

# Agent Based Exploration of Urban Economic Dynamics Under the Rent-Gap Hypotheses

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**Abstract.** We present a stylised agent-based model of housing investment based on the rent gap theory proposed by the late Neil Smith. We couple Smith's supply-side approach to investment, with individual-level residential mobility within a city. The model explores the impact of varying levels of capital flowing in the city and reproduces certain theorised and observed dynamics emerging from the cyclic nature of investment: the tendency of capital to spatially concentrate generating intra-urban inequalities, the occasional formation of persistent pockets of disinvestment and phenomena such as gentrification.

## 1 Introduction

The model presented in this paper is an attempt at approaching urban dynamics integrating structural, supra-individual factors that are sometimes overlooked by modellers in favour of a purely bottom-up vision of cities and their evolution. The tools of complexity science have proven particularly well suited to explore urban dynamics as bottom-up phenomena, as seminal research from Schelling [16] onwards [2], testifies. However the focus on bottom-up generative modelling, centred on individual or household-level agents as the main actors, which is prevalent in most models of residential mobility [11], has the risk of underestimating the broader economic processes that impact the urban form and constrain individual behaviour. A traditional line of research in human geography that has seen recent revival [9, 19] sees the socio-spatial phenomena that shape contemporary cities - suburban sprawl, income segregation, gentrification - as consequences of the varying influx of capital towards urban systems, as opposed to strictly originating from individual-based residential choices. In this work we encode one of the most prominent structuralist theories of housing investment, the rent-gap theory, and couple it with considerations about residential location and cultural transmission, to balance top-down and bottom-up dynamics. The purpose is to build a simple abstract model that integrates the two visions and is capable of reproducing some of the urban dynamics that shape our cities and highlight the structural factors that may be contributing to their emergence.

The model represents a city composed of three layers: (a) the city’s infrastructure; (b) human agents that move through it, interact and influence each other; (c) economic forces that impact on both components, in the form of capital seeking to profit from housing renovation. The model was conceived and designed to investigate two aspects of the relationship between the three components: (1) the economic and spatial dynamics emerging from the interaction between investment/disinvestment cycles and residential mobility patterns; (2) the impact of such dynamics on the city’s cultural fabric - specifically the conditions of emergence and dissolution of pockets of culturally peculiar areas within a city. Due to space constraints this paper will focus only on the first issue, referring to a future article for an extensive joint treatment of the two aspects.

In the next section we briefly discuss the rent-gap hypothesis of housing investment, which informs the economic layer of the model, in Sect. 3 we describe our model of housing investment/residential mobility and cultural exchange. Section 4 presents the outcomes of the model: we will discuss the emerging effects of the spatial distribution of investments and analyse the phenomenon of inner cities decline and subsequent gentrification (Sect. 4.1).

## 2 The “Rent-Gap Theory” and Its Computational Implementations

The theoretical framework that inspires our representation of the economic forces operating in the city is the *rent-gap theory* (RGT): a supply-side approach to housing investment proposed by the late Neil Smith [18], specifically for the study of the phenomenon of gentrification. In Smith’s terms the rent-gap is

the difference between the actual economic return from the rights to use the land that is captured given the present land use and the maximum economic return that can be captured if the land is put to its highest and best use

The gap between *actual* and *potential* economic return is due to progressive decline in maintenance which properties undergo, together with changes in technologies which render dwellings obsolete. Restoration or rebuilding increases the economic return that a portion of land or a dwelling generates, bringing it to the maximum possible. The locations with the highest difference between *actual* and *potential* economic return will be the ones more likely to attract investment capital and be put to “highest and best use”. According to Smith this simple principle explains the sudden inflow of capital towards neglected inner city areas, and the subsequent change of socio-economic composition experienced by such areas, a phenomenon witnessed by US cities since the late 1970s. Although the rent gap theory was proposed to explain a specific phenomenon, gentrification, in our view it can serve as a good conceptualisation of general housing investment behaviour, suitable for a broad exploration and not incompatible with other approaches, including standard economic theory, as pointed out in [4]. A lengthy dispute on the validity of the rent-gap approach took place in the ’90s [5]. The critics pointed out that the notion of “potential economic return

