# **Unsupervised Training of PLDA with Variational Bayes**

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**Abstract.** Speaker recognition relays on models that need a large amount of labeled development data. This models are successful in tasks like NIST SRE where sufficient data is available. However, in real applications, we usually do not have so much data and the speaker labels are unknown. We used a variational Bayes procedure to train PLDA on unlabeled data. The method consisted in a generative model where both the unknown labels and the model parameters are latent variables. We experimented on unlabeled NIST SRE data. The trained models were evaluated on NIST SRE10. Compared to cosine distance, unsupervised PLDA improved EER by 28% and minimum D[CF](#page-8-0) by 36%.

**Keywords:** speaker recognition, PLDA, unsupervised training, variational Bayes, AHC.

# **1 Introduction**

The i-vector approach provides a method to map a speech utterance to a low dimensional fixed length vector while retaining the speaker identity [1]. We can model the i-vector distributions with advanced techniques like probabilistic linear discriminant analysis (P[LD](#page-8-1)A). PLDA is a generative model that decomposes ivectors into a speaker specific part and a channel noise. PLDA models need to be trained on labeled databases with large nu[m](#page-8-2)[be](#page-8-3)r of speakers and sessions per speaker. Unfortunately, in most applications data is scarce and, in many cases, labels are unknown. We intend to train PLDA in t[his](#page-8-4) latter case.

There are previous works that intended to reduce dataset shift to be able to use the same PLDA model in different domains. i-Vector length normalization makes the distributions of different datasets closer. For example, between NIST datasets [2] or between different languages [3]. Bayesian evaluation of likelihood ratios also helps with dataset shift, because the predictive distributions that result, if the amount of training data is [sm](#page-9-0)all, are heavy-tailed [4, 5].

We presented a variational Bayes (VB) method to adapt a full-rank PLDA model from one domain to another with scarce development data  $[6]$ , where

This work has been supported by the Spanish Government and the European Union (FEDER) through projects TIN2011-28169-C05-02 and INNPACTO IPT-2011-1696- 390000.

J.L. Navarro Mesa et al. (Eds.): IberSPEECH 2014, LNAI 8854, pp. 69–78, 2014.

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**Fig. 1.** BN for unsupervised SPLDA

speaker labels were known. Our method was compared with others–parameter or objective function weighting– in the context of the *Domain adaptation challenge* proposed in the 2013 JHU workshop on speaker recognition<sup>1</sup> [7].

The adap[ta](#page-9-1)tion challenge also promoted adapting models using unlabeled data. We adapted a simplified PLDA model from Switchboard data to NIST SRE [8]. The speaker labels and model parameters were hidden variables whose posterior distributions were iteratively estimated by a VB procedure. In this paper, we intend to eva[luat](#page-9-2)e if this procedure is useful to train PLDA from scratch, instead of doing model adaptation. That is, we will not use any labeled data.

Recently, more works about unsupervised adaptation have appeared in re[la](#page-9-3)tion with the challenge. In [9], agglomerative hierarchical clustering (AHC) is used to obtain the speaker labels of the development set. The clustering is based on the pair-wise scores between i-vectors, computed with an out-ofdomain PLDA model. A threshold on the scores, which are unsupervisedly calibrated [10], stops the cluster merging. In [11], several clustering methods were compared (AHC, Markov, infomap). Another approach consists in adding a term accounting for dataset shift to the PLDA model. We can find several flavors of this method [12, 13].

### <span id="page-1-0"></span>**2 Unsupervised SPLDA**

## **2.1 Model Description**

Simplified probabilistic linear discriminant analysis (SPLDA) is a linear gener[ative model that assumes that an i-vector](http://www.clsp.jhu.edu/workshops/archive/ws13-summer-workshop/groups/spk-13/)  $\phi_j$  of speaker i can be written as:

$$
\phi_j = \mu + \mathbf{V} \mathbf{y}_i + \epsilon_j \tag{1}
$$

where  $\mu$  is a speaker independent term, **V** is a low rank eigenvoices matrix,  $\mathbf{y}_i$ is the speaker factor vector, and  $\epsilon_i$  is the within class variability term. We put a standard normal prior on  $y_i$  and normal with zero mean and precision **W** on  $\epsilon_i$ .

