Statistical Estimation of Fluid Flow: An Image Restoration Approach

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Abstract. This paper focuses on Fluid Motion-Field Estimation from video data, which is a useful but challenging problem in environmental monitoring. Rivers are often monitored by flashy hydrographs that exhibit characteristic response times ranging from minutes to hours. In order to estimate the river discharge during a flush flood event, the temporary motion vector field of the river surface is needed. This paper presents a new approach in statistical estimation of fluid flow that calculates a local flow probability distribution function in the frequency domain. Our work improves upon the inefficiencies of spatial estimation of the auto-regressive STAR model and converts motion estimation into a restoration problem, where the local field can be computed fast in the frequency domain, while various natural constraints can be taken into account within the inversion strategy of the motion estimation process.

1 Introduction

Temporary River stream is a general term for all types of intermittent, ephemeral and episodic streams. One of its main characteristics is that its hydrographs exhibit characteristic response times ranging from minutes to hours, as experienced during first flush and storm events. Studies have shown that in the next decades the number of temporary rivers worldwide will increase due to climate changes, but also due to water abstraction interventions [1]. One of their dangerous key characteristics is that they present extreme peaks due to floods. In terms of hydrological features, it is important to have an estimation of the river discharge during a flood event for understanding flood generation and propagation processes, as to prevent their catastrophic consequences. This can be realized through reliable streamflow estimation [2], since discharge can be obtained through integration of the measured surface velocities over the river cross-section. There are many conventional devices for measuring river surface velocities, such as mechanical modules consisting of propellers or acoustic

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Doppler current profilers, but all of these require human presence for operation [3], [4]. The use of camera sensors at the river banks, which can monitor the flood event and transmit important data of the river flow, is much more convenient as long as there are robust methods to extract the surface velocity vectors.

The development of non-rigid motion models for river flow estimation has received considerable attention [5], [6]. In the hydrology community, the problem is often treated through either statistical analysis or the use of Particle Image Velocimetry (PIV) techniques [7]. Since statistical estimation techniques are more computationally efficient, we focus our study on this type of techniques. In particular, we exploit the auto-regressive STAR model [8], which utilizes data from a spatio-temporal neighborhood of each pixel. This model has been considered widely in the hydrologist community; nevertheless, its implementation is based on least squares optimization. As such, it suffers from high computational complexity, high instability of the resulting motion vectors, and limited options for incorporating local constraints. Our approach converts the problem in the form of image restoration, where the probability distribution of the motion field is treated as a spatio-temporal filter applied on the image. Thus, the estimation of the local motion vector at each image pixel is treated as an inversion process aiming to derive the filter structure. The proposed methodology can incorporate local constraints and operates in a computational efficient scheme performed in the frequency domain.

The rest of this paper is organized as follows. Section 2 overviews the STAR model as a state-of-the-art technique for optical flow estimation and introduces our estimation methodology. Section 3 demonstrates the proposed methodology and Section 4 presents some preliminary results with comments on its key features.

2 State of the Art in Optical Flow Techniques

The STAR model, as a three-dimensional extension of Autoregressive models (AR), is widely used in optical flow estimation problems. Generally speaking, it performs prediction using a linear combination of the values in a spatio-temporal neighborhood of appearance state variable s and it has the following form:

$$s(x,y,t)=\sum_{i=1}^{D} A_i s(x+\Delta x_i,\ y+\Delta y_i, t+\Delta t_i)+n(x,y,t) \tag{1}$$

where Δxi, Δyi and Δti specify the neighborhood structure of the motion model, while the term $n(x,y,t)$ is drawn from a zero-mean noise distribution, typically taken as Gaussian white noise [8], [9].

Adapting to image intensity $I(.)$, Equation (1) can be written as:

$$I(x,y,t)=\sum_{i=1}^{M} A_i I(x+\Delta x_i,\ y+\Delta y_i, t+1)+n(x,y,t) \tag{2}$$

or with different temporal notation:

$$I_t(x,y) = \sum_{i=1}^{M} A_i I_{t+1}(x+\Delta x_i, y+\Delta y_i) + n(x,y) \quad (3)$$

$$I(x,y) = \sum_{i=1}^{N} A_i I'(x+\Delta x_i, y+\Delta y_i) + n(x,y) \quad (4)$$

where we denote image intensity I for time t and image intensity I' for time (t+1). The probability A_i is associated with each possible displacement (Δx_i, Δy_i) from time t to ($t+\Delta t$). For each pixel (x,y) there is a preselected "search" area D with cardinality M, which is determined by the max distance $(x+\Delta x, y+\Delta y)$.

