Displacement Influence on Frequencies and Modal Deformations of a Sandwich Beam

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Abstract. Composite structures are used in the aerospace, stars and automotive especially the structures made of composite sandwich panels which are subject to vibration harmful sources of noise and mechanical failures. In this paper, we propose to analyze the influence of the shapes of displacement fields on the vibration behavior of a sandwich beam. For this energy method based on the minimum energy is used to achieve the equations frequencies and a sandwich beam modes. The kinetic and potential energies of the skins are, in turn, derived from the classical laminate theory. Several fields of polynomial movements are tested. Other parts will study the effects of rotational inertia, taking into account the bending energy of the body of sandwich NIDA. We analyzed the frequencies and modes based on different parameters. The experimental data are obtained for recessed-free conditions limits exciting near underrun using an impact hammer. The vibrational response is measured with a laser vibrometer. The natural frequencies are obtained experimentally by modal analysis. Numerical simulations complete this work for two types of sandwich Nomex paper and aluminum. The natural frequencies obtained from the theoretical formulation for numerical solution of the system are compared with experimental results and the results of numerical simulation. The very good agreement between the results shows that the model is correct.

Keywords: Displacement, energy, sandwich, Eigen frequency, modal deformation.

1 Introduction

Energy methods are among the most important methods to study the vibration behavior of beams. The problem with these methods is that they require the

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introduction of a field of kinematic ally admissible displacement or statically admissible stress or both at once.

The accuracy of the method will therefore depend on the right choice of these fields. In the literature, there are several choices of the field selection. But it is difficult to decide on their preferences depending on their application.

Mindlin [1] proposed a model to study the transverse shear of a thick sandwich plate isotope; this model is known by the theory of Mindlin.

Yarlagadda Lesieutre and [2] have developed an analytical method based on the theories of Rayleigh-Ritz to determine the influence of the change of bends orientation directions, temperature and thickness of the laminated plates on the Frequencies and the damping of the bending vibration of a composite plate.

Wang [3] compared the classical theories of thin plates with the theories of Mindlin thick plates then he studied the bending vibrations of a rectangular face and core isotropic sandwich panel.

Meunier and Shenoi [4] introduced the mechanical properties of the elements which form the sandwich plate in the analytical equations of elastic-viscoelastic model and the results of the differential equations give their frequencies and the factors influencing them.

Soula et al. [5] have studied the influence of the vibration on the kinematic behavior of laminated plates. They used three theories plates (Kirchhoff, Mindlin and Reddy) to estimate the natural frequencies and modal damping of symmetric and antisymmetric plates.

Maheri and Adams [6] extended the tests used for measuring the damping of stratified monolithic laminated beams in bending vibration for NIDA sandwiches beams. The contribution due to the damping components, in particular the fiber orientation of the skins, was considered. They compared their experimental results with theoretical study.

Banerjee [7] applied the Timoshenko theory on a beam using the coupling of bending and torsion and taking into consideration the rotational inertia. The characteristics of the composite are taken experimentally as if the material is homogeneous.

Nilsson et al [8] used equivalent homogeneous characteristics to study the bending vibrations of a sandwich beam from the Hamilton integral taking into consideration the transverse shear, the rotational inertia and distortion. They write the dispersion equation giving the wave number for the three modes according to the frequency for some boundary conditions and then they find the modal deformations and dynamic stiffness, the formulation is then validated by a test.

Blevins [9] presented a detailed list of formulas for determining the natural frequencies and their modes corresponding to various structure shapes.

On this paper, we study the influence of the displacement fields shape under different boundary conditions of a sandwich beam on the natural frequencies and modal deformations.

This study which is applied on two different beams shall give us more ideas about the influence of various fields on the modeling accuracy.

An experimental study of two sandwich beams also allows us to understand more this field by comparing the results of tests and calculations on the one hand and various tests according to the excitation and measuring points on the other.

2 Mathematical Formulation

We choose an n order polynomial transversal displacement depending of the constants (1..n = i-1) and the kinetic and potential energies are determined as follows. The potential energy of a sandwich beam is the sum of the bending elastic deformation energy of and traction of both upper and lower faces as well as the shear and bending energy of the body.