<sup>1</sup> http://www.clsp.jhu.edu/workshops/archive/ws13-summer-workshop/ groups/spk-13/

Figure 1 depicts the Bayesian network of this model where the labels  $\theta$  of the training data are hidden.  $\theta$  partitions N i-vectors into M speakers.  $\theta_j$  is a latent variable comprising a 1–of–M binary vector with elements  $\theta_{ji}$  with  $i = 1, \ldots, M$ . Note that the distribution of each speaker is assumed to be Gaussian with mean  $\mu + V y_i$  and precision **W**. The set of all the speakers forms a GMM where  $\theta$ corresponds to the component occupations. The conditional distribution of  $\theta$ given the mixture weights  $\pi_{\theta}$  is

$$
P(\theta|\pi_{\theta}) = \prod_{j=1}^{N} \prod_{i=1}^{M} \pi_{\theta_{i}}^{\theta_{ji}}.
$$
 (2)

We put a Dirichlet prior on the weights:

$$
P(\pi_{\theta}|\tau_{0}) = \text{Dir}(\pi_{\theta}|\tau_{0}) = C(\tau_{0}) \prod_{i=1}^{M} \pi_{\theta_{i}}^{\tau_{0}-1}
$$
(3)

where, by symmetry, we choose the same  $\tau_0$  for all th[e co](#page-9-4)mponents,  $C(\tau_0)$  is the normalization constant,

$$
C(\tau_0) = \frac{\Gamma(M\tau_0)}{\Gamma(\tau_0)^M} \tag{4}
$$

and  $\Gamma$  is the Gamma function.

### **2.2 Model Priors**

We chose the model priors based on Bishop's paper about VB PPCA [14]. We introduced a *hierarchical* prior  $P(\mathbf{V}|\alpha)$  over **V** through a conditional Gaussian distribution of the form:

$$
P(\mathbf{V}|\alpha) = \prod_{q=1}^{n_y} \left(\frac{\alpha_q}{2\pi}\right)^{d/2} \exp\left(-\frac{1}{2}\alpha_q \mathbf{v}_q^T \mathbf{v}_q\right)
$$
(5)

where  $\mathbf{v}_q$  are the columns of **V** and  $n_y$  is the speaker factors dimension. Each  $\alpha_q$ controls the inverse variance of the corresponding  $\mathbf{v}_q$ . If a particular  $\alpha_q$  has a posterior distribution concentrated at large values, the corresponding  $\mathbf{v}_q$  will tend to be small, and that direction of the latent space will be effectively "switched off".

We defined a prior for  $\alpha$ :

$$
P(\alpha) = \prod_{q=1}^{n_y} \mathcal{G}(\alpha_q | a_\alpha, b_\alpha)
$$
 (6)

where  $G$  denotes the Gamma distribution.

We placed a Gaussian prior for the mean  $\mu$ :

$$
P(\mu) = \mathcal{N}(\mu|\mu_0, \beta^{-1}\mathbf{I}) \tag{7}
$$

Low values of  $a_{\alpha}$ ,  $b_{\alpha}$  and  $\beta$  make the priors less informative and vice versa. Finally, we put informative Wishart priors on **W**,

$$
P(\mathbf{W}) = \mathcal{W}(\mathbf{W}|\mathbf{\Psi}_0, \nu_0) .
$$
 (8)

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## **2.3 Variational Bayes with Deterministic Annealing**

We approximated the joint posterior of the latent variables by a factorized distribution of the form:

$$
P(\mathbf{Y}, \theta, \pi_{\theta}, \mu, \mathbf{V}, \mathbf{W}, \alpha | \mathbf{\Phi}) \approx q(\mathbf{Y}) q(\theta) q(\pi_{\theta}) \prod_{r=1}^{d} q(\tilde{\mathbf{v}}'_{r}) q(\mathbf{W}) q(\alpha) \qquad (9)
$$

where  $\tilde{\mathbf{v}}'_r$  is a column vector containing the  $r^{th}$  row of  $\tilde{\mathbf{V}} = [\mathbf{V} \quad \mu]$ . If **W** were diagonal the factorization  $\prod_{r=1}^{d} q(\tilde{\mathbf{v}}'_r)$  would not be necessary because it would arise naturally. However, for full **W**, we have to force the factorization to make the problem tractable.

We computed these factors by using Variational Bayes [15] with deterministic annealing (DA) [16]. The formula to update a factor  $q_i$  is

$$
\ln q_j^* (\mathbf{Z}_j) = \mathrm{E}_{i \neq j} \left[ \kappa \ln P (\mathbf{\Phi}, \mathbf{Z}) \right] + \mathrm{const}
$$
 (10)

where **Z** abbreviates the set of all hidden variables,  $\mathbf{Z}_j$  are the hidden variables corresponding to the  $j<sup>th</sup>$  factor, and  $\kappa$  is the annealing factor; expectations are taken with respect to all the factors  $i \neq j$ . We could prove that Equation (10) optimizes the VB lower bound