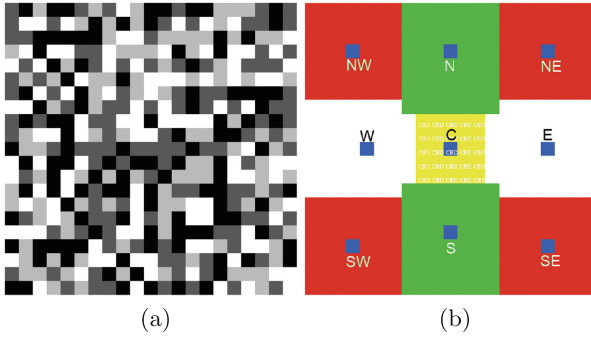
under the best use” is a shading concept, difficult to quantify, and therefore the prediction capabilities of the theory are hampered. Such criticism is far from unjustified, as we will show in Sect. 4.1. Nonetheless, the rent-gap theory proved particularly appealing for computational modelling, where the problem of identifying the *highest and best use* has been addressed by employing the notion of *neighbourhood effect*. Here, the highest possible revenue achievable by a given property after redevelopment is bounded by the average (or maximum, in some implementations) price charged in the vicinity of the redeveloped property, so that, irrespective of the state of the property, the maximum obtainable rent or sale price is practically determined by the overall state of the neighbourhood. This intuition embeds the principle that the state of the surroundings strongly affects a property and builds into the model the “*location! location! location!*” mantra that is familiar to property investors. Such interpretation was proposed by [13] in his abstract, pure cellular automation model of gentrification - the very first computational model to implement a variant of the RGT. Subsequent work [7,8] concentrated on the supply side of an abstract housing market implementing the RGT with a finer-grained set of agents (property units, owner-occupiers, landlords, tenants and developers) and investment capital modelled as an exogenous factor. The authors tested different levels of capital and observed variations in the average price of properties and the share of under maintained properties in the city. This model is to date the most complete implementation of the mechanics of the RGT, although it lacks any consideration of the demand-side of the housing market.

### 3 The Model

The model proposed here implements an entire city, with multiple pre-defined districts. Such an implementation allows exploration of the spatial dynamics emerging from capital circulation at a more fine-grained level and to implement some demand-side dynamics, such as district-level cultural allure. The entities represented in the model are: (a) individual locations (residential properties), defined by their value and repair state; (b) individual agents that represent households, characterised by an income, mobility propensity and cultural configuration; (c) economic forces, represented in the form of exogenous “capital” level, aiming at profiting from redevelopment/restoration of residential locations. Each of the three aspects is described in detail in the following subsections.

#### 3.1 City Structure and Economic Dynamics

We represent a city as a  $21 \times 21$  square grid of 441 residential locations (Fig. 1a) characterised by a value  $V$  and a maintenance level, or repair state,  $r$ , grouped in 9 districts (Fig. 1b).  $r$  is initially set at random in the 0–1 range and  $V$  is set at  $V = r + 0.15$ . Dwellings progressively decay in their condition by a factor  $d = 0.0012$  assuming that, if unmaintained, a location goes from 1 to 0 (becomes inhabitable) in 70 years (1 simulation step = 1 month). In order to match the



**Fig. 1.** The city is composed of 441 residential locations, each with a maintenance level and an economic value, divided in nine neighbourhoods. The colour shade of locations represents maintenance state from white (best condition) to black (worst). Depicted in (a) is the typical model initialization with random values assigned. The nine neighbourhoods (C, N, NW, NE, E, W, S, SE, SW) have a local centre (b). The district boundaries are “soft”, they do not constrain the agents’ behaviour. Only when an *allure* emerges (see Sect. 3.2) is a district represented as a recognisable entity in the agents’ residential decision process (Color figure online).

theoretical assumption of a decline in property price over time, we set the value of the dwelling as decreasing by a depreciation factor of 0.02/year. We also assume that in case of prolonged emptiness of the dwelling (>6 steps) both decay and depreciation factors are increased by 20%.

|            |            |            |
|------------|------------|------------|
| 0.88<br>a  | 0.532<br>b | 0.44<br>c  |
| 0.667<br>d | 0.726<br>e | 0.368<br>f |
| 0.689<br>g | 0.549<br>h | 0.74<br>i  |

**Fig. 2.** Example of price gap formation. The numbers represent locations’ value

The model represents investment in housing renovation/redevelopment as the fundamental economic force operating in the city. This is implemented by the “Capital” parameter,  $K$ , which represents the maximum number of locations that can be redeveloped in the current economic climate, expressed as a fraction of the total number of residential locations of the city, similarly to the approach proposed in [7]. A value of  $K = 0.02$ , as an example, would mean that every 12 steps  $441 * 0.02 = 8$  locations are invested upon and redeveloped in the city. A high level of  $K$  represents a large inflow of investment in the housing market which results in more locations being redeveloped and gaining value. The selection of the locations where the investment lands is carried out deterministically,

based on the value-gap of a location with the neighbouring properties, in accordance with the RGT discussed in Sect. 2. The relevant value gaps are determined in two ways, both in accordance with the *neighbourhood effect*, the principle that the amount of rent or the sale value attainable by a given location is always bounded by the characteristics and the desirability of the area where the property is located. We either set the new value  $nV$  of a redeveloped

