (2010).

With this relatively new and different way of looking at content area literacy, research is needed to inform pedagogy that will promote students' acquisition of mathematical literacies. Lerman (2007) calls for research that investigates how students from diverse backgrounds read what is required of them, and how their reading habits position them to sustain their identities when performing math or science literacies. This type of literacy research, according to Lerman, would inform pedagogy by indicating the degree to which teachers must be explicit in teaching students what is required for them to read texts for appropriate performance, and ultimately to foster equity in mathematics and science teaching.

This chapter provides baseline information on the purposes for which middle school students claim to read their math textbooks. Furthermore, we discuss how teachers might use this information for their own pedagogical practices in support of content area literacies specific to math textbooks. According to Bossé and Faulconer (2008), many who have examined reading mathematics fail to distinguish *reading about* vs. *reading in* mathematics. *Reading about* mathematics makes use of a variety of text types to learn about mathematics (i.e., picture or comic books, novels, biographies), while *reading in* mathematics makes use of mathematical texts to further domain knowledge. Our work addresses the issue of *reading in* mathematics; it is based on a study that explored how sixth through eighth grade students at various English language proficiency and math achievement levels reported using or reading their math textbooks when working independently. We begin by describing how students and teachers use math textbooks in secondary classrooms as demonstrated in previous studies. Next we briefly describe our study and findings to orient the reader for a discussion grounded in the CCSS (2010) and Shepherd's (2005) framework to advocate an active (as opposed to passive) reading of mathematics texts. We conclude with suggestions for teachers' pedagogy to support content area literacies while using mathematics textbooks.

2 How Students and Teachers Use Math Textbooks

According to Love and Pimm (1996), teachers are the mediators between students and their mathematics textbooks; therefore, teachers' interpretations of math textbooks for their students have bearing on how students perceive and ultimately use textbooks (Pepin & Haggarty, 2001). Moreover, teachers' instructional decisions as based on their textbook's curriculum ultimately influence students' opportunities to learn and the way instruction is carried out (Regis, Appova, Reys, & Townsend, 2006; Reys, Reys, Lapan, Holliday, & Wasman, 2003; Valverde et al., 2002).

Although the same mathematics textbooks can be used differently across classrooms, secondary teachers typically use their district-adopted texts as an organizer to make daily decisions about what and when to teach certain grade level content

(Ball & Feiman-Nemser, 1988; Chávez-López, 2003; Grouws & Smith, 2000; Regis et al., 2006). Regis et al. found that of 116 participating middle school mathematics teachers, 39 % of them used the district's mathematics textbook at least 90 % of the instructional days they documented, and over 70 % used the textbook 3 out of 4 days during the documented instructional period. Individual teachers also use the same mathematics textbook in different ways with different groups of students based on the students' learning characteristics (Chávez-López, 2003; Moren, 2000). Draper (1997) states that, in part, students avoid reading their math textbooks because they are unprepared to do so. She argues that since teachers are generally the main source of information in math classrooms, the need for students to read their textbooks for learning mathematics is eliminated. Draper (2002) also points out that methods textbooks for pre-service mathematics teachers rarely discuss reading mathematical texts, and at times when they do, reading in math class is generally given a negative spin by the authors because the approach is associated with more traditional, teacher directed teaching methods.

Shepherd (2005) noted that many of her struggling undergraduate students read mathematics texts as passive receivers of information. Building on the work of Exner (1996), she implemented an approach in her introductory/basic math courses that aimed to convince her students they could successfully read mathematical texts. Shepherd lectured very little during class, only outlining what she felt was important for the students to understand, thus forcing them to read mathematical texts. Specifically, she provided scaffolds to move students from a passive reading of mathematics (e.g., reading a literary text as receiver of information) to an active reading that required students to pause and think critically at certain points of assigned tasks. When students saw a designated symbol (i.e., happy face) in the mathematical text assigned by Shepherd, they were required to do something or take action on the information they were reading before moving on. The action to be taken was suggested by Shepherd as scaffolds for how to approach reading mathematics more actively. Suggested actions involved defining vocabulary, finding or creating examples that would demonstrate what was read, making connections to prior knowledge of mathematics or the context of the task, setting up an equation based on the information, or making notes in the margin. To measure the success of this intervention, Shepherd focused on her students' dispositions towards mathematics because she believed students' feelings about mathematics were a better indication of their success with the subject than passing the course. Students in Shepherd's classes with the reading mathematics text emphasis were generally more engaged learners of mathematics throughout the term and did not give up as easily as previous students in the same introductory course.

Rezat (2009) explored how and when German sixth and twelfth grade high-achieving students consulted their math textbooks when given problem solving tasks by teachers during class. He found that textbooks were primarily used while carrying out four activities termed "utilization scheme types" or USTs (p. 1,264). The first UST, called "solving tasks and problems," had three different UST patterns of use, including: (a) students looked to find worked examples or methods that would assist in solving similar tasks/problems; (b) the tasks students chose to examine more

closely in their textbooks showed similarities to the original task they were trying to solve (i.e., same images graphed or data in similar types of tables/organizers); and (c) students began to look for assistance at the start of the chapter, focusing on headers until useful information was found. Based on student observations, interviews, and student work, Rezat argued that students saw the examples at the beginning of the textbook's lesson to be the most helpful when looking for information to assist them with solving problems. Rezat also found that the students focused more on finding a solution to the problem rather than reading the textbook to develop understanding of the mathematics behind the problem's solution.

The second UST, called "consolidation," Rezat referred to as using the textbooks when students' wanted or needed to learn rules, review or repeat teacher mediated tasks, or solve problems similar (and usually adjacently placed) to teacher-mediated tasks. The overarching purpose for this UST was to go over mathematics already taught by the teacher. Many students in this study also made use of the review sections at the end of the chapter/lesson to consolidate their understanding of the lesson. These patterns of use indicate less dependence on the teacher for understanding and learning mathematics content as students utilized their textbooks independently.