$$
\mathcal{L} = \mathrm{E}\left[\ln P\left(\mathbf{\Phi}, \mathbf{Z}\right)\right] - \mathrm{E}\left[\ln q\left(\mathbf{Z}\right)\right]\right] = \ln P\left(\mathbf{\Phi}\right) - \mathrm{KL}\left(q\left(\mathbf{Z}\right) || P\left(\mathbf{Z} | \mathbf{\Phi}\right)\right) \tag{11}
$$

where expectations are taken with respect to the variational posterior  $q(\mathbf{Z})$ .  $\mathcal{L}$  is an approximation of the marginal likelihood of the data  $\ln P(\Phi)$ , which becomes equality when approximated posterior is equal to the true posterior. Annealing modifies the VB objective in a way that helps to avoid local maxima. We must set  $\kappa < 1$  at the beginning and increase it in each iteration until  $\kappa = 1$ . The full VB equations can be found in our report [17].

## **2.4 Initialization with AHC**

The speaker labels were initialized with Agglomerative hierarchichal clustering (AHC) [18]. AHC is a greedy bottom-up approach. Initially, each i-vector is its own cluster and, then, clusters are progressively merged using a similarity criterion–We used cosine distance. Thus, we started with the pair-wise score matrix between all the development i-vectors. Then, to merge two clusters, A and B, we tried tree linkage criteria: average, complete and single. The linkage criterion determines the similarity between the clusters A and B,  $s(A, B)$ , as a function of the pair-wise scores between their elements  $s(a, b)$ . Thus,

$$
s_{\text{avg}}(A, B) = \frac{1}{|A||B|} \sum_{a \in A} \sum_{b \in B} s(a, b)
$$
 (12)

$$
s_{\text{complete}}(A,B) = \min\left\{s(a,b)|a \in A, b \in B\right\} \tag{13}
$$

$$
s_{\text{single}}(A, B) = \max \{ s(a, b) | a \in A, b \in B \} .
$$
 (14)

#### **2.5 Model Selection**

To select the best model, i.e., the best number of speakers  $M$ ; we used the same method that in our previous work [8]. We ran the AHC+VB algorithm several times, each time hypothesizing a different M. We assumed that the best model is the one that obtains the largest VB lower bound  $\mathcal{L}(M)$ . To fairly compare lower bounds for different M, the Dirichlet prior on the speaker weights needs to be such that the product  $M_{\tau_0}$  is constant. To select the value of that constant, we tried [sev](#page-9-2)eral values and chose the one that maximized the sum  $\sum_M \mathcal{L}(M)$ .

#### **3 Experiments**

### **3.1 Experimental Setup**

We trained PLDA on an unlabeled version of NIST SRE04-08. The i-vectors for this task were provided by the JHU [HL](#page-8-6)T-COE in the 2013 JHU workshop on speaker recognition [11]. The training data consisted of 33125 segments from 3789 speakers. To perform faster experiments, we also created a subset of 500 speakers. The adapted models were evaluated on the NIST SRE10 det5 (tel-tel) extended condition.

The i-vectors were 600 dimensional. Th[ey](#page-9-6) were extracted using 20 MFCC  $+$  $\Delta$  with short time mean and variance normalization. The UBM and i-vector extractor were gender independent and used 2048 Gaussians. We applied centering, whitening and length normalization to the i-vectors [2]. The parameters needed for centering and whitening were trained on all NIST SRE data since speaker labels are not required.

The SPLDA models were gender independent with speaker factors of dimension  $n_y = 400$  when training with 500 speakers; and  $n_y = 600$  when training with all the speakers. Given the results in our previous work [8], we put informative priors on the model parameters. Our priors were based on the average total variance of the data  $s_0^2$ -average across dimensions. From our previous work, we assumed that the average variance of the speaker space was approximately 15% of  $s_0^2$  and the channel variance was the remaining 85%. Thus, we computed  $s_0^2$ from the training data. Then, for  $\alpha$  (prior of the inverse eigenvalues), we placed a wide prior with mode  $1/(0.15s_0^2)$  by setting  $a_\alpha = 2$  and  $b_\alpha = 0.15s_0^2$ . For **W**, we used a Wishart prior with expectation  $1/(0.85s_0^2)$ **I** by setting  $\nu_0 = 602$  and  $\Psi_0 = 1/(0.85s_0^2 \nu_0)$ **I**. Note that, for the Wishart prior to be proper, we need  $\nu_0 > d$ , this means that the prior will have an important influence on the posterior unless we have a number of training segments  $N \gg d$ . We set  $\tau_0 = 400/M$ .

The expectations of the model parameters given the VB posteriors were used to compute the likelihood ratios of the evaluation set in the standard way.

#### **3.2 Experiments Results**

First, we focus on the results obtained by training the PLDA with 500 speakers. Figure 2 plots the EER and VB lower bound against the number of hypothesized speakers M. Each subfigure corresponds to one of the linkage criteria used in



**Fig. 2.** EER(%)/ $\mathcal{L}$  against the number of hypothesized speakers for different initialization methods and using a subset of 500 development speakers



(b) Complete linkage clustering.