property  $p$  at the neighbourhood average, plus 15% (representing a premium for a newly restored property) as in Eq. 1, or at the neighbourhood maximum (Eq. 2). As an example, the price-gap for location  $e$  in Fig. 2, is 0 if Eq. 1 is used ( $1.15 * [(0.88 + 0.532 + 0.44 + 0.667 + 0.368 + 0.74 + 0.549 + 0.689) / 8] < 0.726$ ), and 0.154 ( $0.88 - 0.726 = 0.154$ ) if computed with Eq. 2 (assuming that we are considering the Moore neighbourhood - the eight locations surrounding the central location  $e$  - instead of the whole district for comparison). Therefore, the method based on local maximum will generate a higher number of locations with a positive price-gap, that based on the average will have less, generating, as we will see, more concentration. We choose to test two alternative, but equally plausible, methods because they give rise to somewhat different outcomes, as shown in Sect. 4.1. In order to model the possible varieties of neighbourhood effect, we also consider a vicinity to be either the Moore neighbourhood of a location or the entire district that the location falls in, *whichever is bearing the highest values* and therefore grants the highest return for an investment.

$$nV_p = 1.15 * \max(\text{avg}(V_{moore}), \text{avg}(V_{district})) \quad (1)$$

$$nV_p = \max(\max(V_{moore}), \max(V_{district})) \quad (2)$$

The value gap for location  $p$  will be  $G_p = nV_p - (V_p + C)$ , or 0 if  $G_p < 0$ . Here  $C$  is the cost of removing the present resident if the location is occupied. Once a location is selected for investment its value is set at  $nV_p$  and its repair state is set at  $r = 0.95$ . Table 1 summarises the variables associated with location.

**Table 1.** Location variables

| Name      | Type/range   | Description                                    |
|-----------|--------------|--|
| <b>r</b>  | Float, {0,1} | Maintenance state                              |
| <b>v</b>  | Float, {0,1} | Value  |
| <b>G</b>  | Float, {0,1} | Value-gap: difference with neighbourhood value |
| <b>d</b>  | Integer      | Distance from the centre of town               |
| <b>te</b> | Integer      | Time empty                                     |
| <b>o</b>  | Boolean      | Occupied?                                      |

### 3.2 Agent Model: Cultural Exchange and Residential Mobility

Agents in the model represent individuals or households. They are endowed with an income level,  $i$  a mobility propensity  $m$  and a numeric string that represents their cultural configuration (Table 2). The agent's income level is set at random, normalised to the interval  $\{0,1\}$  and represents the highest price that the agent is able to pay for the right of residing in a property. The model,

**Table 2.** Agent variables

| Name      | Type/range    | Description                                    |
|-----------|---------------|--|
| <b>m</b>  | Float, {0,1}  | Mobility propensity                            |
| <b>c</b>  | List t=10,v=4 | Culture: memetic code                          |
| <b>i</b>  | Float, {0,1}  | Income level                                   |
| <b>d</b>  | Float, {0,1}  | Cognitive dissonance level                     |
| <b>th</b> | Integer       | Time here: steps spent in the current location |

ultimately, implements a pure rental market. The agent’s culture is modelled as a  $n$ -dimensional multi-value string of *traits*, inspired by Robert Axelrod’s classic agent-based model of cultural interaction described in [1] and originally applied to the urban context in [3]. The string represents an individual’s “memetic code”, or “cultural code”: an array of  $t$  cultural traits, each of which can assume  $v$  variations, giving rise to  $v^t$  possible individual combinations. In our model each trait is susceptible to change under the influence of other agents. Cultural influence is localised: agents that have been neighbours for more than 6 consecutive steps are likely to interact and exchange traits, thus rendering the respective cultural strings more similar. At the same time a cultural “cognitive dissonance” effect is at work, implementing a concept proposed by [14, 15] under the label of *spatial cognitive dissonance*: this is, roughly, the frustration of being surrounded by too many culturally distant agents. Similarity between two agents is the proportion of traits they share:

$$sim_{ab} = \frac{\sum_{i=1}^t xor(index(i, agent_a), index(i, agent_b))}{t} \quad (3)$$

Agents who spend more than six months surrounded by neighbours with few common traits ( $sim < 0.3$ ) increase their mobility propensity each subsequent time step. The mobility propensity attribute represents the probability that an agent will abandon the currently occupied location in the subsequent time step. This parameter is set at a low level in the beginning of the simulation, drawn from a Poisson distribution centred at  $m = 0.0016$ , meaning that, on average, agents have a 2% chance of moving each year. Mobility propensity is affected by the conditions of the currently occupied dwelling and the aforementioned cognitive dissonance level. One agent’s  $m$  is increased as follows:  $m_{t+1} = 1.5m_t$  in the following circumstances:

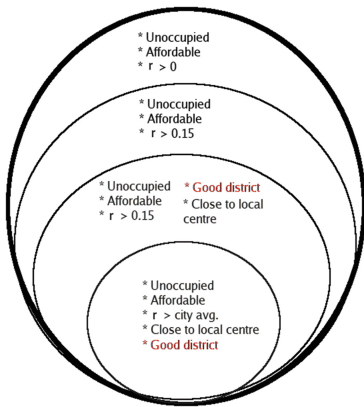
- After 6 months in a dwelling with  $r < 0.15$  (excessive time is spent in a dwelling in excessively bad condition)
- the cultural dissonance level exceeds a threshold for a period of 6 continuous steps.

A special circumstance is when the price of the dwelling currently occupied exceeds the agent’s income. In such case the agent is automatically put in

“seek new place” mode. This represents an excessive rent increase, unsustainable by the agent. The process of finding a new location is bounded by the agent’s income: a new dwelling has to be affordable ( $V \leq i$ ), in relatively good condition, and as close as possible to the centre of the district which contains it. The selection process is represented in Fig. 3. If no affordable and free location is to be found, the agent is forced to leave the city. As Fig. 3 shows, in certain cases the residential choice process of an agent includes the cultural configuration of the district as a factor. A special district-level variable called *allure* is set when the degree of cultural uniformity within a district exceeds a threshold, thus making the area recognisable for some of the features of its inhabitants. We measure cultural uniformity,  $u$ , as the average distance between the  $x$  agents residing in a certain neighbourhood.

$$pairs = \frac{x(x-1)}{2}$$

$$u = \frac{\sum_{i=1}^x \sum_{j=1}^{x-1} sim(agent_i, agent_j)}{t * pairs}$$



**Fig. 3.** The residential choice process. A dwelling has to be affordable, free and habitable ( $r > 0$ ) for an agent to consider moving into it. If these requirements are met, other characteristics are considered. If any district has developed an allure, agents who are relocating consider whether it suits them, based on a homophily preference. When no dwelling meets the out most requirements the agent leaves the city.

The allure of a district is represented as a string of cultural features, similar to that of individuals, where each element of the string is the most common value for that trait in the district population. A district’s allure is therefore an emergent feature of the model, which may or may not appear. This reflects the fact that not every neighbourhood has a special connotation visible to agents, but only those with a recognisable population do. The allure attribute can be thought of as the *reputation* of a neighbourhood in the eyes of agents. The attribute is sticky, after its emergence it is updated seldom and doesn’t necessarily reflect the current composition of a district, representing the fact that reputation is a nearly permanent feature, difficult to eradicate or to replace [6], a characteristic that applies to places’ as well as humans’ reputation. Once a district’s allure has emerged, agents include it in their residential decision under a homophily constraint: the agent will seek to move to a district with an allure similar to her culture string (Table 3).

## 4 Results and Discussion

We run the model for 1200 steps, representing a 100 years timespan, with the assumption of a constant value of  $K$  during the whole simulation time. We leave a systematic exploration of the parameter space to a later paper, here we focus our discussion on the parameter adjustments that

produce some observed urban dynamics. In the next paragraph we will focus on the spatial distribution of investments that different levels of  $K$  and the two systems of computing the rent-gap give rise to, then we will show how the spatial dynamics of capital valorization can determine the familiar phenomena of inner city decay and gentrification. In this model, as in the real world, capital has a dual role: a sufficient amount of capital is needed to ensure that a good proportion of properties in the city is maintained and habitable, but the nomadic nature of capital, which travels across the city in pursuit of the highest profit, generates shocks - in the form of abrupt spikes in prices and cycles of under-maintenance - which affect the ability of (especially least well off, who have limited choice) agents to stay in, or move to, the spot of choice. From this duality arise, ultimately, all the dynamics that we see occurring in the model.