The final two USTs, or "utilization scheme types" from Rezat's work were "acquiring mathematical knowledge" and "activities associated with interest in mathematics." The former was described as students going outside of the lesson or chapter studied during class to an adjacent part of the book that had not been introduced in order to find out about upcoming math lessons, and the latter focused on the images in the math textbook that appealed to the interests of the students. The students did not associate looking at the pictures with learning mathematics; rather, they looked at the pictures and read associated captions out of interest in the images displayed on the page. Rezat and colleagues (2009) state that the students' USTs of textbooks reflected how and what they felt was important when learning mathematics; student dispositions towards learning math were predominately "comprised of learning and applying rules and worked examples to tasks, and developing proficiency in tasks similar to teacher-mediated tasks" (p. 1,267).

More recently, Rezat (2013) reanalyzed his original data to report how students utilized textbooks for the purpose of self-regulated practice of mathematics. He identified three mathematic textbook USTs for the purpose of self-regulated practice (i.e., to improve understanding of concepts and procedures, how to carry out procedures, and grades). He termed these USTs as position-dependent, block-dependent, and salience-dependent. Position-dependent USTs referred to the relative position of the practice problem(s) in relation to the teacher-mediated sections; the positioning of the problems within the text guided the students' choices when determining which problems to focus upon for practice. One of the participating teachers did not mediate or provide worked examples from the textbook until the second week of a construct's instruction. Until this time, the position-dependent practicing scheme does not occur in the data. Only when the teacher mediated textbook problems did her students select adjacent problems for individual practice, indicating how teacher mediation impacts students' textbook use. Block-dependent practice occurred when

students chose specific blocks or sections from the textbook to practice. For example, students selected blocks at the chapter level with rules in boxes or tasks/problems created for students to self-regulate understanding of the chapter's content (e.g., sections entitled, *Practice*). Salience-dependent practice included practicing what was deemed to be salient based on surface level features of the tasks; that is, tasks that were similar in appearance to teacher-mediated tasks but not necessarily close in proximity to the actual problems focused on by teachers. Rezat concluded it was important to note the structure of the mathematics textbook in order to use it efficiently (i.e., for block-dependent USTs or specific sections), yet at the same time, position- (i.e., relative proximity of practice to teacher-mediated problems) and salient-dependent (i.e., tasks that look similar on a surface level) USTs indicated the students' conceptions of what was important for practice based on the focus of the teacher during classroom instruction.

In a study involving undergraduates, over 1,000 students in introductory mathematics classes were surveyed to describe their use of mathematics textbooks (Weinberg, Wiesner, Benesh, & Boester, 2012). Survey responses revealed that students used examples to build mathematical understanding, rather than using expository text for that purpose. The researchers speculated that the reported use of math textbooks may be a result of the textbook structure, along with students' dispositions about reading and their perspectives on the nature of mathematics. Weinberg and colleagues end by advocating that instructors carefully choose mathematics texts and materials to promote mathematical reasoning and to encourage students to read math textbooks to support their developmental reasoning.

In sum, though there are few studies on the comprehension of mathematical texts (and none that we could find focused on diverse secondary populations), the work that has been done indicates mathematics textbooks are heavily relied upon and used by teachers to serve as planning guides for instruction. Teachers tend to mediate the information in mathematics textbooks which effectively eliminates the need for students to critically read mathematics textbooks for understanding. How students perceive the nature of mathematics also has important repercussions for how they use and make sense of mathematical textbooks. When using example and practice problems from their textbooks, most students emphasize solving the problem over understanding how and why the problem is solved.

3 Overview of Our Study: Methods and Findings

The study reported here was part of a larger research and development project that sought to infuse explicit teaching of mathematics-academic language via reading, writing, listening, speaking, and viewing with instruction that developed English language learners' (ELLs') mathematical literacies (Secada & Avalos, 2010). We audio recorded interviews with ELLs and fluent English speakers at various math proficiency levels in the sixth through eighth grade classrooms of teachers,

who were participating in the larger development effort, to answer the following research questions:

- How do middle school students of varying mathematical and English proficiencies report that they read their math textbooks?
 - What text features and types of problems do middle school students report finding most and least helpful when using their math textbooks to solve problems independently (e.g., alone or for homework)?
 - What about these features and types of problems do middle school students report as providing them with support or help that indicates an active approach to reading mathematics textbooks?

3.1 Participants

Our student sample came from two culturally and linguistically diverse, Title I urban middle schools within a large district located in the southeastern U.S. The schools were similar in social/ethnic demographic make-up and at the time of data collection, both had recently implemented magnet school programs with science, technology, engineering, and mathematics (STEM) curricular emphases. We were specifically interested in how students of varying math achievement levels and English proficiencies reported that they used their math textbooks to determine if there were differences in use by achievement and/or English proficiency. Purposeful sampling was used to select students for lower-achieving or higher-achieving groups. Within the lower-achieving group, students were further sorted into ELLs or FES (fluent English speaking) categories.

The majority of participating students in sixth and seventh grades scored below the 32nd percentile on the Test of Mathematics Achievement-2 (TOMA-2; Brown, Cronin, & McEntire, 2009) administered in the spring of the year that they were interviewed; these students' interview data were grouped in a *lower achieving* (LA) category for the purpose of this study. Sixteen of the LA students in our sample were identified and receiving services as ELLs (hereafter referred to as LA-ELLs) by the district. These students primarily spoke Spanish as their first language (L1) and had intermediate proficiency in English; however, within this sample there were also a couple of L1 Haitian Creole speakers with early intermediate English proficiency, and one student who spoke Urdu (L1) with intermediate English proficiency. Most of the LA-ELLs ($n = 13$) were in remedial (intensive) math classes for 1.5 h each day, and the remaining LA-ELLs ($n = 3$) were placed in a STEM class that integrated math instruction with science, technology, and engineering content. The mean TOMA-2 score for the LA-ELL group was at the 6th percentile.

Twenty-one of the LA participants were fluent English speakers (hereafter referred to as LA-FES), meaning that they had never received ELL-related services or that they had exited from ELL designation based on passing the state test for this purpose. The majority of the 21 LA-FES students were in an intensive math

class ($n = 17$) and the rest in the STEM class ($n = 4$). The mean TOMA-2 score for LA-FES group was at the 3rd percentile.

Eight students in the sample were in an algebra class, which was considered to be advanced math for their grade/age ($n = 1$ in seventh grade; $n = 7$ in eighth grade). All of the participating algebra students were FES (ALG-FES with a mean TOMA-2 score at the 72nd percentile. Because there was only one ELL in the algebra class (i.e., considered higher-achieving), we excluded this student's interview data from our sample.