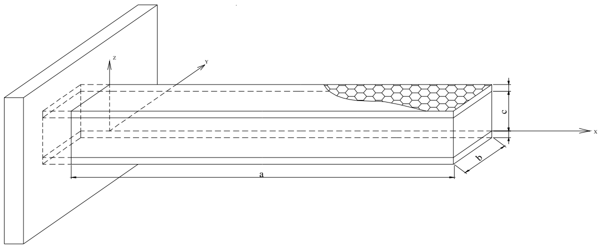

Fig. 1 Nida Sandwich beam

The elastic deformation of the lower side can be written as follows:

$$\varepsilon_{xB} = (z + \frac{f_l}{2})(\frac{\partial^2 W_c}{\partial x^2}) \tag{1}$$

$$\varepsilon'_{xB} = (z - c - \frac{f_u}{2})(\frac{\partial^2 W_c}{\partial x^2}) \tag{2}$$

$$V_{BF} = \frac{E}{2(1-\nu^2)} \int_{-f_l}^{0} dz \int_0^a (\varepsilon_{xB})^2 dx \tag{3}$$

With the same method we determine the energy of elastic bending deformation of the upper layer:

$$V'_{BF} = \frac{E}{2(1-\nu^2)} \int_{c}^{c+f_u} dz \int_0^a (\varepsilon'_{xB})^2 dx \tag{4}$$

To determine the tensile energy of one of the faces we must first determine the normal force to this surface and its extension in the parallel axis to the face.

N_{FL} : The normal Force at the lower face and N_{FU} : the normal force at the upper face

$$N_{FL}=\int_{-f_l}^{0}\sigma_f dz \quad (5) \qquad \text{And} \qquad N_{FU}=\int_{c}^{c+f_u}\sigma_f dz \quad (6)$$

The Formula of σ_f which is the stress in an arbitrary face of the fiber is as follows:

$$\sigma_f=-E(z-\frac{c}{2})(\frac{\partial^2 W_c}{\partial x^2}) \quad (7)$$

Substitute (7) in (5) and (6) we obtain:

$$N_{FL}=-\int_{-f_l}^{0}E(z-\frac{c}{2})(\frac{\partial^2 W_c}{\partial x^2})dz \quad (8) \qquad \text{And} \qquad N_{FU}=-\int_{c}^{c+f_u}E(z-\frac{c}{2})(\frac{\partial^2 W_c}{\partial x^2})dz \quad (9)$$

Using the integral we determine the normal forces at the both the upper and the lower faces:

$$N_{FL}=E\frac{f_l}{2}(c+f_l)(\frac{\partial^2 W_c}{\partial x^2}) \quad (10) \qquad N_{FU}=-E\frac{f_u}{2}(c+f_u)(\frac{\partial^2 W_c}{\partial x^2}) \quad (11)$$

V_{NF} : is the tensile energy of the lower layer of the normal force of this face.

ε_{xL} : The elastic deformation in the x direction of the lower face.

ε_{xU} : The elastic deformation in the x direction of the upper face.

$$V_{NF}=\frac{1}{2}\int_0^a N_{FL}\varepsilon_{xL}dx \quad (12) \qquad V'_{NF}=\frac{1}{2}\int_0^a(-N_{FL}).(-\varepsilon_{xL})dx \quad (13)$$

The bending elastic deformation energy of the bottom layer per unit width is as the following:

$$\varepsilon_{xL}=\frac{1}{2}(c+f_l)(\frac{\partial^2 W_c}{\partial x^2}) \quad (14) \quad \text{and} \quad \varepsilon_{xU}=-\frac{1}{2}(c+f_u)(\frac{\partial^2 W_c}{\partial x^2}) \quad (15)$$

Substituting (10) and (14) in (12) we obtain:

$$V_{NF}=\frac{E.f_l.(c+f_l)^2}{8}\int_0^a(\frac{\partial^2 W_c}{\partial x^2})^2dx \quad (16)$$

Substituting (11) and (15) in (13) we obtain:

$$V'_{NF}=\frac{E.f_u.(c+f_u)^2}{8}\int_0^a(\frac{\partial^2 W_c}{\partial x^2})^2dx \quad (17)$$