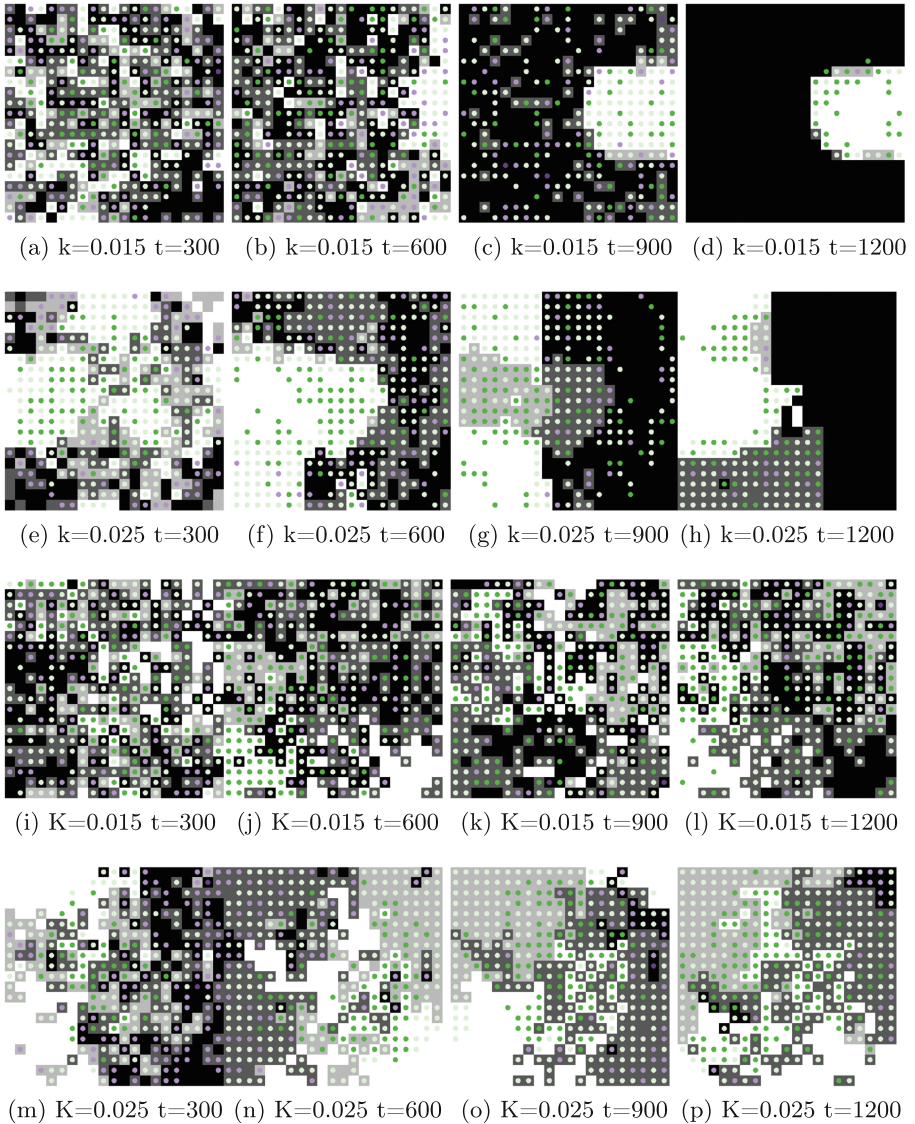
**Table 3.** District-level variables

| Name     | Type/range   | Description              |
|----------|--------------|--------------------------|
| <b>u</b> | float, {0,1} | Cultural uniformity      |
| <b>a</b> | list         | Allure (cultural makeup) |

### 4.1 Uneven Development: Spatial Dynamics of Capital and Pockets of Disinvestment

The first noteworthy dynamic produced by the model has to do with the distribution of the redeveloped locations in the city throughout the simulation. All simulation runs start with a random distribution of prices and maintenance conditions across the city: the situation at  $t = 0$  is similar to that represented in Fig. 1. We observe that, regardless of the price-gap computation mechanism, the model shows a tendency of capital to first concentrate spatially, and subsequently moving “in bulk” across the city, in pursuit of the widest gaps between actual and potential prices. The level of capital determines the speed and the scope of the process, that can involve only certain areas or the entire city. Figures 4, 5 and 6 represent the spatial evolution of maintenance conditions and the corresponding price dynamics for different levels of capital and for the two gap-setting mechanisms that we considered. As Fig. 4 shows, after an initial period during which the locations attracting investment are scattered throughout the city, strong clustering emerges, visible as wide white areas representing areas of high maintenance and high price. This happens because the locations receiving investment increase their value and, when a large enough number of locations is increasing value in a small area, the rent-gaps of neighbouring locations widen, making them more likely to attract further investment themselves, thus generating a feedback loop. However, as capital is limited, if investments