Essentially, as in optical flow estimation, the STAR model follows two basic assumptions:

- Brightness Constancy Constraint: the brightness (intensity) of a pixel being displaced remains constant over a short period of time.
- Spatial Smoothness Constraint: Neighboring pixels experience nearly the same motion field.

If the estimation of the motion vector at (x,y) is extended over a local region G around this pixel with cardinality N, under the second assumption of "consistent" motion, then the search window at time ($t+\Delta t$) is expanded over a region Γ determined by the overlay of D over G.

In vector form, Equ. (4) for the entire local estimation region G can be written as:

$$\underline{\mathrm{I}} = \mathrm{I}' \underline{a} + \underline{n} \quad (5)$$

where:

- $\underline{a}$: (1×M) vector of displacement probabilities of pixel (x,y); probability distribution according to which the intensity at pixel $(x+\Delta x_i, y+\Delta x_i)$ in the next frame equals the intensity at pixel (x,y) in the current frame. A block region around pixel (x,y) of size G has the same motion field, i.e. same probability vector, and the estimation or search space covers all possible displacements, i.e. a block-region D over each pixel of this region.
- $\underline{I}$: Current-frame Intensity values within a ($m \times m$) Local Neighborhood of cardinality D around the pixel of interest (x,y).
- I': Next-frame Intensity values at a ($n \times n$) Destination Neighborhood of cardinality G around each pixel of the Local Neighborhood ($m \times m$).

All pixels in the Local Neighborhood D share the same motion distribution function. As a consequence, we get an equation in the form of (5) for every pixel of interest in the current frame, within its local neighborhood D. The solution of the normal least-squares equations system derives the STAR distribution coefficients, i.e. the transition probabilities of the Destination neighborhood Γ for each Search neighborhood D. The motion vector assigned at the central pixel of the Local Neighborhood D is the vector with the highest probability.

Fig. 1 presents an example of STAR model for the case of $m=5$ and $n=3$. For the computation of motion probabilities at I_1 we engage the entire search region at the next frame composed of all pixels I'_{1i}.

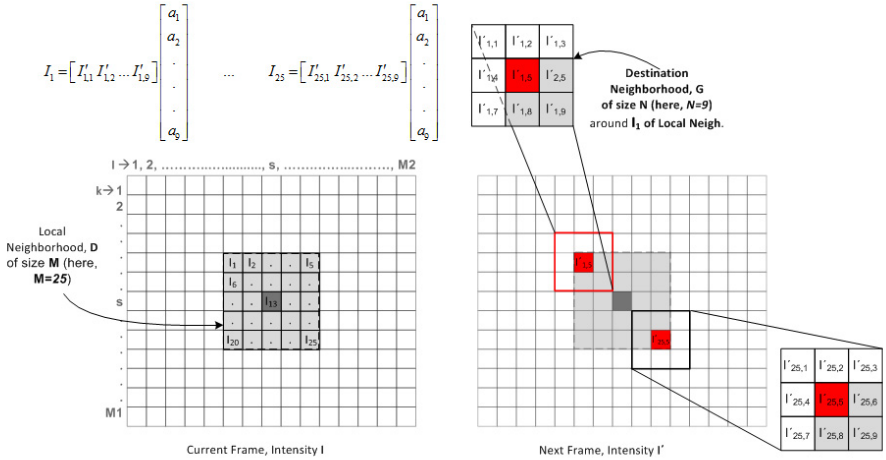


Fig. 1. Example of STAR model

Alleviating the non-stable nature of the least squares estimation, Chang et al. [5] proposed to feed the set of probability distribution coefficients into a global optimization framework in order to return a smooth motion field with a regularized velocity vector at each pixel [5]. This leads to a dense motion field with a unique velocity vector in each local neighborhood from the set of local distribution functions extracted from the previous step.

In our approach, we notice that the formulation of the search and destination neighborhoods for the derivation of Equ. (5) closely resembles a filtering operation of the latter with a filter composed of the motion probability coefficients in $\underline{a}$. The result of this filtering operation is identical to the intensity field in the original Search area *I* of cardinality D, obtained from the application of the constant-coefficient filter $\underline{a}$ on the entire destination area *I'* of expanded cardinality Γ due to the size G of the filter. This formulation can derive the unknown filter coefficients from the solution of the inverse problem in (5) formulated under the periodic expansion assumption as an inversion in the DFT domain.

