An Overview of the Black-Scholes-Merton Model After the 2008 Credit Crisis

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Abstract The 2008 credit crisis exposed the over-simplified assumptions of the Black-Scholes-Merton (BSM) model. This paper provides an overview of some of the adjustments forced on the BSM model by the 2008 credit crisis to maintain the relevance of the model. The inclusion of credit value adjustment (CVA), debit value adjustment (DVA), funding value adjustment (FVA) and the posting of collateral in the BSM model are discussed.

1 Introduction

The credit crisis of 2008 was a dramatic event for financial markets. This was the beginning of the financial tsunami that would plague and force changes in global markets for many years to come. The economic meltdown that followed had massive effects on many everyday issues such as house prices, interest rates and inflation. Investment banks were also affected and numerous investment banks either defaulted or were taken over by the U.S. Federal Reserve to avoid default. The impact on financial derivative pricing did not escape the 2008 credit crisis.

Prior to the 2008 credit crisis, pricing the value of a derivative was relatively straightforward. Universally, practitioners and many academics agreed on the pricing method used to price a derivative. The method was well-known: discount future expected cash flows under the risk-neutral measure to the present date using the

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risk-free rate. This method was derived from the fundamental theory laid down by Black, Scholes and Merton in the 1970s (see Black and Scholes [3] and Merton [26]).

Although there are many known approaches to option pricing, which includes heavy-tailed distribution techniques, the continuous time Black-Scholes-Merton (BSM) model is considered by many financial practitioners to be adequate for option pricing, irrespective of its over-simplified assumptions. It was and still is, widely used in practice, as it is well understood and yields a framework in which both pricing and hedging is possible. The deep-rooted acceptance of the BSM model is further cemented by the fact that the discrete time Cox, Ross and Rubinstein (CRR) model, which is a discretisation of the BSM model, is very useful and easy to implement in practice (see Cox et al. [10]).

The 2008 credit crisis drove home the fact that what was used in practice prior to the crisis as an approximation (also called a proxy) for the theoretical notion of a risk-free interest rate, as required by the BSM model, is totally inadequate to yield realistic results.

The myth that banks are risk-free was disproved by the 2008 credit crisis. The default of what we used to call *too big to fail* banks, such as Lehman Brothers and Bear Stearns, which defaulted in the 2008 credit crisis, disproved the myth that banks are risk-free (see Gregory [15]).

The 2008 credit crisis also exposed the inadequate management of counterparty credit risk. Counterparty credit risk (also known as default risk) between two parties, is the two-sided risk that one of the counterparties will not pay, as obligated on a trade or a transaction between the two parties.

Changes need to be made to the usual ways in which "business was conducted" prior to the 2008 credit crisis and these changes need to be addressed and incorporated in the models used prior to the 2008 credit crisis.

The aim of this paper is to present the current state of affairs with regard to the BSM model.

For terminology not explained in the paper, the reader is referred to Alexander [1] or Hull [18].

2 Credit Value Adjustment (CVA) and Debit Value Adjustment (DVA)

Over-the-counter (OTC) derivative trading is done directly between two parties without any supervision on an exchange (this, however, is going to change in the future due to stipulations in the Basel III Accord).

Even before the 2008 credit crisis, banks realised that many corporate clients are not risk-free; therefore, in OTC derivative trades, banks charged their clients a credit value adjustment (CVA). CVA is defined as the fair market value of the expected loss of an OTC derivative trade given that the opposite counterparty defaults.

Many of the banks' clients believed that banks were risk-free; therefore, the clients would accept the price that banks offered them and in turn did not charge banks a

CVA on the trade. Reference papers on CVA include Sorensen and Bollier [29], Jarrow and Turnbull [24] and Duffie and Huang [13].

As a result of the 2008 credit crisis, banks are not seen as risk-free anymore. One implication of this is the inclusion of a debit value adjustment (DVA) in the derivative's price. DVA is defined as the fair market value of the expected gain of an OTC derivative given own default. The origins of DVA are found in Duffie and Huang [13]; however, their paper deals mostly with swaps. Gregory [14] and Brigo and Capponi [4] examine bilateral credit risk in general and derive DVA formally. In essence, DVA is the adjustment clients charge the bank for the bank's own credit risk. Therefore, from the client's point of view, the adjustment is known as CVA and from the bank's point of view the adjustment is known as DVA.

One aspect of pricing with CVA and DVA is that it allows two credit-risky counterparties to trade with each other. If two counterparties charged each other a CVA and do not include the offsetting DVA term, then the two counterparties would not agree on the price of the derivative. The inclusion of DVA allows symmetric prices. The concept of symmetric prices means that two counterparties will price the derivative at the same price.

DVA is a hotly debated and controversial quantity. The reason for the controversy is that the DVA amount can only be realised when the bank defaults. If the bank is out-of-the-money on a trade and defaults, then the bank only needs to pay a recovery of the mark-to-market (MTM); therefore, the bank benefits from its own default. It can be compared to buying life insurance. The policy will only be realised after the death of the policy holder. Some practitioners argue that DVA simply cannot be hedged effectively.

Gregory and German [16] describe DVA as a double edged sword. On the one hand, it creates a symmetric world where counterparties can readily trade with one another, even when their underlying default probabilities are high. On the other hand, the nature of DVA and its implications and potential unintended consequences create some additional complexity and potential discomfort. From an accounting point of view, adding DVA to the price makes sense; however, the regulators are not so sure. Risk Magazine on 6th February 2012 reported that

> Accountants want banks to report as profits the impact of widening credit spreads on their liabilities, but regulators are moving in the other direction.

