

# A Novel Robot Leg Designed by Compliant Mechanism

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**Abstract.** A novel flexible robot leg is designed by compliant mechanism, which provides a solution for stable walking. A pseudo-rigid-body model (PRBM) is built to analyze and verify the compliant mechanism. The simulation result indicates that the mechanism can approximately meet the requirements. Furthermore, the robot leg can work in two states by bistable design, namely “open state” and “contraction state”. This robot leg is applicable to biped robots or multiped robots in small or micro scale.

**Keywords:** Compliant mechanism, Pseudo-rigid-body model (PRBM), Bistable, Robot leg.

## 1 Introduction

Mobile robots may be classified into wheeled robots, tracked robots, legged robots [1], etc. Legged robots move most commonly by means of bionic legs, such as the one-legged hopping robot, Kenken [2], the planar bipedal compliant legged robot, Spring Flamingo [3], the goat-imitated quadruped legged robot, KOLT [4, 5], the ockroach-inspired hexapod legged robots RHex [6] and iSprawl [7, 8]. Many of them are either multi-actuated or complicatedly designed, which may lead to cumbersome, inconvenience for optimization or limitation in size.

Compliant mechanisms are flexible mechanisms that transfer an input force or displacement to another point through elastic body deformation [9]. Compared to traditional rigid-body mechanisms, compliant mechanisms have advantages in simplifying processing and manufacturing, reducing the number of components and assembly time, reducing wear and tear, etc.

This paper presents a novel compliant leg mechanism [12], which is designed and analyzed by means of the pseudo-rigid-body model.

## 2 A Novel Robot Leg Design

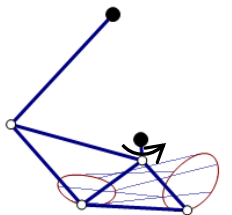
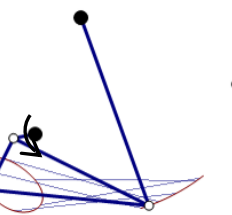
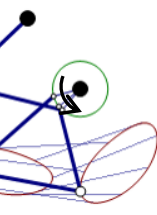
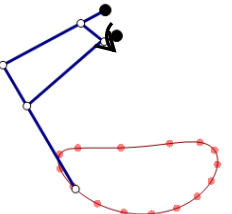
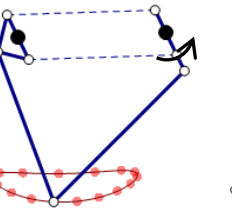
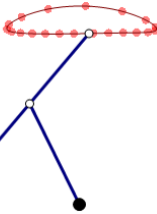
In order to design a reliable and efficient leg mechanism for small and micro robot, five objectives are set as follows:

- (1) A single leg applicable to biped robot, quadruped robot or multiped robot;
- (2) Driven by only one motor for less weight;
- (3) Having a bionic movement locus and pose of foot, which means the back swing of the leg should possibly be stable through the time after touching and before leaving the ground;
- (4) Capable of reducing volume, weight, wear and tear by means of simple mechanisms;
- (5) Simple but artistic.

**2.1 Traditional Mechanical Designs of a Single Leg**

To contrast with the compliant mechanism presented in this paper, several traditional mechanical designs of a single leg are proposed as shown in Table 1.

**Table 1.** Traditional mechanical designs of a single leg

Foot shape	Schemes of traditional mechanical designs		
Planar foot	 <p style="text-align: center;">(a)</p>	 <p style="text-align: center;">(b)</p>	 <p style="text-align: center;">(c)</p>
Point foot	 <p style="text-align: center;">(d)</p>	 <p style="text-align: center;">(e)</p>	 <p style="text-align: center;">(f)</p>

From Table1, the conclusions can be given as follows:

To scheme (a), (b) and (d), the movement of the foot part is barely acceptable;

