Vibro-Acoustic Analysis of Laminated Double-Wall: Finite Element Formulation and Reduced-Order Model

Walid Larbi, Jean-François Deü, and Roger Ohayon

Conservatoire National des Arts et Métiers (Cnam),
Structural Mechanics and Coupled Systems Laboratory,
2 rue Conté, 75003 Paris, France
walid.larbi@cnam.fr

Abstract. This paper presents a finite element model for sound transmission analysis through a double sandwich panels with viscoelastic core inserted in an infinite baffle. The proposed model is derived from a multi-field variational principle involving structural displacement of the panels and acoustic pressure inside the fluid cavity. To solve the vibro-acoustic problem, the plate displacements are expanded as a modal summation of the plate's real eigenfunctions in vacuo. Similarly, the cavity pressure is expanded as a summation over the modes of the cavity with rigid boundaries. Then, an appropriate reduced-order model with mode acceleration method by adding quasi-static corrections is introduced. The structure is excited by a plane wave. The radiated sound power is calculated by means of a discrete solution of the Rayleigh Integral. Fluid loading is neglected. Various results are presented in order to validate and illustrate the efficiency of the proposed reduced finite element formulation.

Keywords: double-wall, viscoelastic, vibroacoustic, finite element, modal reduction.

1 Introduction

Double-wall structures are widely used in noise control due to their superiority over single-leaf structures in providing better acoustic insulation. Typical examples include double glazed windows, fuselage of airplanes and vehicles, etc. Various theoretical, experimental and numerical approaches have been investigated to predict the sound transmission through double walls [1, 2, 3].

By introducing a thin viscoelastic interlayer within the panels, a better acoustic insulation is obtained. In fact, sandwich structures with viscoelastic layer are commonly used in many systems for vibration damping and noise control. In such structures, the main energy loss mechanism is due to the transverse shear of the viscoelastic core [4, 5].

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In the first part of this paper, a non-symmetric finite element formulation of double-wall sandwich panels with viscoelastic core is derived from a variational principle involving structural displacement and acoustic pressure in the fluid cavity. Since the elasticity modulus of the viscoelastic core is complex and frequency dependent, this formulation is complex and nonlinear in frequency. Therefore, the direct solution of this problem can be considered only for small size models. This has severe limitations in attaining adequate accuracy and wider frequency ranges of interest. An original reduced order-model is then proposed to solve the problem at a lower cost. The proposed methodology, based on a normal mode expansion, requires the computation of the uncoupled structural and acoustic modes. The uncoupled structural modes are the real and undamped modes of the sandwich panels without fluid pressure loading at fluid-structure interface, whereas the uncoupled acoustic modes are the cavity modes with rigid wall boundary conditions at the fluid-structure interface. It is shown that the projection of the full-order coupled finite element model on the uncoupled bases, leads to a reduced order model in which the main parameters are the classical fluid-structure and residual stiffness complex coupling factors. Moreover, the effects of the higher modes of each subsystem is taken into account through an appropriate so-called static correction. Despite its reduced size, this model is proved to be very efficient for simulations of steady-state and frequency analyses of the coupled structural-acoustic system with viscoelastic damping and the computational effort is significantly reduced.

In the last part, numerical examples are presented in order to validate and analyse results computed from the proposed formulation.

2 Finite Element Formulation of the Coupled Problem

2.1 Local Equations

Consider a double-wall structure coupled to an acoustic enclosure shown in Fig. 1. Each wall occupies a domain Ω_{Si}, $i \in \{1,2\}$ such that $\Omega_S = (\Omega_{S1}, \Omega_{S2})$ is a partition of the whole structure domain. A prescribed surface force density $\mathbf{F}^d$ is applied to the external boundary Γ_t of Ω_S and a prescribed displacement $\mathbf{u}^d$ is applied on a part Γ_u of Ω_S. The acoustic enclosure is filled with a compressible and

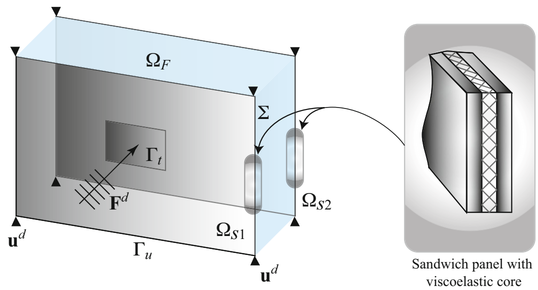

Fig. 1 Double sandwich wall structure coupled to an acoustic enclosure

inviscid fluid occupying the domain Ω_F. The cavity walls are rigid except those in contact with the flexible wall structures noted Σ.