3.2 Procedure

A typical lesson was selected from a seventh grade textbook in the district's adopted math series, and a structured interview protocol was created based on the problem types and text features from the lesson. The sixth through eighth grade district-adopted math textbook series shared the same format and structure. Therefore, the students were not necessarily familiar with the math content of the selected lesson, but they were familiar with the way the lesson was structured; how it proceeded from page to page; and how sidebars, highlighted font, and other text features are used for each lesson. Although most texts in the U.S. follow the exposition—examples—exercises structure, the adopted textbook used by the participating teachers and students started with a worked-out word problem. Individual interviews were conducted with participating students using a structured protocol to ask if they paid attention to each text feature or revisited each problem on every page of the lesson when working independently using their textbook. As a follow up question, we asked the students why they thought they did or did not pay attention or would/would not revisit that text feature or problem type (i.e., *How is this helpful?*). The students were not asked to work out math problems or explain concepts during the interviews. Examples of questions/prompts from the protocol about the structure of the text (e.g., worked out problem, practice problem) include: *“Now let's look at a different section on this page. Do you read or look at X without your teacher asking you to do so?”* If so, the interviewer prompted with, *“When or why would you?”* If the student stated that the section was helpful, the interviewer asked, *“How does it help you?”* An example from the protocol asking about text features stated, *“Headers, bolded or highlighted font, pictures, and boxes with problems inside are all text features used in math books. Which of these text features help you when you are reading your math book without your teacher helping or asking your class to look at them?”* Then the interviewer would point out each feature individually, according to what was on each of the selected textbook pages (i.e., *“Does the highlighted text or words help you when you are working without your teacher or help from someone?”*). If the student responded positively, the interviewer prompted, *“How does that text feature help you?”*

Not all of the participating students articulated why the text features or problems were helpful or unhelpful, so our results report the responses that students clearly

articulated as helpful or not helpful. When possible, we included elaborations on those responses that explained why students stated they would or would not go back to the problem or did/did not pay attention to the text feature. A total of 45 students (as previously described) were interviewed, with each interview lasting approximately 35–40 min.

3.3 Data Analysis

After transcribing the interviews, two researchers worked together initially and then separately, coding line by line to develop and verify open codes that emerged from the students' responses (Glaser & Strauss, 1967). We then revisited and articulated more than half of each other's coded transcripts. Categories and themes emerged for the LA-ELL, LA-FES, and ALG-FES groups based on codes from students' elaborated responses for helpfulness (or unhelpfulness) for each problem type and text feature (Strauss & Corbin, 1998). Both researchers articulated their coded findings until agreement was reached to finalize the coding themes across and within groups.

3.3.1 Helpfulness and Active Versus. Passive Reading

Upon reviewing the analyzed data, we found it helpful to use Shepherd's (2005) work on active versus passive reading of mathematics as a framework to further analyze these data and present the results. We build on Shepherd's work and define *active reading* of mathematics as the reader actively attempting to make meaning from the text to build understanding of mathematics knowledge (e.g., reading that *further*s understanding of lesson concept; *makes connections* across problems/problem types, real world applications). It is a far-sighted view of reading math that emphasizes creating meaning and building on prior knowledge to reason through problems. On the other hand, *passive reading* is a focus on getting through the mathematical text for the primary purpose of solving a task or problem (i.e., focus on a solution over understanding of the solution process) rather than actively making meaning from a text in a holistic way. It is a myopic or compartmentalized view of reading math by which the reader relies less on reading for meaning and understanding of mathematical concepts, which could result in plugging in numbers to an algorithm without much use of reasoning through problems. According to our definition of active and passive reading, we defined what this may look like for the text features and selected problem types based on Shepherd's work (Table 3). We then separated the themes for each text feature and problem type into two groups. For themes that indicated action was taken to promote understanding of

the mathematics concept while using their textbooks independently, we designated active characteristics. For themes that demonstrated a passive stance (recipient of information), we designated passive characteristics.

3.4 Results

A summary of results for selected text features¹ and themes indicating “helpfulness” can be found in subsequent tables. The selected text features and problem types are reflective of what was mentioned in the literature as utilized by students when working in their mathematics textbooks (i.e., headers, font variation, vocabulary definitions, example, and practice problems). Shepherd (2005) in particular mentions the need to write in order to promote understanding of mathematics, so we also included the single problem from the textbook that requests students to write a description of two approaches to solve a problem and explain which they prefer. There were many common themes across groups as to how students reported these text features or problem types helpful or unhelpful when working independently, but there were also important differences when framing the themes using an active versus passive reading stance. Table 1 provides helpful themes from selected text features across groups, and Table 2 lays out helpful themes from selected problem types across groups.

3.5 Reported Themes for Text Features

3.5.1 Font Variation

All groups saw the function of varying font as providing important information. LA student responses indicated they used bolded font to *direct* them, and the ALG-FES students indicated bolded font *guided* them. Only two LA-ELLs stated they did not pay attention to font variation when reading the math textbook; otherwise, the responses to questions concerning this text feature demonstrated it was helpful for all students when reading mathematics textbooks.

3.5.2 Headers

All three subgroups of students saw headers as important for reading math textbooks because the headers told what the section was about and what the reader would be

¹Space limitations preclude us from presenting results for all text features and problem types of the lesson.