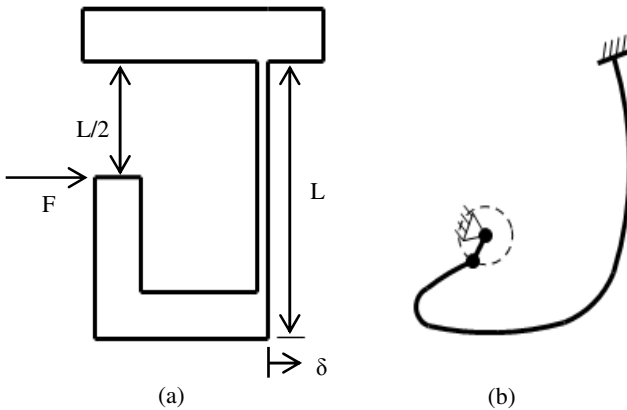
To scheme (c) and (e), the mechanisms are more valuable because they are conveniently adjustable for optimization. However, there are too many hinges for further machining and assembling in small or micro size.

To scheme (f), the planar four-bar linkage provides the most ideal result of movement locus and speed, but the end of the output link is beyond the whole mechanism. Therefore a parallelogram mechanism is needed for the motion transmission, which will bring the mechanism intricacy as well.

## 2.2 The Design of Compliant Mechanism

To find a mechanism being both simple and capable of meeting the requirements, a mechanism working as scheme (b) in Table 1, is proposed in Fig. 1(b), which is an almost evolution of a normal compliant mechanism in Fig. 1(a) [9]. The mechanism in Fig. 1 (a) is used to obtain an approximate linear displacement. It predicts that the mechanism in Fig. 1(b) works in a similar way. In addition, it can imitate the behavior of an animal's leg: swing backward—raise the foot—swing forward—lower the foot.

This mechanism is not only simple, but also convenient for optimization by adjusting the length and the thickness. Additionally, it can be multistable by design, which may offer multifunction.



**Fig. 1.** (a) A normal compliant mechanism: the easiest way to obtain an approximate linear displacement from a cantilever beam. (b) A compliant mechanism similar to the mechanism shown in Fig. 1(a)

## 3 Pseudo-Rigid-Body Model (PRBM) of the Robot Leg

In order to design, analyze and optimize the mechanism, a simplified scheme is proposed, as shown in Fig. 2(a). The compliant mechanism is equivalent to three fixed-pinned segments (initially curved cantilever beams) of lengths  $l_1$ ,  $l_2$  and  $l_3$  respectively, whose joints are added by torsional springs to retain the energy storage properties of compliant mechanisms.

The pseudo-rigid-body model [9-11] is shown in Fig. 2(b).

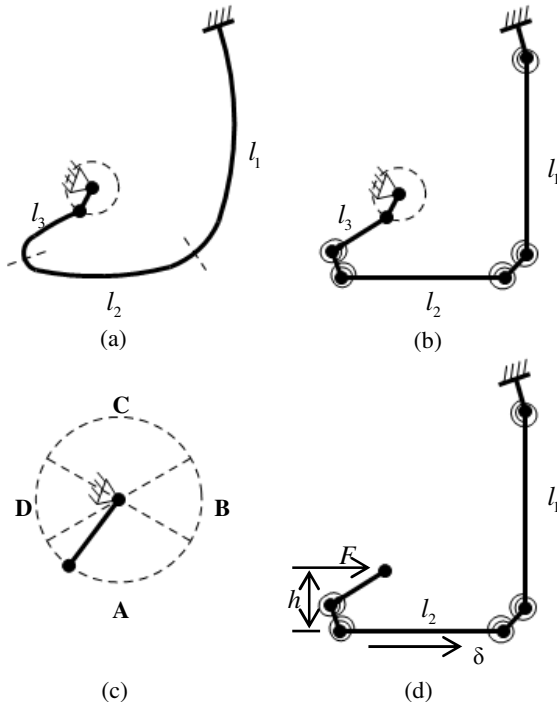


Fig. 2. The pseudo-rigid-body model of the mechanism

The upside end of the PRBM is completely fixed, while the other end is connected to a crank by a hinge, which offers a circular motion. This cyclic circular motion can be divided into four phases according to its different effects, as shown in Fig. 2(c). The compliant leg mechanism swings backward during phase A and swing forward during phase C. Besides, it raises the foot from the ground during phase B and lowers during phase D. To simplify the calculation and analysis, approximately assume that the direction of the input displacement is rightward, as well as the input force. And if the stiffness coefficient of the material is suitable, the support force of the ground can be ignored.