The harmonic local equations of this structural-acoustic coupled problem can be written in terms of structure displacement $\mathbf{u}$ and fluid pressure field p [6]

$$\mathrm{div}\boldsymbol{\sigma}(\mathbf{u}) + \rho_S\omega^2\mathbf{u} = \mathbf{0} \qquad \text{in } \Omega_S \tag{1}$$

$$\boldsymbol{\sigma}(\mathbf{u})\mathbf{n}_S = \mathbf{F}^d \qquad \text{on } \Gamma_t \tag{2}$$

$$\boldsymbol{\sigma}(\mathbf{u})\mathbf{n}_S = p\mathbf{n} \qquad \text{on } \Sigma \tag{3}$$

$$\mathbf{u} = \mathbf{u}^d \qquad \text{on } \Gamma_u \tag{4}$$

$$\Delta p + \frac{\omega^2}{c_F^2}p = \mathbf{0} \qquad \text{in } \Omega_F \tag{5}$$

$$\nabla p \cdot \mathbf{n} = \rho_F\omega^2\mathbf{u}\cdot\mathbf{n} \qquad \text{on } \Sigma \tag{6}$$

where ω is the angular frequency, $\mathbf{n}_S$ and $\mathbf{n}$ are the external unit normal to Ω_S and Ω_F; ρ_S and ρ_F are the structure and fluid mass densities; c_F is the speed of sound in the fluid; and $\boldsymbol{\sigma}$ is the structure stress tensor.

2.2 Constitutive Relation for Viscoelastic Core

In order to provide better acoustic insulation, damped sandwich panels with a thin layer of viscoelastic core are used in this study (Fig. 1). When subjected to mechanical vibrations, the viscoelastic layer absorbs part of the vibratory energy in the form of heat. Another part of this energy is dissipated in the constrained core due to the shear motion.

The constitutive relation for a viscoelastic material subjected to a sinusoidal strain is written in the following form:

$$\boldsymbol{\sigma} = \mathbf{C}^*(\omega)\boldsymbol{\varepsilon} \tag{7}$$

where $\boldsymbol{\varepsilon}$ denotes the strain tensor and $\mathbf{C}^*(\omega)$ is the complex elasticity tensor. It is generally complex and frequency dependent ($*$ denotes complex quantities). It can be written as:

$$\mathbf{C}^*(\omega) = \mathbf{C}^{'}(\omega) + i\mathbf{C}^{''}(\omega) \tag{8}$$

where $i = \sqrt{-1}$.

Furthermore, for simplicity, a linear, homogeneous, viscoelastic constitutive equation will be used in this work. In the isotropic case, the viscoelastic material is defined by a complex and frequency dependence shear modulus in the form:

$$G^*(\omega) = G^{'}(\omega) + iG^{''}(\omega) \tag{9}$$

where $G^{'}(\omega)$ is know as shear storage modulus, as it is related to storing energy and $G^{''}(\omega)$ is the shear loss modulus, which represents the energy dissipation effects.

With these assumptions, the stress tensor of the sandwich structure is complex and frequency dependent.

2.3 Finite Element Equation

After variational formulation using the test-function method [6, 7] and discretization by the finite element method, we obtain the following matrix system of the fluid/elastic structure with viscoelastic damping coupled problem:

$$\left[\begin{pmatrix} \mathbf{K}_u^*(\omega) & -\mathbf{C}_{up} \\ \mathbf{0} & \mathbf{K}_p \end{pmatrix} - \omega^2 \begin{pmatrix} \mathbf{M}_u & \mathbf{0} \\ \mathbf{C}_{up}^T & \mathbf{M}_p \end{pmatrix}\right] \begin{pmatrix} \mathbf{U} \\ \mathbf{P} \end{pmatrix} = \begin{pmatrix} \mathbf{F} \\ \mathbf{0} \end{pmatrix} \tag{10}$$

where $\mathbf{U}$ and $\mathbf{P}$ are the vectors of nodal values of $\mathbf{u}$ and p respectively; $\mathbf{M}_u$ and $\mathbf{K}_u^*(\omega)$ are the mass and stiffness matrices of the structure; $\mathbf{M}_p$and $\mathbf{K}_p$ are the mass and stiffness matrices of the fluid; $\mathbf{C}_{up}$ is the fluid-structure coupled matrix; $\mathbf{F}$ is the applied mechanical force vector. Note that since the elasticity modulus of the viscoelastic core of the sandwich panels is complex and frequency dependent, the stiffness matrix $\mathbf{K}_u^*(\omega)$ is also complex and frequency dependent.