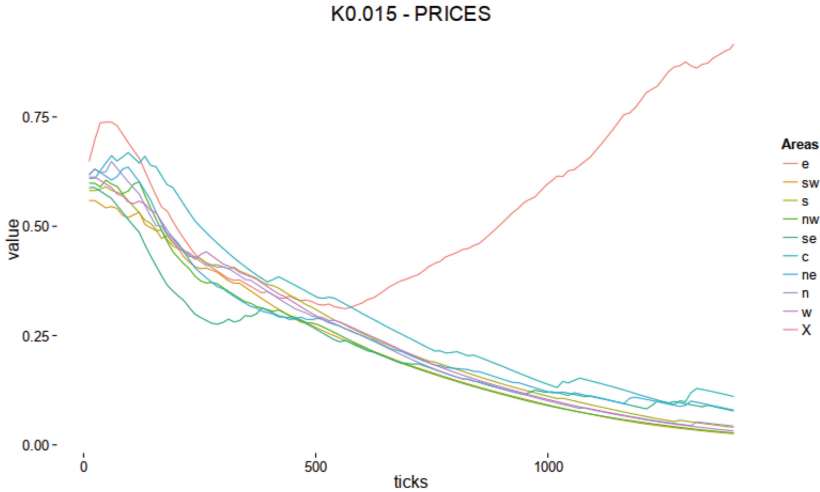




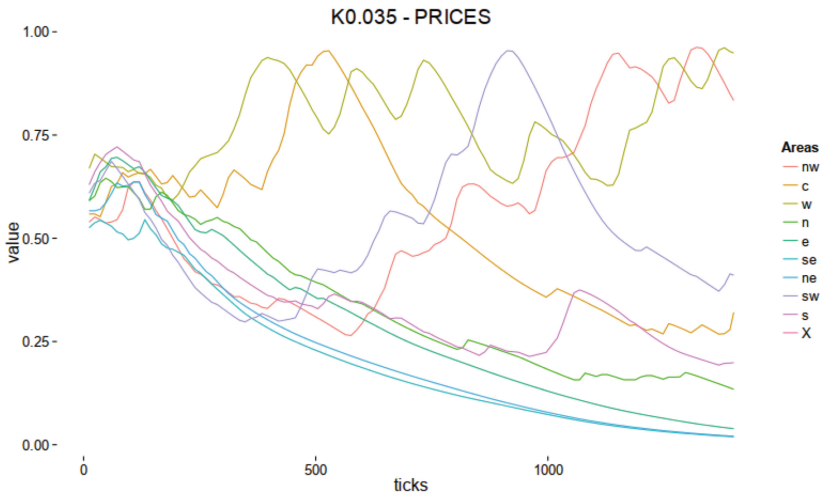
**Fig. 4.** Evolution of maintenance condition for different levels of  $K$  and price-gap setting mechanism. (a–h) is based on average, (i–p) on maximum local prices. The circles represent agents, colour represents income in 4 shades: dark green, light green, dark violet, light violet in decreasing order (Color figure online).

start to concentrate in an area, inevitably other areas experience neglect, and a phase of decline starts elsewhere in the city. The decline ends when the price-gaps become “competitive” again, which happens mostly when all the gaps are closed in the previously “successful” area, and provided that enough capital is

available. If so, investment moves away to settle in another area, generating the typical development cycles shown in Figs. 5 and 6, matching Neil Smith’s assertion that “urban development in capitalist economies tends to involve a cyclical process of investment, disinvestment and reinvestment”. The overall effect is that

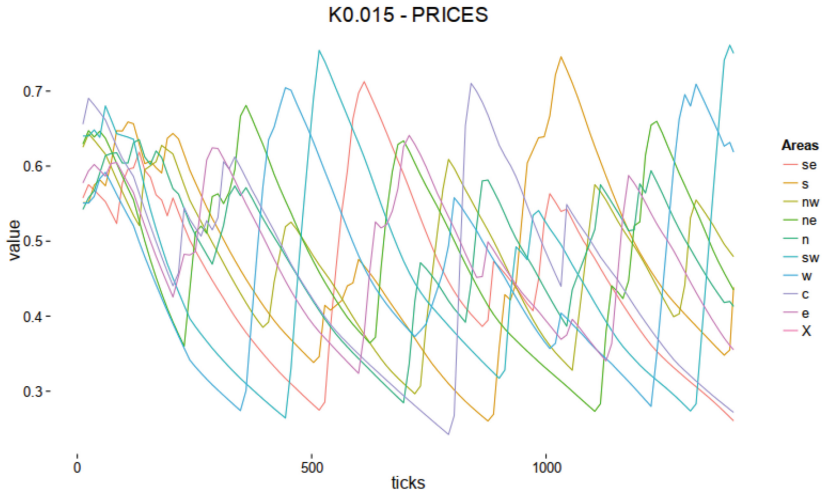
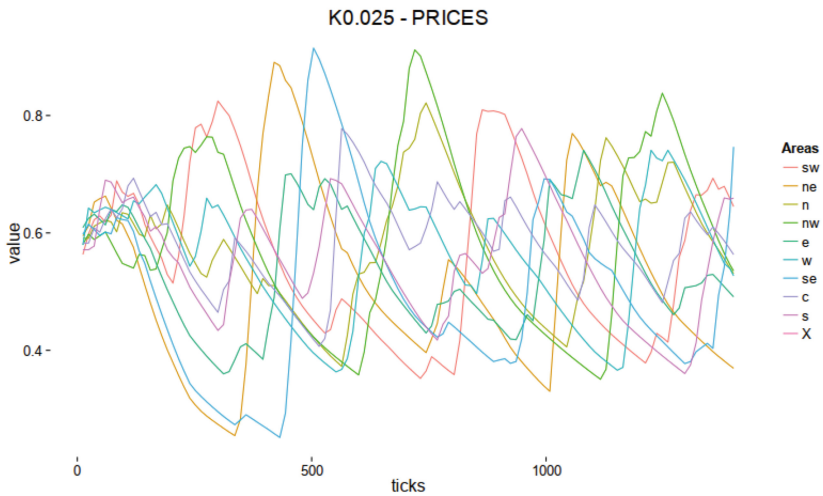