## 4 Calculation and Analysis

As shown in Fig. 2(d), the input force is  $F$ , which is equivalent to a force  $F$  and a moment  $M_1$  loading at the free end of segment  $l_1$ . To segment  $l_2$ ,  $F$  is equivalent to a force  $F$  and a moment  $M_2$  loading at the free end as well. It is shown in Fig. 3(a) and Fig. 3(b).

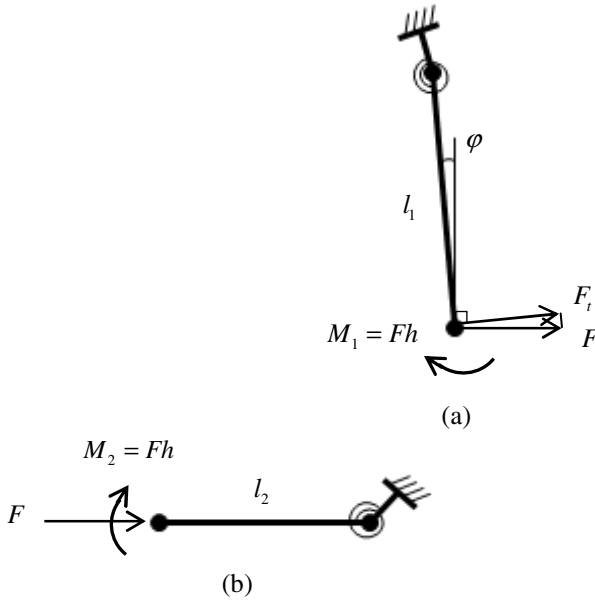


Fig. 3. Force analysis of the mechanism

(1) In Fig. 3(a), the beam end rotation angle of  $l_1$  resulting from  $F$  would be obtained by equations [9]:

$$\begin{cases} T = \rho l_1 F_t = \rho l_1 F \cos \varphi \\ T = K_{1F} \theta_{1F} \\ K_{1F} = \rho K_{\theta 1F} \frac{E_1 I_1}{l_1} \\ \theta_{1F-end} = c_{\theta F} \theta_{1F} \end{cases},$$

where  $T$  is the torque on the torsional spring resulting from  $F$  at the beam end of  $l_1$ ,  $\rho$  is the new characteristic radius factor,  $l_1$  is the beam length,  $F_t$  is the transverse force of  $F$ ,  $\varphi$  is the angle between  $F$  and  $F_t$ ,  $K_{1F}$  is the torsional spring constant,  $\theta_{1F}$  is the pseudo-rigid-body angle,  $K_{\theta 1F}$  is the stiffness coefficient,  $E_1$  is the modulus of elasticity,  $I_1$  is the moment of inertia of the cross-section,  $\theta_{1F-end}$  is the beam end angle, the constant  $c_{\theta F}$  is termed as the parametric angle coefficient.  $\varphi$  ranges from  $-10^\circ$  to  $10^\circ$ , so  $\cos \varphi \approx 1$ .

Therefore, the beam end rotation angle of  $l_1$  resulting from  $F$  is

$$\theta_{1F-end} = \frac{c_{\theta F}}{K_{\theta 1F} E_1 I_1} F l_1^2 \tag{1}$$

(2) In Fig. 3(a), the beam end rotation angle of  $l_1$  resulting from  $M_1$  would be obtained by equations [9]:

$$\begin{cases} M_1 = Fh \\ M_1 = K_{1M}\theta_{1M} \\ K_{1M} = \gamma_1 K_{\theta 1M} \frac{E_1 I_1}{l_1} \\ \theta_{1M-end} = c_{\theta M} \theta_{1M} \end{cases}$$

where  $M_1$  is an equivalent moment by translation of  $F$ ,  $h$  is the height of the original  $F$  above the ground,  $K_{1M}$  is the torsional spring constant,  $\theta_{1M}$  is the pseudo-rigid-body angle,  $\gamma_1$  is the characteristic radius factor,  $K_{\theta 1M}$  is the stiffness coefficient,  $\theta_{1M-end}$  is the beam end angle, the constant  $c_{\theta M}$  is the parametric angle coefficient.