3 Reduced Order Model

In this section, we introduce a reduced-order formulation based on a normal mode expansion with an appropriate static correction.

3.1 Eigenmodes of the Structure in Vacuo

In a first phase, the first N_s eigenmodes of the structure *in vacuo* are obtained from

$$\left[\mathbf{K}_u^*(\omega) - \omega^2 \mathbf{M}_u\right] \mathbf{U} = \mathbf{0} \tag{11}$$

Due to the frequency dependent of the stiffness matrix, this eigenvalue problem is complex and nonlinear. It is assumed that vibrations of the damped structure can be represented in terms of the real modes of the associated undamped system if appropriate damping terms are inserted into the uncoupled modal equations of motion. Thus, the complex stiffness matrix is decomposed in the sum of two matrices:

$$\mathbf{K}_u^*(\omega) = \mathbf{K}_{u0} + \delta\mathbf{K}_u^*(\omega) \tag{12}$$

where $\mathbf{K}_{u0} = \mathrm{Re}[\mathbf{K}_u^*(0)]$ is the real and frequency-independent stiffness matrix calculated with a constant Young module's of the viscoelastic core and $\delta\mathbf{K}_u^*(\omega)$ is the residual stiffness matrix.

The ith real eigenmode is obtained from the following equation

$$\left[\mathbf{K}_{u0} - \omega_{si}^2 \mathbf{M}_u\right] \mathbf{\Phi}_{si} = \mathbf{0} \quad \text{for } i \in \{1, \cdots, N_s\} \tag{13}$$

where $(\omega_{si}, \mathbf{\Phi}_{si})$ are the natural frequency and eigenvector for the ith structural mode.

3.2 Eigenmodes of the Internal Acoustic Cavity with Rigid Walls

In this second phase, the first N_f eigenmodes of the acoustic cavity with rigid boundary conditions are obtained from the following equation

$$\left[\mathbf{K}_p - \omega_{fi}^2 \mathbf{M}_p\right] \mathbf{\Phi}_{fi} = \mathbf{0} \quad \text{for } i \in \{1, \cdots, N_f\} \tag{14}$$

where $(\omega_{fi}, \mathbf{\Phi}_{fi})$ are the natural frequency and eigenvector for the ith acoustic mode.

It is important to note that the physical acoustic modes in a rigid fixed cavity are such that $\int_{\Omega_F} p \; \mathrm{d}v = 0$ which excludes the $\omega_{fi} = 0$, $p = constant$ solution which is not physical but had to be introduced in the formulation of the coupled problem. Thus, the variational formulation of this kind of system, in order to be regularized for zero frequency situation, i.e. valid for a static problem, has to be modified by adding the following constraint $\rho_F c_F^2 \int_\Sigma \mathbf{u}.\mathbf{n} \, \mathrm{d}s + \int_{\Omega_F} p \, \mathrm{d}v = 0$ (see [6] for details). When doing this, on one hand the static pressure is defined precisely by

$$p^s = -\frac{\rho_F c_F^2}{|\Omega_F|} \int_\Sigma \mathbf{u}.\mathbf{n} \, \mathrm{d}s \tag{15}$$

and on the other hand, the reduced order formulation will be carried only by projection on the physical acoustic modes.