(a)  $K=0.015$ , average-based gaps



(b)  $K=0.025$ , average-based gaps

**Fig. 5.** Price dynamics by district for different levels of capital under average based gap-setting mechanism. The tendency towards concentration and the cyclic trend of investment, disinvestment and reinvestment are evident. In the case of average-based gap setting, higher levels of capital correspond to more districts being involved in the cycles.

(a)  $K=0.015$ , maximum based gaps(b)  $K=0.035$ , maximum based gaps

**Fig. 6.** Price dynamics by district for different levels of capital under maximum based gap-setting mechanism. When using maximum-based gaps, all the districts participate in the economic cycles even at lower prices of capital. Here higher capital corresponds to wider oscillations and higher prices

of white areas “moving” across the city from neighbourhoods with narrow price gaps to those with wider price gaps. The dynamics produced by the model are a powerful intra-urban depiction of what David Harvey calls the *spatial fix*, or “the need of capital to try and displace systemic pressures onto other geographical

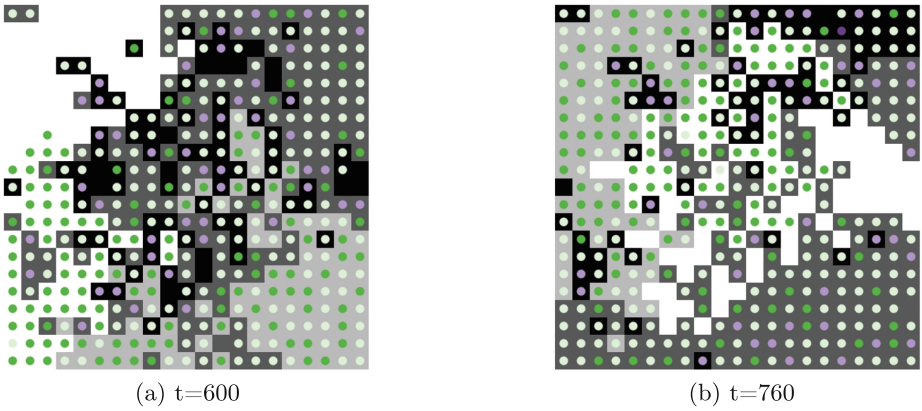
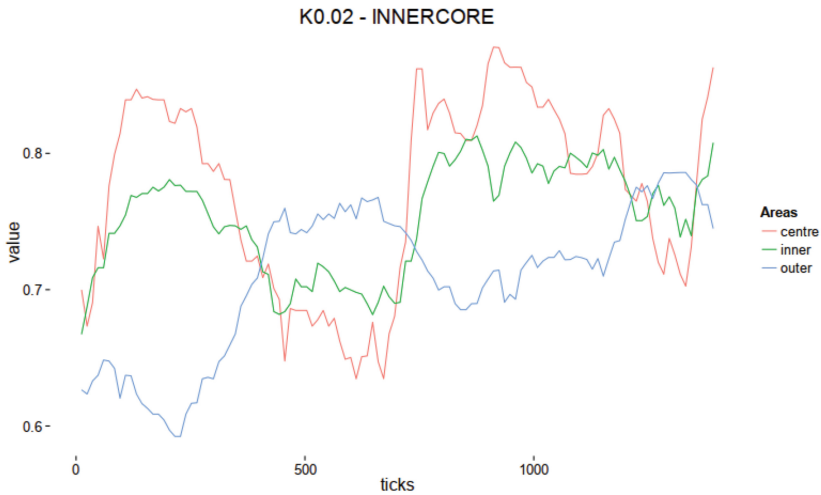
areas” [10]: when investment becomes unprofitable in an area, because the existing rent gaps do not grant enough yield any more, capital has to move to a new area. This mechanism is ultimately the source of unevenness in the development of different areas in the same city.