The body Shear energy (Vc) par unit width is as following:

$$V_c=\frac{1}{2}\int_0^c dz\int_0^a G_{XZ}\cdot(\gamma_{XZ})^2dx \quad (18)$$

With: $\gamma_{XZ} = \frac{1}{G_{XZ}} \cdot \tau_{XZ}$ (19) and $\tau_{XZ} dx = -\frac{\partial}{\partial x} N_{FL} dx$ (20)

The body bending energy is as following:

$$V_{FC} = \frac{1}{2}\int_0^a E_c.I.\left(\frac{\partial^2 W_c}{\partial x^2}\right)^2 dx \quad \text{With} \quad I = \frac{c^3.b}{12} \tag{21}$$

E_C : Young Modulus of core

From the previous formula we determine the total potential energy which is:

$$V = V_{CC} + V_{FC} + V_{NF} + V'_{NF} + V_{BF} + V'_{BF} \tag{22}$$

The kinetic energy of the vibration of the sandwich panel is equal to the sum of kinetic energy of translational and rotational kinetic energy.

$$T = E_{CT} + E_{CR} \tag{23}$$

Where: $E_{CT} = \frac{1}{2}\rho\int_0^a \left(\frac{\partial W}{\partial t}\right)^2 dx$ (24) And $E_{CR} = \frac{1}{2}\int_0^a I\left(\frac{\partial}{\partial t}\left(\frac{\partial W}{\partial x}\right)\right)^2 dx$ (25)

The vibration system is considered as conservative where A_i (i =1..n-1) are constants. The resolve of the obtained system allows us to determine (n-1) natural frequencies and their shape mode.

3 The Beam Results

First, we presented the mechanical and geometrical characteristic of test specimens analyzed

Table 1 Geometrical and mechanical characteristics of used specimen

	NIDA in Al	NIDA in NOMEX
Length (a) in mm	250	*250*
Width (b) in mm	53	*53*
Thickness core (c) in mm	5	*8*
Each layer thickness $f_u = f_l$ in mm	0.75	*0.5*
Density ρ in Kg/m3	573	*221*
E_c in Pa	130.10^6	$2.5\ .10^6$
E_p in Pa	70.10^9	$8.63\ .10^6$
G_{XZ} in Pa	5600.10^6	70.10^9
ν	0.33	*0.33*

Second, we take a beam where field checking the no translation and not the non-rotation on the fixed end section x=0 is:

We present only the results where the differences between fields are observable. We mention that the fields are the following:

Field 1: order 2 polynomial
Field 2: order 3 polynomial
Field 3: order 4 polynomial
Field 4: order 5 polynomial
Field 5: order 6 polynomial

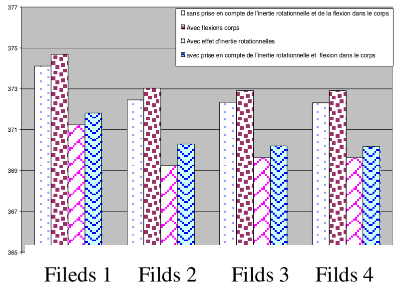

Fig. 2 First natural frequency depending of several parameters

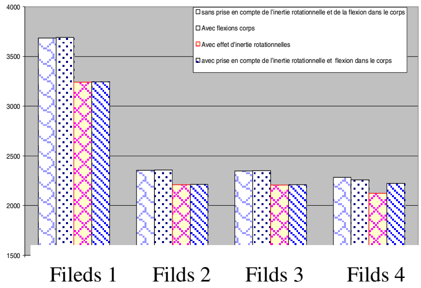

Fig. 3 Second natural frequency depending of several parameters

The first pulse from the first displacement fields (order 2 polynomial) with only one variable is widely superior to other values which tend to the same region limit 370 rad/s if we consider both the effect of of rotational inertia and the bending body. The value of the 2nd natural frequency tends to a limit close to 2300 rad/s without the effect of the rotational inertia and converges to 2200 rad / s from the third field (order 4 polynomial). The value of the third natural frequency converges only from the fourth field (order 5 polynomial).