Table 1 Themes based on “Helpful” responses per selected text features across groups

Text feature and description of use in mathematics textbook	Algebra FES	LA-FES	LA-ELLs
<p>Font Variation (Color, Size, Bold): Font variation was used to call attention to directions for completing different sections, example problems or tasks and steps used to solve them, headers for each section, sidebar information, and cross references to other textbook sections with similar problems</p>	<p>Tells important information; Serves as a guide</p>	<p>Tells important information Important to learn what lesson is about Bolder font indicates exact procedures and supports understanding</p>	<p>Tells important information Tells you what to do (i.e., directions to follow)</p>
<p>Headers: Labeled each section of the textbook (i.e., Example, Practice, Real World Application)</p>	<p>Tells you what you'll be doing</p>	<p>Tells what you're going to learn/do, what section is about</p>	<p>Explains what you are learning</p>
<p>Highlighted Font:</p>	<p>Writes or reads vocab specifically to learn and use</p>	<p>Provides words for lesson</p>	<p>Identifies examples Vocabulary is important-it helps to understand what lesson is about</p>
<p>Vocabulary specific to understanding the math content for each lesson was highlighted and defined within the text</p>	<p>Helps understanding Must be important since highlighted</p>		<p>Words we need to learn</p>

Table 2 Themes based on “Helpful” responses per selected problem types across groups

Problem type and description	Algebra FES	LA-FES	LA-ELLs
Worked Examples	Reference for solving other problems	Reads repeatedly to better understand how to solve problems	Enhances understanding when unsure how to solve problems
	Helps independent problem-solving	Previews the lesson and test	Previews the lesson
	Copies methods in notes and refers to them	Reference for solving other problems independently	Provides practice
	Reviews often to further understanding	Shows how to solve the problems	Shows how to solve the problems
Semi-Worked Examples	Challenges you because requires independent set-up of equation	Provides reference for other problems	Helps understanding of real world applications of mathematics
	Provides opportunity of real world application	Relates to real world situations	Provides an example
	Provides practice	Provides example for other problems	Provides practice
Procedural Practice Problems	Checks understanding	Provides practice to deepen math understanding	Supports/checks understanding
		Checks understanding	Draws connections from examples to complete practice problems
Applied Practice Problems	Provides practice related to example	Provides more practice	Provides practice for test
	Checks understanding	Independent solving to check understanding in relation to other problems	Supports/checks understanding of the lesson
		Helps determine which method to use to solve	Helps determine which method to use to solve
		Deepens understanding of word problems and lesson	Draws connections from examples to complete the word problems
Write Math	ALG-FES students did not find this helpful overall	Deepens understanding when math told through statements and words	Prepares you for a test

doing. The LA-ELLs also reported headers as helpful because the headers identified examples. The only reports of headers not being helpful (i.e., did not pay attention to headers) came from the same LA-ELL students that did not find font variation helpful.

3.5.3 Highlighted Font-Vocabulary

All groups indicated they understood the importance of vocabulary in comprehending mathematics texts and concepts. There were no reports of highlighted vocabulary being unhelpful across groups (Table 2).

3.6 Reported Themes for Problem Types

3.6.1 Worked Examples

The *Worked Examples* were problems with explicit steps completed for worked solutions. These appeared to be most helpful for all students across groups, who reported the worked examples helped with understanding how to solve other problems in the chapter. Both FES groups reported these problems were used as a reference for others, and the ALG-FES group indicated they reviewed the worked examples often to help them understand the lesson. Both LA groups stated the problems provided a preview of the lesson, and FES students also added that the worked examples served as a preview of the test. Only a few students in each LA group did not find these problems to be helpful and reported they did not read them. Additionally, the LA-ELL group reported they would need teacher assistance to understand and did not find these problems helpful when working independently.

3.6.2 Semi-worked Example

When students were asked about a semi-worked out problem (partially worked solutions to scaffold parts of the solution) requiring them to set up the equation for solving (i.e., real world example), they all believed this type of problem supported their understanding for other parts of the chapter and provided opportunities to apply mathematics to real world situations. Interestingly, there were a couple in the ALG-FES group who reported reading this problem type but not finding these problems helpful. This was because the semi-worked examples required too much effort to set up the equation to solve, or the students simply disregarded the problem unless directed to it by their teacher.

3.6.3 Procedural and Applied Practice Problems

Procedural Problems had limited amounts of written words, were similar and adjacent to worked examples, or were provided in a section for students to practice and check content understanding. *Applied Practice Problems* were general word or story problems requiring students to apply procedures to a specific situation in order to solve the problem. All students across groups agreed that both kinds of practice problems provided practice and opportunities to check understanding. Some students also mentioned that both kinds of problems prepared them for tests (LA-ELL) or deepened students' understanding through independent problem solving opportunities (LA-FES). When explicitly asked about which problems students preferred (*Procedural* vs. *Applied* or word problems), all students preferred the *Procedural Problems* in this section but with different rationales. The LA-FES students stated that they relied on teacher assistance or an example to follow in order to solve word problems, and they felt more independently able to solve procedural problems. The LA-ELL students specifically mentioned that word problems were more difficult because they must "look at every word and detail" and, thus, preferred "less wordy" problems. A couple of the ALG-FES students reported they preferred procedural problems because these problems were more difficult to understand and, thus, challenged them. But, the majority of these students preferred procedural problems because word problems were more difficult to solve.

3.6.4 Write Math

The *Write Math* problem required students to describe two possible methods for solving problems in writing and explain which they prefer. The majority of students across all groups either did not find this problem type helpful (i.e., they did not read or pay attention to it) or did not articulate if it was helpful or not helpful. The LA-ELL group, who did report this problem to be helpful, stated that it prepared them for tests. A few of the LA-FES group stated that *Write Math* helped them better understand when math was told with words and statements. Most of the LA students in this sample were in participating Language in Math intervention classrooms and wrote more than the other students in the sample; this may have impacted their responses simply because most LA students (in the intensive classes) had more experience writing in math class than the other students. One ALG-FES student indicated that *Write Math* was helpful because it was good practice for using vocabulary in the explanation. Others who articulated why this problem wasn't helpful stated it was not reflective of real life to write in math or that they did not like to write (Table 3).

Students across groups demonstrated active and passive reading approaches in their responses (Table 4). The lower-achieving students had more themes categorized as a passive stance; however, when comparing the two lower-achieving student groups, overall the LA-FES students had fewer themes emerge in this category.

Table 3 Examples demonstrating active and passive reading characteristics of mathematics texts

Text feature or problem type	Active reading	Passive reading
Font Variation/Headers	Reads to understand what to do (directions)	Doesn't read or pay attention
Highlighted Font Vocabulary	Notices and pays attention to find examples and non-examples of the vocabulary word or concept;	Doesn't read or pay attention
	Makes connections to other math concepts or vocabulary	
Worked or	Reads actively (reads for understanding;	Plugs in steps from one problem without attempting to understand why steps make sense or are used
Semi-Worked Example, and Practice Problems	Copies example in notebook; writes notations;	
	Makes connections with other concepts/problems; Constructs own examples	
Write Math	Writes to clarify or understand more about math problem/solution process	Doesn't see value or benefit of writing in math

The LA-FES students in our sample also appeared to understand the purpose of text features (font variations, headers, highlighted font) when comparing responses and themes in those categories. Though the data demonstrate there were active readers across groups, the ALG-FES students did not have any statements identified to indicate there were passive readers within that group.