**Fig. 3.** EER(%)/ $\mathcal{L}$  against the number of hypothesized speakers for different initialization methods and using all the development speakers

the AHC initialization. The left y-axes show the scale of the EER, and the right y-axes show the scale of the lower bound. The horizontal dashed line indicates the baseline–cosine similarity– and the vertical dashed line indicates the point where  $\mathcal L$  is maximum. Regarding the detection of the number of speakers in the development set, average and complete linkage criteria had their  $\mathcal L$  maxima

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	M Actual				M Max $\mathcal L$			
		$EER(\%)$ MinDCF		M $\mathcal{L} \times 10^{-6}$ EER(%) MinDCF			M	$\mathcal{L} \times 10^{-6}$
Baseline (Cosine)	5.96	0.66			5.96	0.66		
Oracle labels	3.02	0.50	500		3.02	0.50	500	
VB average link	4.96	0.58	500	5.8060	5.00	0.58	508	5.8066
VB complete link	5.16	0.60	500	5.8048	5.35	0.61	520	5.8054
VB single link	7.61	0.77	500	5.7830	5.37	0.61	925	5.7987
Oracle labels	2.19	0.42	3789		2.19	0.42	3789	
VB average link	3.14	0.44	3789	36.977	4.53	0.47	7000	37.036
VB complete link	3.67	0.46	3789	37.014	4.25	0.47	5500	37.041

**Table 1.** EER(%)/MinDCF for different initialization of the VB. The table blocks correspond to training 500 or all the speakers.

close to the actual value. In contrast, single linkage almost doubled the speakers. Nevertheless, the maximum  $\mathcal L$  was a good criterion to select a model with low EER for the three cases. For average and complete linkage, it selected the point of minimum EER. For complete linkage it did not choose the minimum EER but a point quite near of it. In the three cases, it significantly improved the baseline.

Table 1 compares EER and minimum DCF for multiple cases. The table also shows the number of speakers and the  $\mathcal L$  obtained in each case. The table has two column blocks. The left block shows results for the model corresponding to the actual M; and the right block to the model that maximizes  $\mathcal{L}$ . The upper block of rows correspond to the development set of 500 speakers. Average linkage obtained the lowest EER and DCF for both model selection methods (M actual and max.  $\mathcal{L}$ ). Also for both methods average linkage obtained the highest  $\mathcal{L}$ , so we can use  $\mathcal L$  to choose the best initialization. The Max  $\mathcal L$  model improved the baseline by 16% and 12% in terms of EER and DCF. With respect to training with oracle labels, we still have a margin of improvement of 39% and 14% respectively. Single linkage was the worst initialization method so we discarded it for the following experiments.

Figure 3 plots EER and  $\mathcal L$  against M when training with all the development speakers (3789). In this case,  $\mathcal{L}$  was maximum for M much higher than its actual value–almost twice for average linkage. The reason is that, when we increase the number of speakers, we increase the probability of finding speakers with overlapping i-vector distributions and clustering becomes harder. Despite of that, the selected models outperformed the baseline. Average linkage obtained the best EER and DCF for the models with oracle M but complete linkage was better for the model maximizing  $\mathcal{L} \mathcal{L}$  was also higher for complete linkage. With respect to the baseline, EER improved by 28.7% and DCF by 36.4%. With respect to oracle model selection, we have a margin for improvement of 26% and 6% respectively; and with respect to oracle labels, margins of 48.5% and 10%. We can see that those margins are still very high. As we increase the amount of data, the margin between the unsupervised and supervised models also increases. As clustering becomes harder, there is a point where increasing the amount of unsupervised data is not beneficial.

# **4 Conclusions**

We presented a method to train SPLDA models with unsupervised labels. We designed a generative model where labels and model parameters are hidden variables that are updated iteratively with a variational Bayes procedure. The speaker labels were initialized using AHC with different linkage criteria. The best criteria were average and complete linkage. The VB procedure was run several times, each time hypothesizing a different number of development speakers. We selected the model that maximized the VB lower bound.

<span id="page-8-0"></span>We experimented training on unlabeled NIST SRE04-08 data. We evaluated the resulting model on the NIST SRE10 det5 condition. For training with 500 speakers, the algorithm was able to select almost the best model. Compared to cosine distance, EER improved by 16% and minimum DCF by 16%. For training with 3789 speakers, clustering becomes harder and we did not selected the best model. However, EER improved by 28% and DCF by 36%. Despite that these gains were significant, there is still a large margin of improvement to match the results of supervised training.

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