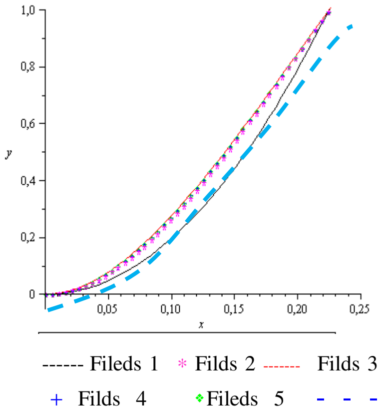

Fig. 4 first modal deformation comparison between different fields

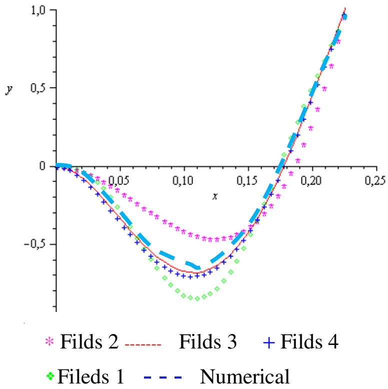

Fig. 5 second modal deformation comparison between different fields

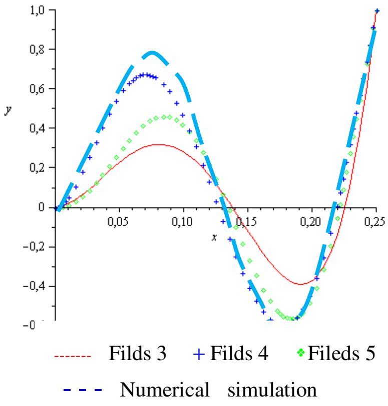

Fig. 6 Third modal deformation comparison between different fields

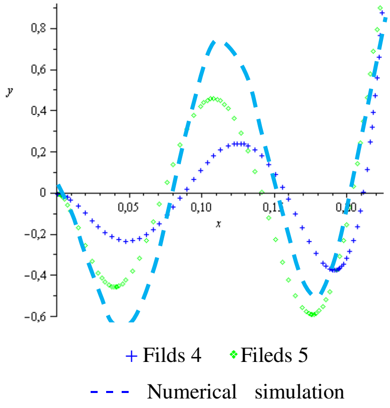

Fig. 7 Fourth modal deformation comparison between different fields

It can be concluded that the high order frequencies require more order to stabilize and more the mode order rises more we need a higher order polynomial.

We notice that the first modal deformation obtained from the field (1) is not close to other deformed because of the function that characterizes the displacement field does not define the shear energy of the body $\left(\partial^3 W / \partial x^3 = 0\right)$. Even the first obtained natural frequency is clearly superior to the others results, therefore this result will be eliminated from our comparative study.

When we increase the number of parameters that characterize the displacement fields we notice that the natural frequencies and modal deformed curves converge to a common solution.

Each time we increase the number of parameters that characterize the displacement fields, the calculations become more and more difficult and sometimes we can‘t solve the systems of the obtained equations where we find unusable results (problems with the field (5)), that’ s why we will consider the results obtained from the field (4) as a reference.

From figure 5, 6 and 7 we remark that the deformed reach their max and min in the same x-axis and also get canceled at the same points.

4 Experimental Study

4.1 Test Presentation

4.1.1 Used Material Presentation

The test rig, shown in Figure 8, consists of a massive steel structure for embedding beams and insulation from external vibrations. The beams are excited by an instrumented hammer connected to the acquisition system. The vibrational response of the beam is itself measured with a laser vibrometer.

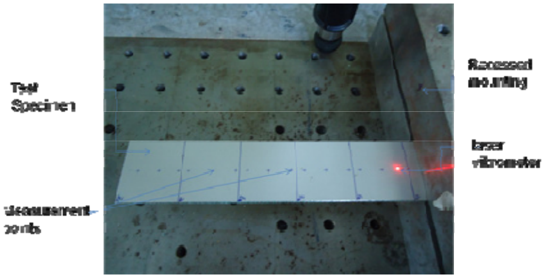

Fig. 8 Test rig with vibrometer laser

Performing a test of static bending with considering the effect of shear force (the thickness of the specimen is relatively large) allows us to determine the static stiffness and EIz. In these calculations the specimen is assumed to be elastic homogeneous isotropic and linear.

The last two assumptions are verified in the used loading area.