3.3 Modal Expansion of the General Problem

By introducing the matrices $\mathbf{\Phi}_s = [\mathbf{\Phi}_{s1} \cdots \mathbf{\Phi}_{sN_s}]$ of size $(M_s \times N_s)$ and $\mathbf{\Phi}_f = \left[\mathbf{\Phi}_{f1} \cdots \mathbf{\Phi}_{fN_f}\right]$ of size $(M_f \times N_f)$ corresponding to the uncoupled bases (M_s and M_f are the total number of degrees of freedom in the finite elements model associated to the structure and the acoustic domains respectively), the displacement and pressure are sought as

$$\mathbf{U} = \mathbf{\Phi}_s \mathbf{q}_s(t) \quad \text{and} \quad \mathbf{P} = \mathbf{\Phi}_f \mathbf{q}_f(t) \tag{16}$$

where the vectors $\mathbf{q}_s = [q_{s1} \cdots q_{sN_s}]^T$ and $\mathbf{q}_f = \left[q_{f1} \cdots q_{fN_f}\right]^T$ are the modal amplitudes of the structure displacement and the fluid pressure respectively.

Substituting these relations into Eq. (10) and pre-multiplying the first row by $\mathbf{\Phi}_s^T$ and the second one by $\mathbf{\Phi}_f^T$, we obtain

- N_s mechanical equations

$$-\omega^2 q_{si} + \sum_{k=1}^{N_s} \gamma_{ik}^*(\omega) q_{si} + \omega_{si}^2 q_{si} - \sum_{j=1}^{N_f} \beta_{ij} q_{fj} = F_i \tag{17}$$

- N_f acoustic equations

$$-\omega^2 q_{fi} + \omega_{fi}^2 q_{fi} - \omega^2 \sum_{j=1}^{N_s} \beta_{ij} q_{sj} = 0 \tag{18}$$

where $F_i(t) = \boldsymbol{\Phi}_{si}^T \mathbf{F}$ is the mechanical excitation of the ith mode; $\beta_{ij} = \boldsymbol{\Phi}_{si}^T \mathbf{C}_{up} \boldsymbol{\Phi}_{fj}$ is the fluid structure coupling coefficient and $\gamma_{ik}^*(\omega) = \boldsymbol{\Phi}_{si}^T \delta \mathbf{K}_u^*(\omega) \boldsymbol{\Phi}_{sk}$ the reduced residual stiffness complex coefficient.

At each frequency step, the reduced system (Eqs. (17) and (18)) is solved by updating $\gamma_{ik}^*(\omega)$. After determining the complex amplitude vectors q_{si} and q_{fi}, the displacement and pressure fields are reconstructed using the modal expansion (Eqs. (16)).

3.4 Static Corrections

The process of mode truncation introduces some errors in the response that can be controlled or minimized by a modal truncation augmentation method. In this method, the effects of the truncated modes are considered by their static effect only. First the applied loading vector $\mathbf{F}$ is composed as:

$$\mathbf{F} = \sum_{i=1}^{L} \alpha_i(t) \mathbf{F}_{0i} \tag{19}$$

where $\mathbf{F}_{0i}$ is the invariant spatial portion and $\alpha_j(t)$ is the time varying portion. For each invariant spatial load, the static modal eigenvector $\boldsymbol{\Psi}_{si}$ is given by:

$$\boldsymbol{\Psi}_{si} = \mathbf{K}_{u0}^{-1} \mathbf{F}_{0i} \tag{20}$$

The truncated basis containing the real and undamped structure modes is enriched by the static modal eigenvectors such that

$$\bar{\boldsymbol{\Phi}}_s = [\boldsymbol{\Phi}_{s1} \cdots \boldsymbol{\Phi}_{sN_s}, \boldsymbol{\Psi}_{si}, \cdots \boldsymbol{\Psi}_{sl}] = [\boldsymbol{\Phi}_s \, \boldsymbol{\Psi}_s] \tag{21}$$

The truncated fluid basis is enriched with the static pressure $\mathbf{P}_s$ computed from Eq. (15):

$$\bar{\boldsymbol{\Phi}}_f = [\boldsymbol{\Phi}_f \, \mathbf{P}_s] \tag{22}$$

Thus, the displacement and pressure are sought as

$$\mathbf{U} = \boldsymbol{\Phi}_s \mathbf{q}_s(t) + \boldsymbol{\Psi}_s \mathbf{q}_s^0(t) \quad \text{and} \quad \mathbf{P} = \boldsymbol{\Phi}_f \mathbf{q}_f(t) + \mathbf{P}_s q_f^0(t) \tag{23}$$

where the vectors $\mathbf{q}_s^0$ and q_f^0 are the quasi-static modal amplitudes of the structure displacement and the fluid pressure respectively. Similar coupled differential equations than Eqs. (17) and (18) can of course be obtained after modal projection on the enriched bases $\bar{\boldsymbol{\Phi}}_s$ and $\bar{\boldsymbol{\Phi}}_f$.