Utilising the neighbourhood maximum, instead of average, in the gaps-setting mechanism (i.e. using Eq. 2 in Sect. 3.1) generates a more fluid movement of capital that flows in the whole city even at low levels (Fig. 6), while the average has a constricting effect, due to the lower number of location developing a price gap, that concentrates the gaps - and therefore the profitable locations - in a limited area. In this case, for low level of capital only a limited set of districts are able to generate price-gaps wide enough to attract investment, and few districts participate in the economic cycles, while some others fall in permanent disrepair.

The fact that the model produces substantially different outcomes when using local maximum or local average as the price-gap setting mechanism - a difference not so fundamental, after all - seems to support the criticism that the RGT is too vaguely defined. It is also true that a clear-cut distinction between mean and maximum based price gaps is largely arbitrary. The two mechanisms could be at work at the same time in different areas of a city, for example responding to different demand levels: in popular, desirable areas an investor could charge the maximum local price for a restored property, whereas in areas of lower demand only the average could be successfully achieved. On the other hand, the model seems to disprove the argument of one of the fiercest critics of the rent-gap theory, Steven Bourassa. He pointed out that the existence of neighbourhoods which seem to never experience disinvestment contradicts the theory of the cyclical process of investment [4]. However the model shows that, in certain cases, a district can constantly achieve the highest rent-gaps within itself, and thus receive constant investment at the expense of the rest of the city, as is the case shown in Fig. 5a. Also, for higher levels of capital, the emerging cyclical process of investment generates oscillations of different magnitude in different areas, so that some areas never reach a substantial level of disinvestment.

## 4.2 Decay and Gentrification of the Inner City Core

One of the dynamics that have affected many cities in the Anglo-Saxon world for most of the 20th century is the slow decay of the inner core to the advantage of a sprawling and wealthy periphery. The “doughnut” cities have most of the wealth concentrated in the suburbs and an inner core in disrepair and populated by a low income, often predominantly immigrant, population. This tendency seems to have been reversed in the last decades, with the rapid gentrification of inner city areas. Most explanations of this phenomenon focus on the change of the social composition of cities and a consequent change of preferences in the younger population that now favours “city living” [12]. Another explanation sees this movement as supply, rather than demand, driven. It’s the position that Neil Smith advanced in his 1979 paper, titled in the most self-explanatory fashion, “gentrification: a back to the city movement by capital, not people” [17]. Figure 7c illustrates the emergence of this dynamic in the model: in this instance

(a)  $t=600$ (b)  $t=760$ 

(c) Income levels of residents of the central, semicentral and peripheral areas

**Fig. 7. Decay and gentrification of the inner city** ( $K=0.02$ , max-based price gaps). In (a) the “doughnut” is formed and visible: the centre of the city is in bad repair state and populated by middle-low income agents. In (b) capital *moves back to the city centre*. The decline of incomes in the central area is steady for the first 500 ticks (red line in (c)), with the corresponding rise of wealthy agents in the periphery. The process of gentrification lasts less than 100 ticks, then a new cycle starts (Color figure online).

agents have a preference towards living near the core of the city, nonetheless the trajectories of capital make the best housing available at the periphery of the city for a substantial amount of time, and therefore the wealthy agents concentrate in the suburbs. When investing in the centre becomes profitable again, the reverse movement materializes and the inner city gentrifies. While

the historic emergence of the doughnut effect took place in a phase of urban expansion, not implemented in this model which only considers a fixed urban area with immutable boundaries, the model suggests that a cyclical “doughnut effect” can emerge purely as a consequence of capital movements, without having to rely on demand-side explanations.

## 5 Conclusion and Future Work

The model presented here falls squarely in the near end of the continuum between abstract/pedagogic and realistic models. The main aim has been to implement in code the assumptions of a particular socioeconomic theory, the RGT, and employ the model to clarify and visualise certain mechanics that geographers had described in theory.

In related ongoing work we look at further implications of the original theory: some non immediately obvious consequences of capital circulation, i.e. those that affect the cultural look and feel of a city. The idea upon which this work is conceived is that the city is the product of agents of different nature and the stress on bottom-up emergence of phenomena should not over-represent the role of individuals and households. A good model of urban dynamics should include agents of different magnitude and account for the mutation of the micro-level *scenario*, or context, that often derives from processes unfolding independently, at the macro-level. The model shown here also serves as a basis for the development of a more realistic model, currently in the works: one that integrates a wider set of agents, the entire geography of an actual city as well as the income distribution of its residents and the maintenance state of its dwellings. The aim will be to test and validate theoretical predictions against actual data and, possibly, to highlight new implications and extend the theory.

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