4.1.2 Experimental Device

The specimen which is fixed at one end and free on the other is suspended vertically to avoid the effect of static deflection.

The excitation hammer (Figure 10) with rubber tip and incorporated force sensor excites the bar horizontally with a transient load of small amplitude.

The bi- channels spectrum analyzer calculates the transfer function averaged over several samples. The coherence function is checked at each test and is close to one.

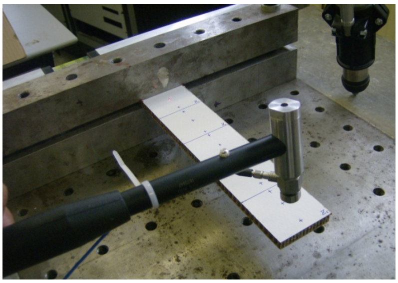

Fig. 9 Experimental device- Hammer shock sensor

Fig. 10 Experimental device- Hammer shock sensor

4.2 Test Results

During the tests we used a single excitation point (at x = a) is the free end and the acceleration is measured in a sufficiently close to the fixed end (x = a / 6).

The results of the transfer functions in amplitude and phase are shown in the following section.

The choice of the frequency band is imposed by the bandwidth of the accelerometer concerning the adhesive assembly.

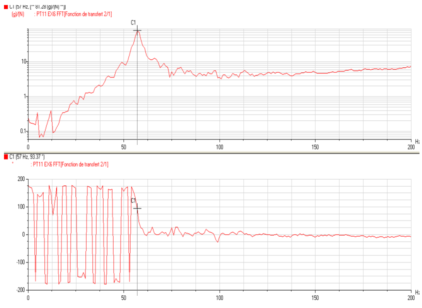

Fig. 11 Transfer function magnitude and phase of the Al beam

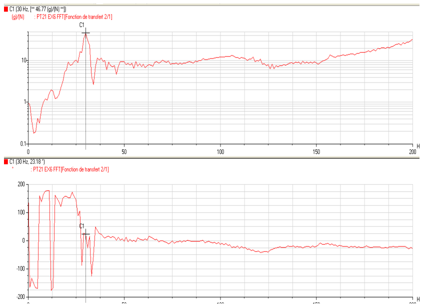

Fig. 12 Transfer function magnitude and phase of the Nomex

5 Numerical Simulation

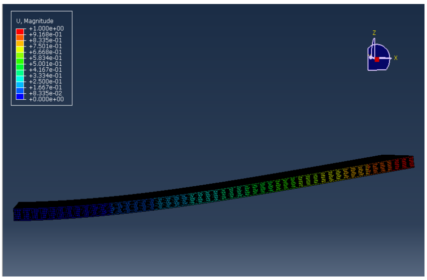

Fig. 13 First modal deformation of flexion in plan (o,x,y)

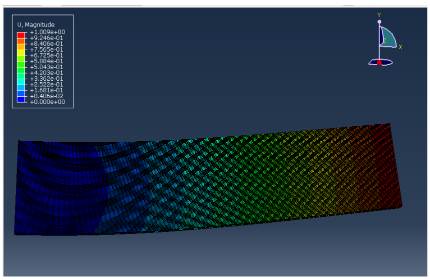

Fig. 14 First modal deformation of flexion in plan (o,x,z)

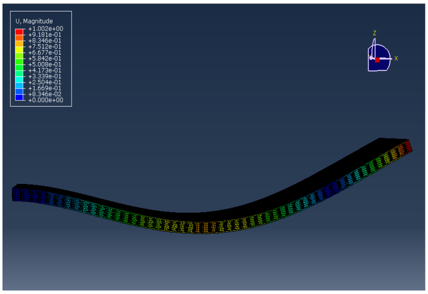

Fig. 15 second modal deformation of flexion in plan (o,x,y)