4 Summary, Discussion, and Implications

To summarize our main results: the *Worked Examples* were helpful for most students. However, the entire low-achieving (LA) sample had themes that indicated teacher assistance would be necessary in order for this problem type to be helpful when working independently, indicating more of a passive approach to reading. The *Semi-worked Example* did not have any themes indicating students read these problem types passively. It could be that the structure of the problem required them to read actively in order to solve it, which in turn promoted understanding. Themes from the *Procedural Practice* problems indicate that LA-ELLs read both actively and passively, and the FES groups demonstrated themes of active reading characteristics. The *Applied Practice Problem* category indicates word problems were difficult for the entire LA sample who relied heavily on support from others to solve; the ALG-FES group did not articulate much concerning word problems—except for the theme indicating they preferred the problems for the challenge they

Table 4 Themes demonstrating active and passive reading characteristics across groups

Text feature or problem type	Algebra FES		LA-FES		LA-ELLs	
	Active	Passive	Active	Passive	Active	Passive
Font Variation	Tells important information;	Serves as a guide	Tells important information;	Important to learn what lesson is about	Tells important information	Doesn't Pay Attention
	Serves as a guide		Bolded font indicates exact procedures and supports understanding		Tells you what to do (i.e., directions to follow)	
Headers	Tells you what you'll be doing		Tells what you're going to learn/do, what section is about		Explains what you are learning; Identifies examples	Doesn't Pay Attention
Highlighted Font	Writes or reads vocab specifically to learn and use		Provides words for lesson		Vocabulary is important-it helps to understand what lesson is about	
	Helps understanding; Must be important since highlighted				Words we need to learn	
Worked Examples	References for solving other problems		Reads repeatedly to better understand how to solve problems		Enhances understanding when unsure how to solve problems	Doesn't read
	Helps independent problem-solving		Previews the lesson and test		Previews the lesson	Would do this with teacher assistance

Semi-Worked Examples	Provides practice	Provides reference for other problems		Helps understanding of real world applications of mathematics	
	Checks understanding	Relates to real world situations		Provides an example	
	Challenging; requires effort to set up equation	Provides example for other problems			
Procedural Practice Problems	Provides practice	Provides practice to deepen math understanding		Provides practice	Reviews answers with teacher
	Checks understanding	Checks understanding		Supports/checks understanding	Doesn't pay attention
Applied Practice Problems	Provides practice related to example	Provides more practice	Relies on teacher	Supports/checks understanding of the lesson	Wouldn't attempt without teacher
	Checks understanding	Independent solving to check understanding in relation to other problems	Doesn't read or pay attention	Helps determine which method to use to solve	Provides practice for test
	Prefers word problems for challenge	Helps determine which method to use to solve	Too difficult	Draws connections from the word problems	More difficult because must pay attention to every word and detail
		Deepens understanding of word problems and lesson	Reads only when assigned by teacher		Doesn't read or pay attention
					Too wordy

posed. Finally, results from the *Write Math* problem show that this problem type was not thought of as helpful to promote understanding of math by most students. Other than the LA-ELLs who used the problem to prepare for a test, students indicated they did not see the value of writing for mathematics understanding. There was one theme from the LA-FES group indicating they did see the value of writing for math, probably due to the fact that the majority of these students had more experience writing during math class.

Student self-reports across groups, particularly those in the LA groups, as to why certain problem types were helpful or unhelpful when working independently indicate that the students are unaccustomed to reading mathematics for meaning and persevering through problems to solve them. LA students found the *Applied Practice* word problems to be especially difficult, and as a result, they must rely on teacher support to solve the problems. In contrast, at least some of the ALG-FES group preferred the challenge of word problems over *Procedural Practice* problems. The student self-reports also reflect their dispositions towards mathematics and what they perceived to be the purpose for learning math (i.e., solving a problem for a test vs. developing reasoning skills to solve problems in general).

Student self-reports about how they use and read their textbooks, along with why students perceive certain text features and problem types as “helpful” or “unhelpful,” point to the possibility that teacher mediation of the curriculum could be an important factor in assisting students’ to adopt more active reading behaviors to comprehend mathematics textbooks. Teacher scaffolding of active-reading behaviors may also impact students’ dispositions about the nature and purpose of learning mathematics as reported by Rezat (2009, 2013) and Weinberg et al. (2012). We use Rosenblatt’s (1978, 1982), Kintsch’s (1988), and Halliday’s (1978) theories to discuss how reading with a more active stance could be helpful in comprehending mathematics texts for understanding.

Rosenblatt’s (1978, 1982) theory treats reading as a two-way transactional process in which a reader and the text are central to the meaning-making process. According to Rosenblatt’s (1978, 1982) theory, the reader is to be an active participant in making learning meaningful, filling in the missing pieces of text with a variety of responses. Kintsch’s (1988) theory assumes that a given text is processed and represented at different levels: (a) at the propositional level, expressing semantic content (meaning) of the text, and (b) at the situational level, conceptualizing and creating a model of a situation based on explicit information with the support of prior knowledge. Textbase understanding is sometimes disjointed or incomprehensible, depending on linguistic complexity, and requires situational understanding to aid comprehension. Halliday’s (1978) notion of social semiotics describes language as a social phenomenon that is rooted in social and cultural contexts where lexicogrammatical (linguistic and grammar) choices are made; thus, this theory explicitly deals with linguistic complexity.

These combined reading theories call for a change in the way teachers approach mathematics instruction, using textbooks to support reading comprehension of mathematical textbooks *while* fostering and developing mathematical understanding. The structure of most mathematics textbooks in the U.S.

(exposition—examples—exercises) lends itself to this combined theoretical approach. However, the focus of what is mediated by teachers during instruction, using the exposition and examples sections of the textbook, is what we feel should change in order to comprehend textbooks, increase mathematics understanding, and meet more rigorous learning standards. Rezat's (2013) and Weinberg et al.'s (2012) work demonstrates how students approach the exercises section of mathematics textbooks is dependent upon how teachers mediate the exposition and example sections. Based on student self-reports, our results support their findings.