4 Numerical Examples

4.1 Sound Transmission through an Elastic Double-Panel System

In this section, the validation of the proposed coupled finite elements formulation for sound radiation is presented. The problem under consideration is shown in Fig. 2. A normal incidence plane wave excites a double-plate system filled with air (density ρ_F = 1.21 kg/m^3 and speed of sound c_F = 340 m/s). The plane wave has a pressure amplitude of 1 N/m^2 and is applied to plate 1 as the only external force to the system. The plates are identical and simply supported with thicknesses of 1 mm. The density of the plates is 2814 kg/m^3, the Youngs modulus is 71 GPa, the Loss factor is 0.01 and Poisson ratio 0.33. The surrounding fluid is the air. This example was originally proposed by Panneton in [2].

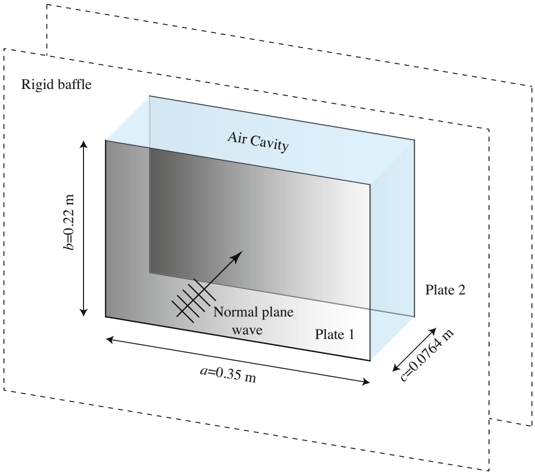

Fig. 2 Double-plate system filled with air: geometric data

When the excitation is applied to the first plate, the second one vibrates and radiates sound caused by the coupling of air and plate 1. The normal incidence sound transmission loss is then computed using the Rayleigh's integral method [1] which needs the finite element solution of surface velocities of plate 2. For this purpose, the resolution of the coupled system is done with a modal reduction approach using the first 10 in vacuo structural modes and the first 10 acoustic modes of the fluid in rigid cavity. The truncated bases are enriched by the static modal eigenvectors.

Fig. 3 shows the normal incidence transmission loss through a simply supported plate (dashed line). Due to the modal behavior of the plate, dips in the transmission loss curve are observed at its resonance frequencies (modes (1, 1), (3, 1) and (1, 3)). When a second plate is used to form an airtight cavity (continuous line), an increase in the transmission loss is achieved except in the region of the so-called plate-cavity-plate resonance (mode (1, 1)*). At this frequency, the two plates move

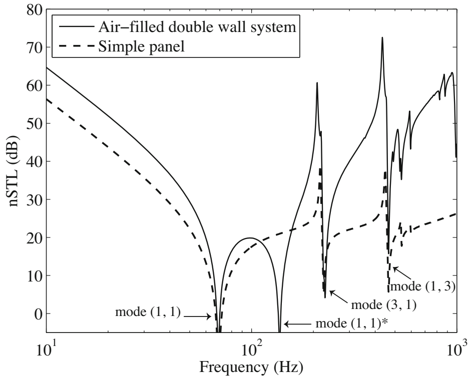

Fig. 3 Comparison of the normal incidence sound transmission (nSTL) through an air-filled double panel and a simple panel

out of phase with each other and the effect of the cavity on the plates is mostly one of added stiffness. This frequency is similar to the mass-air-mass resonance of unbounded double panels.

In addition, the variation of the nSTL of an air-filled panels and a simple panel is in very good agreement with the published date from [2].