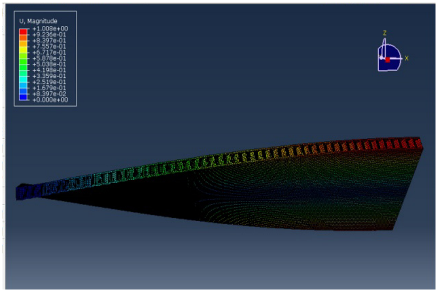

Fig. 16 First modal deformation of torsion

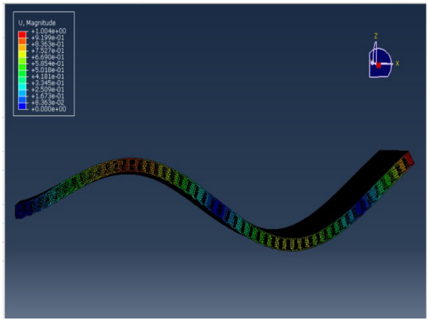

Fig. 17 Third modal deformation of flexion in plan (o,x,y)

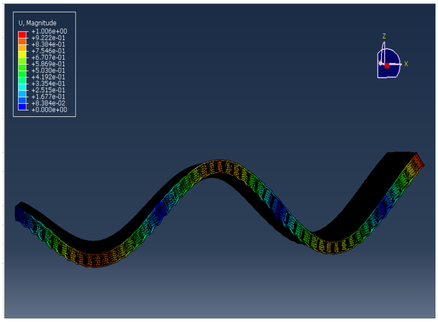

Fig. 18 Fourth modal deformation of flexion in plan (o,x,z)

6 Comparative Analysis

Comparing results among them, we see a slight variation between the natural frequencies and modal deformation determined digital simulation compared with analytical and experimental results this variation may be due to the not taken into account the effect of the glue between the layers has a significant effect known vibration damping.

7 Conclusion

The shape of the displacement field has a great influence on the natural frequency, the right choice is essential to reach a common solution. The second value of the first and the third natural frequency converge respectively from a displacement field of 3, 4 and 5 order polynomial.

For each form of the displacement field we notice the relationship between the coefficients of the matrix which may be written as follows

For n order polynomial:

$$M_{ij} = K_{ij} - L_{ij} \quad \text{Avec} \quad \begin{matrix} i = 1....(n-1) \\ j = 1....(n-1) \end{matrix}$$

Where L_{ij} can be in a general form.

$$L_{ij} = \frac{1}{2}\rho\omega^2 \frac{2}{(3+i+j)} a^{(3+i+j)}$$

$$K_{1j} = 4(1+j)a^j \cdot C \qquad K_{2j} = N_{2j} + (12 \cdot j) \cdot a^{(j+1)} \cdot C$$

With relations between the N_{ij} coefficients.

We can suggest to increase the degree of polynomial that characterizes the displacement field or to modify its shape in order to reach at a general relationship.

We can also choose a field of displacement according to the three direction (x, y, z) wherein for better vibration behavior of our beam while taking account of the torsion and bending in both horizontal and vertical plane.

References

[1] Mindlin, R.D.: Influence of rotatory inertia and shear on flexural motions of isotropic, elastic plates. J. Appl. Mech., 1831–1838 (1951)

[2] Yarlagadda, S., Lesieutre, G.: Fiber contribution to modal damping of polymer matrix composite panels. Journal of Spacecraft and Rockets 32(5), 825–831 (1995)

[3] Wang Deducing, C.M.: thick plate solutions from classical thin plate solutions. Struct. Eng. Mech. 11(1), 89–104 (2001)

[4] Meunier, M., Shenoi, R.A.: Dynamic analysis of composite sandwich plates with damping modeled using higher-order shear deformation theory. Composite Structures 54, 243–254 (2001)

[5] Soula, M., Nasri, R., Ghazel, M.A., Chevalier, Y.: The effects of kine-matic model approximations on natural frequencies and modal damping of laminated composite plates. Journal of Sound and Vibration 297, 315–328 (2006)

[6] Maheri, M.R., Adams, R.D.: Steady-state flexural vibration damping of honeycomb sandwich beams. Composites Science and Technology 52(3), 333–347 (1994)

[7] Banerjee Frequency, J.R.: equation and modal deformation formulae for composite Timoshenko beams. Composite Structures 51, 381–388 (2001)

[8] Nilsson, E., Nilsson Prediction, A.C.: Measurement of some dynamic properties of sandwich structures with honeycomb and foam cores. Journal of Sound and Vibration 251(3), 409–430 (2002)

[9] Blevins, R.D.: Formulas for Natural Frequency and Modal deformation. Krieger Publishing Company, Florida (2001)