Therefore, the beam end rotation angle of  $l_1$  resulting from  $M_1$  is

$$\theta_{1M-end} = \frac{c_{\theta M}}{\gamma_1 K_{\theta 1M} E_1 I_1} Fl_1 h \tag{2}$$

(3) In Fig. 3(b),  $F_i = 0$ , and the pseudo-rigid-body angle of  $l_2$  resulting from  $M_2$  would be obtained by equations [9]:

$$\begin{cases} M_2 = Fh \\ M_2 = K_{2M}\theta_{2M} \\ K_{2M} = \gamma_2 K_{\theta 2M} \frac{E_2 I_2}{l_2} \end{cases}$$

where  $M_2$  is an equivalent moment by translation of  $F$ ,  $K_{2M}$  is the torsional spring constant,  $\theta_{2M}$  is the pseudo-rigid-body angle,  $\gamma_2$  is the characteristic radius factor,  $K_{\theta 2M}$  is the stiffness coefficient.

Therefore, the pseudo-rigid-body angle of  $l_2$  resulting from  $M_2$  is

$$\theta_{2M} = \frac{1}{\gamma_2 K_{\theta 2M} E_2 I_2} Fl_2 h \tag{3}$$

(4) If this compliant leg mechanism works stably in walking, the foot part should keep level in a certain range. That means, the rotation angle of the pseudo-rigid-body segment  $l_2$  is equal to zero:

$$\theta_{1F-end} - \theta_{1M-end} - \theta_{2M} = 0 \tag{4}$$

The design equation is given by substituting Eq. (1), Eq. (2) and Eq. (3) into Eq. (4):

$$A \cdot l_1^2 - B \cdot l_1 h - C \cdot l_2 h = 0 \quad (5)$$

where  $A = \frac{c_{\theta F}}{K_{\theta F}} \cdot \frac{1}{E_1 I_1}$ ,  $B = \frac{c_{\theta M}}{\gamma_1 K_{\theta 1M}} \cdot \frac{1}{E_1 I_1}$ ,  $C = \frac{1}{\gamma_2 K_{\theta 2M}} \cdot \frac{1}{E_2 I_2}$ .

Especially, if the materials and cross-sections of segment  $l_1$  and segment  $l_2$  are identical, then  $E_1 = E_2$  and  $I_1 = I_2$ .

As a result, the mentioned mechanism that meets the condition of Eq. (5) will have a stable walk on the level.

At the same time, the J-shaped component can be tightly bound around with a caterpillar band, driving by another motor, to move not only on foot but also on the crawler.

## 5 Kinematics and Static Simulation

The input displacement has been assumed to be in the horizontal direction for simplification. When such an input is given, the output is presented by means of finite-element analysis in Fig. 4. The result indicates that the design equation, Eq.(5), approximately meets the requirements.

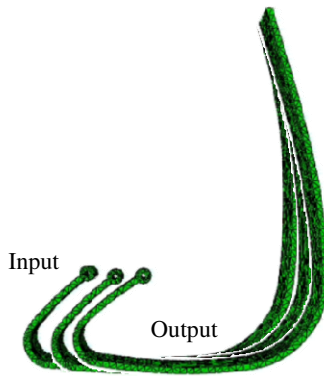
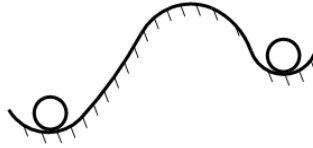