3 Proposed Methodology

The proposed formulation treats Fluid Motion Field Estimation as an Image Restoration problem. Accordingly, the output pixel *y(i,j)* is determined through the convolution of the filter *a(i,j)* with the local area of the input image *f(i,j)* as:

$$g(i,j) = a(i,j) * f(i,j) + n(i,j) \tag{6}$$

where *n(i,j)* is the external noise process, assumed to be image-independent. In the estimation of the motion vector field as expressed in the previous section, the input image area *f(i,j)* is defined on the next frame *I'(x,y)*, whereas the output image area *g(i,j)* is defined on the current frame *I(x,y)*.

Formulating the convolution process for the entire local search window, where the same filter structure is moved over all positions of the search window, we can formulate the problem in vector form as:

$$\underline{\mathrm{I}} = \mathrm{A}\,\underline{\mathrm{I}'} + \underline{n} \tag{7}$$

where vectors $\underline{\mathrm{I}}$ and $\underline{\mathrm{I}'}$ denote the "input and "output" image windows, respectively, and matrix A implements the locally shifted structure of the filter coefficients. With a closer examination of the local positioning of the filter on the intensities of $\underline{\mathrm{I}'}$, as shown in Fig. 2, we can verify the block Toeplitz structure of A. Furthermore, assuming periodic expandability of image windows, the matrix A assumes a block-circulant structure, which can be implemented in the DFT domain.

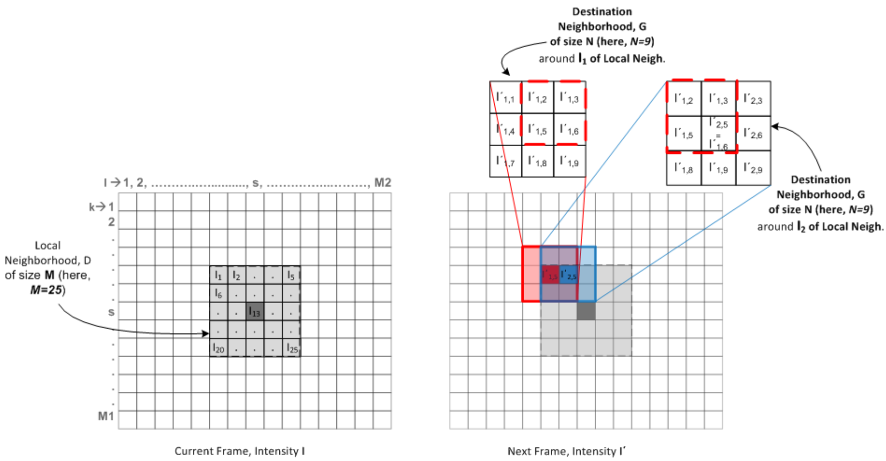


Fig. 2. Estimation Neighborhood Structure

The proposed formulation exports all advantages of the field of image restoration to that of non-rigid motion estimation. First, the DFT implementation provides increased computational efficiency in the inversion process. Then, many other deterministic and/or stochastic inversion methods can be easily implemented, such as constrained, Wiener and robust estimation. Furthermore, spatio-temporal estimation, not only in two subsequent frames but in a sequence of frames, can be modeled as a three-dimensional restoration problem with the expansion of the local spatial input area to a local spatio-temporal area. This expansion closely resembles the formulation of the color image restoration problem as a 3D estimation problem to exploit intra-frame correlation, as efficiently modeled in local block areas.

If we consider the entire destination window *Γ* for the computation of motion probability vector for the central pixel (*x,y*), the entire operation for the local search neighborhood G can be seen as a convolution between the counterpart of the area *G* at time *t+1* with the constant-coefficient "filter" or motion probability field of size *D*, composing the search window for each pixel. Fig. 3 illustrates the calculation of the velocity field that is computed on the search area G placed at central pixel (*x*,*y*).

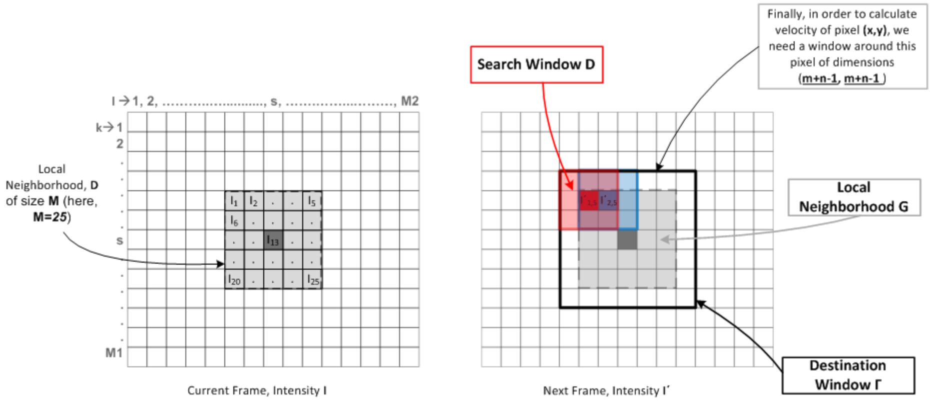


Fig. 3. Example of pixel velocity calculation

In the spatial domain, for each pixel in the local neighborhood *G* (time *t*) we perform convolution of destination neighborhood (time *t+1*) with the filter of size D (coefficients *α*), expanding the computation to size Γ. After expanding all areas to size Γ and applying the DFT operation, domain we perform point-by-point multiplication in the frequency as follows:

$$\mathrm{I}_G(u,v) \;=\; \mathrm{I}'_\Gamma(u,v)\cdot A_D(u,v) \tag{8}$$

where the subscripts denote the regions of initial support of the operators.

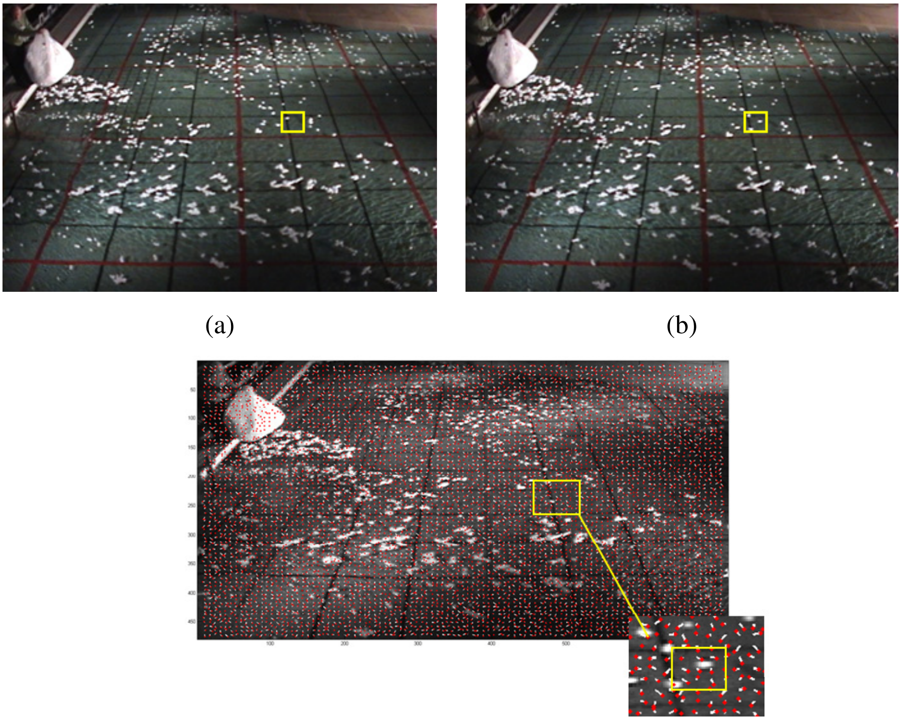
The solution of (7) in the spectral domain through pseudo-inversion, i.e. the solution of Equation (8) using a system of normal equations for least-squares, would derive the same estimate as the STAR probability vector $\underline{a}$, resulting from the solution of the normal least-squares equations system. Constrained or regularized inversion schemes for Eq. (6) can derive more robust estimates of the motion field, without the need of a subsequent step of global optimization, as in Chang et al. [6].

The proposed solution follows the solution of image restoration problem (in the presence of noise, as Equ. (6) denotes), where techniques like Wiener filter of CLS are used. In Wiener filtering case, the objective is to find a restored image, such that the expected error is minimized $\min_{\hat{f}} E\{(f-\hat{f})^2\}$, where *f* is the uncorrupted image and $\hat{f}$ is the estimated image. In the CLS case, we minimize the Laplacian $\sum_x\sum_y[\nabla^2\hat{f}(x,y)]^2$ in order to have a result "as smooth as possible".

4 Results

As a first step of testing our proposed methodology, we consider a set of fluid and river flow video datasets, in order to assess the main trend of the output motion-vector field as along with the effects of different sizes of the local neighborhood and search windows.

Fig. 4 (a) and (b) illustrate a sequence of two successive frames presenting our testbed image dataset, which was observed in LS-PIV S/W developed by the IHR-Hydroscience and Engineering Dept. at University of Iowa. In Fig. 4(c), the motion vectors that were produced by the proposed methodology are presented, where white lines represent the motion vectors and the red points represent the motion direction.