4.2 Sound Transmission through a Double Laminated Glazing Window

The proposed reduced order finite-elements formulation is applied now to calculate the transmission loss factor of a double laminated glazing window. The system consists of two identical clamped laminated panels of glass separated by an air cavity of 12 mm thickness. Each laminated glass is composed of two glass plates bonded together by a Polyvinyl Butyral (PVB) interlayer. The thickness of outer and inner glass ply is $h_1 = h_3 = 3$ mm and those of the PVB interlayer is $h_2 = 1.14$ mm. The glass ply is modeled as linear elastic material (density 2500 kg/m^3, Youngs modulus 72 GPa, and Poisson ratio 0.22). The material properties of the PVB are both thermal and frequency dependent. From dynamic and thermal tests, Havrillak and Negami have found an empirical law describing this dependence. The resulting complex frequency dependent shear modulus of the PVB is given at 20°C as [8]:

$$G^*(\omega) = G_\infty + (G_0 - G_\infty)\left[1 + (i\omega\tau_0)^{1-\alpha_0}\right]^{-\beta_0} \tag{24}$$

where $G_\infty = 0.235$ GPa, $G_0 = 0.479$ Mpa, $\alpha_0 = 0.46$, $\beta_0 = 0.1946$, $\tau_0 = 0.3979$. The Poisson ratio of the PVB is 0.4 and density is 999 kg/m^3. Concerning the excitation and the finite element discretization, we used the same ones as in the previous example.

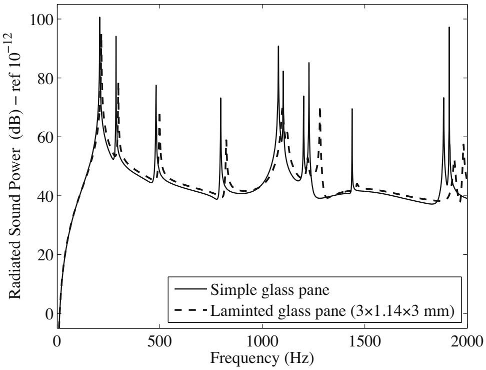

Fig. 4 Comparison of radiated sound power from a simple glass pane and a laminated glass with the same mass

A comparison between a simple glass and a laminated glass with PVB interlayer with an equivalent surface mass is shown in Fig. 4. Calculation was limited to 2000 Hz maximum. This comparison shows that laminated glass has a much lower acoustic radiation compared to conventional glass at resonance frequencies du to the effect of the viscoelastic layer. The reduction of sound radiation power is around 10 dB in lower frequencies and around 20 dB in higher frequencies. In fact, at low frequencies, the viscoelastic material is soft and the damping is small. At higher frequencies, the stiffness decreases rapidly and the damping is highest. Moreover, flexural vibrations causes shear strain in the viscoelastic core which dissipates energy and reduces vibration and noise radiation. Note that the thickness of the viscoelastic layer has a significant influence in terms of attenuation.

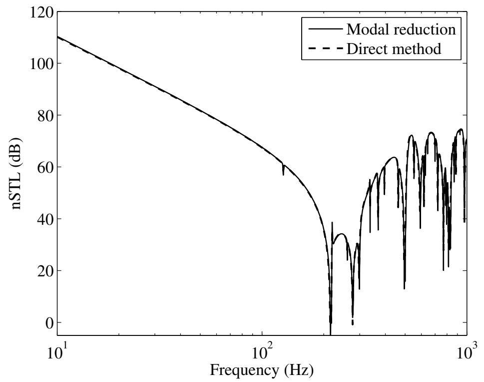

Fig. 5 nSTL through an air-filled double panel: comparison between the modal reduction approach and the direct nodal method

Fig. 5 shows a comparison between the nSTL of the coupled problem, obtained with the proposed accelerated modal reduction approach with a truncation on the first twenty structural modes ($N_s = 20$) and first twenty acoustic modes ($N_f = 20$) and the direct nodal method (Eq. (10)) where the displacement and pressure vectors are calculated for each frequency step. The structural modes are calculated from Eq. (13) using the constant shear storage modulus G_∞. As can be seen, a very good agreement between the two methods is proved. In this respect, it should be noted that the resulting reduction of the model size and the computational effort using the reduced order method are very significant compared to those of the direct approach.

5 Conclusions

In this paper, a finite element formulation for sound transmission through double wall sandwich panels with viscoelastic core is presented. A reduced-order model, based on a normal mode expansion, is then developed. The proposed methodology requires the computation of the eigenmodes of the undamped structure, and the rigid acoustic cavity. Quasi-static corrections are introduced in order to accelerate the convergence. Despite its reduced size, this model is proved to be very efficient for simulations of steady-state analyses of structural-acoustic coupled systems with viscoelastic interlayers when appropriate damping terms are inserted into the modal equations of motion.

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