The first part of the lesson, the exposition portion of the mathematics textbook, was designed to build students' background knowledge of the mathematics domain. Previous studies indicate students do not pay attention to the exposition portion of mathematics textbooks (Rezat, 2009, 2013; Weinberg et al., 2012). The textbook pages we selected for our study did not have an exposition section, which was typical for this textbook. Instead, the lesson began with a word problem that included a worked example. Although the students reported the worked example to be helpful, the header and sidebar information (lesson objective/main idea, undefined vocabulary words, and written state standard) were the only references indicating what the lesson would be about and triggering prior knowledge. Though not reported here, students in our study generally did not pay attention to the lesson objective/main idea, undefined words, or state standard in the sidebar. Moreover, even if students reported that they *did* pay attention to those sections, these features would be of little use to activating prior knowledge in a meaningful way without teacher mediation for those with limited or no prior experiences with the mathematics concept—in much the same way that simply stating what the lesson is about does not help students without some prior knowledge or schema to build upon (Kintsch, 1988).

A common recommendation for mainstream mathematics instruction is for teachers to mediate the textbook's lesson introduction in order to ascertain students' prior knowledge and experiences (both within and outside of school) with the concept and lesson objective to drive their instruction. More time can be spent on building students' prior knowledge and conceptual understandings of the math content if the teacher discovers students are not familiar with the concept. Our results expand that recommendation to incorporate teacher attention to and mediation of the language structures used by the author to convey mathematical meaning, which otherwise may constrain students' comprehension of the text. ELLs specifically need support with the language of mathematics and academic discourse as they generally do not learn this outside of school (Zwiers, 2008, 2009).

The examples section of the textbook should also be approached differently so that teachers no longer emphasize the solving of problems only to find answers; our results suggest that students will use their books for the same narrow purpose. Instead, we recommend that teachers focus student attention on understanding the problem solving process in the hopes of expanding the reading practices of their students. If teachers would approach examples in mathematics textbooks as providing opportunities for students to use and develop their conceptual understanding and reasoning, we hypothesize that teachers would be more likely to scaffold how

students read examples and encourage reading more meaningfully rather than solely looking for the steps and procedures to follow in order to solve problems.

5 Conclusion

When coupled with previous research on how teachers mediate and how students use mathematics textbooks, our study's findings suggest that mathematics textbooks should be used differently than what seems to be regular classroom practice in order to better support all students' understandings of mathematics texts, particularly lower-achieving FES students and ELLs (Regis et al., 2006). We advocate a more active stance for reading mathematics in conjunction with instruction that meaningfully makes connections to students' previous experiences with the mathematics content and language features specific to mathematics texts. In so doing, we proposed the merging of three language comprehension theories (Halliday, 1978; Kintsch, 1988; Rosenblatt, 1978, 1982) to guide instruction using mathematics texts.

The combined theories emphasize the two-way transactional processing of texts at different levels (semantic and situational), based on lexico-grammatical choices of the author. Although more research in this area is needed, we hypothesize that a dual focus on reading for meaning and the comprehension of language used to convey meaning in and for mathematics could be scaffolded by teachers' mediation of the textbook to promote an active reading stance. This would help to build students' reasoning skills and develop dispositions that seek to understand and approach mathematics textbooks and problems for reasons beyond tests, graded assignments, and "school-related" motives. Ultimately, the mediation of mathematics textbooks by teachers using such an approach could enable communication of mathematics while developing disciplinary literacies and practices for deep understandings of the content that CCSS and other rigorous learning standards expect.

References

- Alvermann, D., & Moore, D. (1991). Secondary school reading. In R. Barr, M. Kamil, P. Mosenthal, & P. D. Person (Eds.), *Handbook of reading research* (Vol. II, pp. 951–983). New York, NY: Longman.
- Ball, D. L., & Feiman-Nemser, S. (1988). Using textbooks and teachers' guides: A dilemma for beginning teachers and teacher educators. *Curriculum Inquiry*, 18, 401–423.
- Barbu, O. C., & Beal, C. R. (2010). Effects of linguistic complexity and math difficulty on word problem solving by English learners. *International Journal of Education*, 2(2), 1–19.
- Barton, M. L., & Heidema, C. (2000). *Teaching reading in mathematics: A supplement to teaching reading in the content areas teacher's manual*. Aurora, CO: Mid-continent Research for Education and Learning. Retrieved from ERIC Document Reproduction Service ED 469 111.
- Bossé, M. J., & Faulconer, J. (2008). Learning and assessing mathematics through reading and writing. *School Science and Mathematics*, 108(1), 8–19.