Fig. 4. Result of computer simulation

## 6 Bistable Design

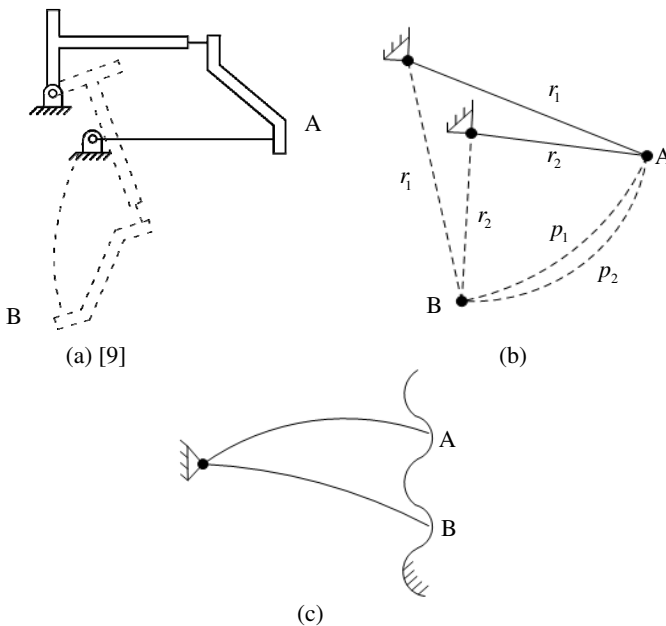
Moreover, the robot leg is designed as a bistable mechanism. A bistable compliant mechanism experiences the following changes when working: the potential energy of the system increases from a minimum value at first and then decreases to another [9], as shown in Fig. 5.



**Fig. 5.** Schematic diagram of bistable characteristic [9]

The design shown in Fig. 6(a) is a common bistable compliant mechanism [9], which has two stable states A and B. A schematic mechanism can be generalized from this type of bistable compliant mechanism, as shown in Fig. 6(b). When transferring from state A to state B, because of the limitation of the length of rod  $r_1$ , the flexible rod end of  $r_2$  changes its path from  $p_2$  to  $p_1$ . Therefore rod  $r_2$  go through a process in which its potential energy falls after rising. So does the potential energy of the entire system.

According to this principle, a bistable mechanism may be as simple as shown in Fig.6(c), whose energy changing process has been set up.



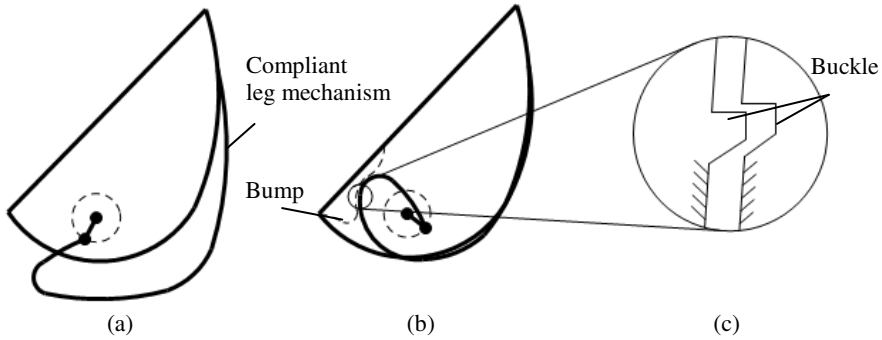
**Fig. 6.** Simplification of a common bistable mechanism

Similarly, the proposed robot leg is designed as a bistable mechanism, with two stable states named “open state” and “contraction state”. “Contraction state” makes it much more portable.

As shown in Fig. 7(a), the compliant leg mechanism generally works in the “open state”. When the mechanism is forced into the casing, it is affected by the bump



shown in Fig. 7(b), which results in a process that the potential energy increases first and then decreases to another state, “contraction state”. Moreover, a buckle is set for a firmer state, as shown in Fig. 7(c). It is because a sudden decrease of potential energy at the minimum value point helps to separate the two states.



**Fig. 7.** Bistable compliant leg mechanism

## 7 Conclusion

This paper presents a novel robot leg designed by a bistable compliant mechanism being applicable to small or micro robot for stable walk. It is designed by means of the pseudo-rigid-body model and analyzed by computer simulation. The result shows that it approximately meets the requirements, and needs further optimization.

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