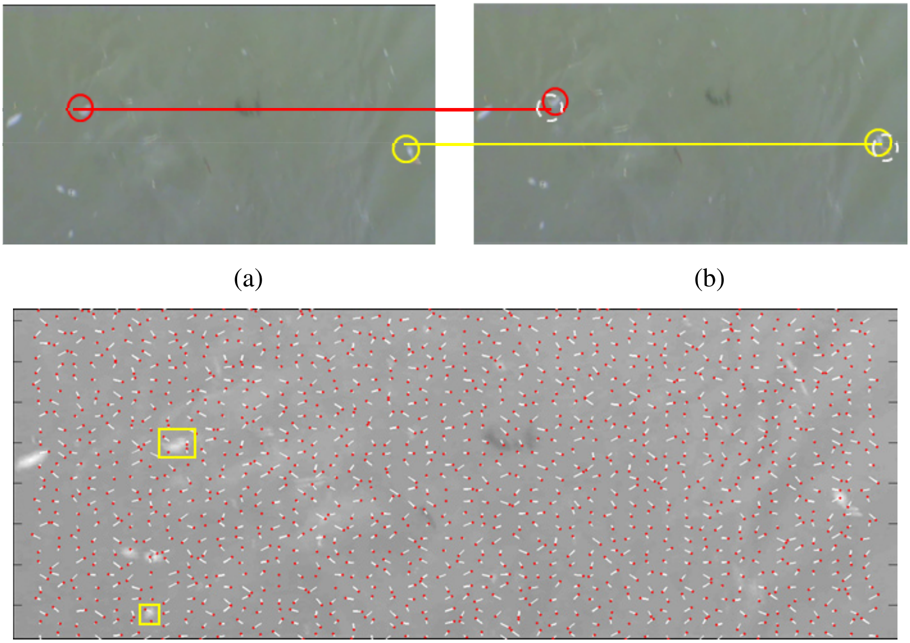
(a) (b)

(c)

Fig. 4. (a), (b) Two successive frames of water flow, (c) Motion Vector Field with m=5 & n=3

It is clear that the main trend of the flow field, with the largest density of local shift-vectors is towards the right direction, which coincides with the actual motion trend of this experiment. The local neighborhood area has size of m = 5, while the search window has size of n = 3. The reason for choosing such small window sizes is that the flow is characterized by dense white ball distributions, so larger window sizes would probably engage neighboring white balls, outside the motion region of the current ball under consideration.

In Fig. 5 we consider a video of river flow with floating foam and sediments. The main motion trend that appears in Fig. 5(c) as a result of the proposed methodology, follows the river flow, which has an up and left or right mixed direction (yellow and red line indicate both cases of flow direction) More specifically, Fig. 5(a) and 5(b) present two successive frames at time *t* and (*t+1*) respectively. The foam item inside the red box moves towards up and right during *Δt,* while the foam item inside the yellow box moves towards up and left during the same time period. In both cases, the dashed white circles denote the foam item position in frame at time *t*. Here, the window parameters m and n were selected to have the values of 11 and 5 respectively, since the position shift of the same intensity data blocks is not too dense.



(a) (b)

(c)

Fig. 5. (a), (b) Two successive frames of river flow, (c) Motion Vector Field with m=11 & n=5

An interesting remark here relates to the observation that within the closed sediment area the position shift is near to zero, i.e. the relative motion vector has a short magnitude, if the center of the local neighborhood is inside the sediment; however, if it is located near the edge of the sediment, the motion vector follows the flow direction, as the example cases in the yellow boxes indicate. This point demonstrates the effect of the local neighborhood size and its relation to the river flow characteristics, such as the existence of floating tracers, the water quality in terms of transparency, etc.

5 Conclusions

The proposed methodology provides a computationally efficient, non-intrusive way to estimate the river flow surface motion-vector field. The key features of the methodology are not bounded to floating tracers, in contrast to many literature methodologies. Instead, the use of statistical analysis of the river flow exploits only the image intensity, irrespective of the content of the river-flow images. The novelty of our methodology is that computations (including inversion processes) are efficiently performed in the frequency domain by converting motion estimation into a restoration problem. The major advantage of this consideration is that the local field is computed fast, while several inversion strategies can be exploited.

In future work, we will also consider a classification method for isolating the main flow trend. Furthermore, we will focus efforts into calculating the surface river flow velocities that are vertical to the downstream flow, again exploiting the classification outcome. In this way we will attempt to calculate the river discharge during the video recording period of the river flow. We also plan to collect flood event data and further analyze the effects of the local neighborhood and search window sizes in respect to the efficiency of early detection of flood signs.

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