- Brown, V. L., Cronin, M. E., & McEntire, E. (2009). *Test of mathematical abilities* (2nd ed.). Austin, TX: Pro-Ed.
- Buehl, D. (2011). *Developing readers in the academic disciplines*. Newark, DE: International Reading Association.
- Carter, T. A., & Dean, E. O. (2006). Mathematics intervention for grades 5–11: Teaching mathematics, reading, or both? *Reading Psychology*, 27, 127–146. doi:10.1080/02702710600640248.
- Chávez-López, O. (2003). *From the textbook to the enacted curriculum*. Unpublished doctoral dissertation, University of Missouri, Columbia.
- Conley, M. (2008). Cognitive strategy instruction for adolescents: What we know about the promise, what we don't know about the potential. *Harvard Educational Review*, 78(1), 84–106.
- Crandall, J., Dale, T. C., Rhodes, N. C., & Spanos, G. (1989). *English skills for algebra*. Englewood Cliffs, NJ: Prentice-Hall.
- Draper, R. J. (1997). Jigsaw: Because reading your math book shouldn't be a puzzle. *Clearing House*, 71(1), 33–36. doi:10.1080/00098659709599319.
- Draper, R. J. (2002). School mathematics reform, constructivism, and literacy: A case for literacy instruction in the reform-oriented classroom. *Journal of Adolescent & Adult Literacy*, 45, 520–529.
- Draper, R. J., Broomhead, P., Jensen, A. P., Nokes, J. D., & Siebert, D. (Eds.). (2010). *(Re)Imagining content area literacy instruction*. New York, NY: Teachers College Press.
- Draper, R. J., & Siebert, D. (2004). Different goals, similar practices: Making sense of the mathematics and literacy instruction in a standards-based mathematics classroom. *American Educational Research Journal*, 41, 927–962.
- Draper, R. J., & Siebert, D. (2010). Rethinking texts, literacies, and literacy across the curriculum. In R. J. Draper, P. Broomhead, A. P. Jensen, J. D. Nokes, & D. Siebert (Eds.), *(Re)Imagining content area literacy instruction* (pp. 20–39). New York, NY: Teachers College Press.
- Exner, G. (1996). *An accompaniment to higher mathematics*. New York, NY: Springer.
- Glaser, B. G., & Strauss, A. L. (1967). *The discovery of grounded theory: Strategies for qualitative research*. Chicago, IL: Aldine.
- Grouws, D., & Smith, M. S. (2000). Findings from NAEP on the preparation and practices of mathematics teachers. In P. Kenney & E. Silver (Eds.), *Results from the seventh mathematics assessment of the National Assessment of Educational Progress* (pp. 107–140). Reston, VA: National Council of Teachers of Mathematics.
- Halliday, M. A. K. (1978). *Language as a social semiotic*. London, UK: Edward Arnold.
- Heller, R., & Greenleaf, C. (2007). *Literacy instruction in the content areas: Getting to the core of middle and high school improvement*. Washington, DC: Alliance for Excellent Education. Retrieved online <http://all4ed.org/wp-content/uploads/2007/06/LitCon.pdf>
- Hoffert, S. B. (2009). Mathematics: The universal language? *The Mathematics Teacher*, 103, 130–139.
- Huang, J., & Normandia, B. (2008). Comprehending and solving word problems in mathematics: Beyond key words. In Z. Fang & M. Schleppegrell (Eds.), *Language and reading in content areas: Toward a linguistically informed secondary reading pedagogy* (pp. 64–83). Ann Arbor, MI: The University of Michigan Press.
- Jackson, K., Garrison, A., Wilson, J., Gibbons, L., & Shahan, E. (2013). Exploring relationships between setting up complex tasks and opportunities to learn in concluding whole-class discussions in middle-grades mathematics instruction. *Journal for Research in Mathematics Education*, 44, 646–682.
- Kersaint, G., Thompson, D. R., & Petkova, M. (2009). *Teaching mathematics to English language learners*. New York, NY: Routledge.
- Kintsch, W. (1988). The role of knowledge in discourse comprehension: A construction-integration model. *Psychological Review*, 95, 163–182.
- Langer, J. A. (2011). *Envisioning knowledge: Building literacy in the academic disciplines*. New York, NY: Teachers College Press.
- Lerman, S. (2007). Directions for literacy research in science and mathematics education. *International Journal of Science and Mathematics Education*, 5, 755–759.

- Love, E., & Pimm, D. (1996). "This is so": A text on texts. In A. Bishop, K. Clements, C. Keitel, J. Kilpatrick, & C. Laborde (Eds.), *International handbook of mathematics education* (pp. 371–409). Dordrecht, The Netherlands: Kluwer.
- McKenna, M. C., & Robinson, R. D. (1990). Content literacy: A definition and implications. *Journal of Reading*, *34*, 184–186.
- Metsisto, D. (2005). Reading in the mathematics classroom. In J. M. Kenny, E. Hancewicz, L. Heuer, D. Metsisto, & C. L. Tuttle's (Eds.), *Literacy strategies for improving mathematics instruction* (pp. 9–23). Alexandria, VA: Association for Supervision and Curriculum Development.
- Moje, E. B. (2008). Foregrounding the disciplines in secondary literacy teaching and learning: A call for change. *Journal of Adolescent & Adult Literacy*, *52*, 96–107.
- Moore, R. C. (1994). Making the transition to formal proof. *Educational Studies in Mathematics*, *27*, 249–266.
- Moren, E. (2000). *Categorizing different ways secondary school mathematics teachers use written materials for classroom work*. Doctoral thesis, King's College, University of London. Retrieved from <http://ethos.bl.uk>
- Moschkovich, J. N. (2010). *Language and mathematics education: Multiple perspectives and directions for research*. Charlotte, NC: Information Age Publishing.
- National Council of Teachers of Mathematics. (1991). *Professional standards for teaching mathematics*. Reston, VA: Author.
- National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.
- National Governors Association Center for Best Practices, Council of Chief State School Officers. (2010). *Common core state standards for mathematics*. Washington, DC: Author.
- Ogle, D. (2009). Reading comprehension across the disciplines: Commonalities and content challenges. In S. R. Parris, D. Fisher, & K. Headley (Eds.), *Adolescent literacy, field tested: Effective solutions for every classroom* (pp. 34–46). Newark, DE: International Reading Association.
- Olson, M. R., & Truxaw, M. P. (2009). Preservice science and mathematics teachers and discursive metaknowledge of text. *Journal of Adolescent & Adult Literacy*, *52*(5), 422–431. doi:10.1598/JAAL.52.5.6.
- Österholm, M. (2006). Characterizing reading comprehension of mathematical texts. *Educational Studies in Mathematics*, *63*, 325–346. doi:10.1007/s10649-005-9016-y.
- Österholm, M. (2008). Do students need to learn how to use their mathematics textbooks? The case of reading comprehension. *Nordisk matematikdidaktick*, *13*, 53–73.
- Parris, S. R., Fisher, D., & Headley, K. (2009). *Adolescent literacy, field tested*. Newark, DE: International Reading Association.
- Pearson, P. D., & Fielding, L. (1991). Comprehension instruction. In R. Barr, M. Kamil, P. Mosenthal, & P. D. Pearson (Eds.), *Handbook of reading research* (Vol. II, pp. 815–860). New York, NY: Longman.
- Pepin, B., & Haggarty, L. (2001). Mathematics textbooks and their use in English, French, and German classrooms: A way to understand teaching and learning cultures. *Zentralblatt für Didaktik der Mathematik*, *33*, 158–175.
- Regis, T. P., Appova, A., Reys, B. J., & Townsend, B. E. (2006). What role do textbooks play in U.S. middle school mathematics classrooms? *Taiwan Journal of Mathematics Teachers*, *5*, 10–20. Retrieved from <http://mathforum.org/regis/REGIS-Taiwan.pdf>
- Reys, R. E., Reys, B. J., Lapan, R., Holliday, G., & Wasman, D. (2003). Assessing the impact of standards-based middle grades mathematics curriculum materials on student achievement. *Journal for Research in Mathematics Education*, *34*(1), 74–95.
- Rezat, S. (2009). The utilization of mathematics textbooks as instruments for learning. In V. Durand-Guerrier, S. Soury-Lavergne, & F. Arzarello's (Eds.), *Proceedings of the 6th congress of the European society for research in mathematics education*. Lyon, France. Retrieved from <http://ife.ens-lyon.fr/publications/edition-electronique/cerme6/wg7-22-rezat.pdf>

- Rezat, S. (2013). The textbook-in-use: Students utilization schemes of mathematics related to self-regulated practicing. *Zentralblatt für Didaktik der Mathematik*, 45, 659–670.
- Rosenblatt, L. M. (1978). *The reader, the text, the poem*. Carbondale, IL: Southern Illinois University Press.
- Rosenblatt, L. M. (1982). The literary transaction: Evocation and response. *Theory Into Practice*, 21(4), 268–277.
- Schell, V. (1982). Learning partners: Reading and mathematics. *The Reading Teacher*, 35, 544–548.
- Schleppegrell, M. J. (2007). The linguistic challenges of mathematics teaching and learning: A research review. *Reading & Writing Quarterly*, 23, 139–159. doi:10.1080/10573560601158461.
- Secada, W. G., & Avalos, M. A. (2010–2013). *Language in math*. Washington, DC: U. S. Department of Education, The Institute of Education Sciences. Retrieved from <http://ies.ed.gov/ncer/projects/grant.asp?ProgID=59&grantid=1034>
- Secada, W. G., Zisselsberger, M., Langer-Osuna, J., & Avalos, M. A. (2011). Developing teachers' repertoires for language in the mathematics classroom. In M. Setati, T. Nkambule, & L. Goosen (Eds.), *Proceedings of the ICMI study mathematics and language diversity*. São Paulo, Brazil: International Commission on Mathematical Instruction.
- Shanahan, C., Shanahan, T., & Mischia, C. (2011). Analysis of expert readers in three disciplines: History, mathematics, and chemistry. *Journal of Literacy Research*, 43, 393–429. doi:10.1177/1086296X11424071.
- Shanahan, T., & Shanahan, C. (2012). What is disciplinary literacy and why does it matter? *Topics in Language Disorders*, 32(1), 7–18. doi:10.1097/TLD.0b013e318244557a.
- Shepherd, M. D. (2005). Encouraging students to read mathematics. *Problems, Resources, and Issues in Mathematics Undergraduate Studies*, 15, 124–144.
- Siebert, D., & Draper, R. J. (2008). Why content-area literacy messages do not speak to mathematics teachers: A critical content analysis. *Literacy Research and Instruction*, 47, 229–245. doi:10.1080/19388070802300314.
- Siebert, D., & Hendrickson, S. (2010). (Re)Imagining literacies for mathematics classrooms. In R. J. Draper, P. Broomhead, A. P. Jensen, J. D. Nokes, & D. Siebert (Eds.), *(Re)Imagining content-area literacy instruction* (pp. 40–53). New York, NY: Teachers College Press.
- Siegel, M., Borasi, R., & Smith, C. (1989). A critical review of reading in mathematics instruction: The need for a new synthesis. In S. McCormick & J. Zutell (Eds.), *Cognitive and social perspectives for literacy research and instruction, thirty-eighth yearbook of the National Reading Conference* (pp. 269–277). Chicago, IL: National Reading Conference.
- Siegel, M., & Fonzi, J. (1995). The practice of reading in an inquiry-oriented mathematics class. *Reading Research Quarterly*, 30, 632–673.
- Sizer, T. R. (1984). *Horace's compromise: The dilemma of the American high school*. Boston, MA: Houghton Mifflin.
- Spanos, G., Rhodes, N., Dale, T., & Crandall, J. (1988). Linguistic features of mathematical problem-solving: Insights and applications. In J. P. Mestre & R. R. Cocking (Eds.), *Linguistic and cultural influences on learning mathematics* (pp. 221–240). Hillsdale, NJ: Lawrence Erlbaum.
- Strauss, A., & Corbin, J. (1998). *Basics of qualitative research: Techniques and procedures for developing grounded theory*. Thousand Oaks, CA: Sage.
- Valverde, G. A., Bianchi, L. J., Wolfe, R. G., Schmidt, W. H., & Houang, R. T. (2002). *According to the book: Using TIMSS to investigate the translation of policy into practice through the world of textbooks*. Dordrecht, The Netherlands: Kluwer.
- Van Garderen, D. (2004). Reciprocal teaching as a comprehension strategy for understanding mathematical word problems. *Reading & Writing Quarterly*, 20, 225–229. doi:10.1080/10573560490272702.
- Weinberg, A., Wiesner, E., Benesh, B., & Boester, T. (2012). Undergraduate students' self-reported use of mathematics textbooks. *Problems, Resources, and Issues in Mathematics Undergraduate Studies*, 22, 152–175. doi:10.1080/10511970.2010.509336.

- Wright, W. E., & Li, X. (2008). High-stakes math tests: How No Child Left Behind leaves newcomer English language learners behind. *Language Policy*, 7, 237–266. doi:[10.1007/s10993-008-9099-2](https://doi.org/10.1007/s10993-008-9099-2).
- Zisselsberger, M., Avalos, M. A., & Secada, W. G. (2012, April). *Operationalizing a linguistic framework for math texts to inform and improve instruction for English learners*. Paper presented at the 2012 annual meeting of the American Educational Research Association, Vancouver, BC, Canada.
- Zwiers, J. (2008). *Building academic language: Essential practices for content classrooms*. San Francisco, CA: Jossey-Bass Publishers.
- Zwiers, J. (2009, April). *Academic conversation assessment: A window into oral academic language proficiency, communication skills, and content understanding*. Paper presented at the 2009 annual meeting of the American Educational Research Association, San Diego, CA.