(see Carver [8]). The accounting rules International Financial Reporting Standards (IRFS) 13 and Financial Accounting Standards Board (FASB) 157 require DVA. However, the Basel III committee has decided to ignore any DVA relief in capital calculations.

3 The Risk-Free Rate: The Proxies LIBOR Versus OIS

The BSM model requires that one has to discount future expected cash flows under the risk-neutral measure using the risk-free rate. The risk-free rate is the theoretical rate of return on an investment with no risk of financial loss. The risk-free rate defines the expected growth rates of market variables in a risk-neutral world.

In practice, the pertinent question is: which interest rate should be used as a proxy for the risk-free rate?

The London Interbank Offered Rate (LIBOR) rate is the rate that banks could freely borrow and lend at. Prior to the 2008 credit crisis, practitioners constructed a curve from LIBOR rates, Eurodollar futures and swap rates, which Hull and White [21] refer to as a LIBOR swap curve. The 3-month LIBOR swap curve was used by practitioners as a proxy for the risk-free rate.

An overnight interest rate swap (OIS) is a swap for which the overnight rate is exchanged for a fixed interest rate for a certain tenor (also known as maturity). An overnight index swap references an overnight rate index, such as the Fed funds rate, as the underlying for its floating leg, while the fixed leg would be set at an assumed rate.

Before the 2008 credit crisis, the LIBOR-OIS spread, which is the difference between the LIBOR rate and the OIS rate, was only a few basis points. It was stable and not significant (see Gregory [15]).

The 2008 credit crisis caused a significant spread between 3-month LIBOR and the OIS rate. The LIBOR-OIS spread spiked to hundreds of basis points in the aftermath of the default of Lehman Brothers in September 2008 and has remained significant ever since. Many practitioners believe that the spread between LIBOR and OIS rates describes the health of the banking industry. As one can see from Fig. 1 the banks in 2008 were not "in good shape" during the crisis. The fact that the LIBOR-OIS spread has remained significant illustrates why banks are not risk-free. These shifts made it apparent that LIBOR incorporates an adjustment for the credit risk of banks and swap

Fig. 1 The spread between 3-month LIBOR and OIS during the 2008 crisis. *Source* http://www.soberlook.com, May 31, 2014

rates correspond to the risk of unsecured short-term loans to financial institutions; therefore, the LIBOR swap curve is an imperfect proxy for the risk-free rate. The OIS rate appears to be the preferred choice as a proxy for the risk-free rate (see Hull and White [21] and Hunzinger and Labuschagne [23]).

4 Collateral and Funding Costs

The 2008 credit crisis emphasised the importance of the managing of counterparty credit risk.

One of the ways to mitigate counterparty credit risk is by posting collateral in a derivative trade. Collateral is a borrower's pledge of specific assets to a lender, to secure repayment of a liability.

Banks required collateral posted from their counterparties on certain trades prior to the 2008 credit crisis. But as it became apparent that banks are not risk-free, clients require that banks now also post collateral on some transactions. For exchange traded derivatives, i.e. stock option, counterparty credit risk is not an issue, because the two counterparties in the trade are required to post margins to the exchange.

The posting of collateral in a derivative trade is regulated by a Credit Support Annex (CSA). A CSA is a contract that documents collateral agreements between counterparties in trading OTC derivative securities. The trade is documented under a standard contract called a Master Agreement, developed by the International Swaps and Derivatives Association (ISDA). The 2010 ISDA margin survey suggests that 70 % of net exposure arising from OTC derivative transactions are collateralised (source: www2.isda.org).

After the 2008 credit crisis many banks have started to use OIS rates for discounting collateralised transactions and LIBOR swap rates for discounting non-collateralised transactions. This can be clarified by considering the fundamental paper of Piterbarg [27] in which he notes fundamental facts regarding derivative pricing when collateral is posted.

Piterbarg [27] notes that when pricing a zero-threshold CSA trade, where the collateral is cash and in the the same currency as the derivative, the cash flows should be discounted using the collateral rate of that particular currency. Collateral posted overnight will earn a rate similar to the index rate referenced in an OIS. Furthermore, when the trade is not collateralised, then the cash flows should be discounted using the funding rate of the bank. He also notes that one may price a derivative trade by always discounting the future expected cash flows using a collateral rate and making a funding value adjustment (FVA). FVA is a correction made to the risk-free price of an OTC derivative to account for the funding cost in a financial institution.

Posting collateral in an OTC trade may mitigate counterparty credit risk and funding costs; however, this depends on the collateral posted in the trade and how often this collateral is readjusted according to market movements. Collateral can be changed daily, weekly or monthly, which will affect the exposures of the two counterparties.

The 2008 credit crisis drove home the realisation that banks are not risk-free. This resulted in banks becoming reluctant to lend to each other and banks became unable to borrow at preferential rates. This resulted in banks charging a FVA on transactions. When managing a trading position, one needs cash to conduct operations such as hedging or posting collateral. This shortfall of cash can be obtained from the treasury of the bank. The funding cost adjustment (FCA) is the cost of lending money at a funding rate which is higher than the risk-free rate. The firm may also receive cash in the form of collateral or a premium. The funding benefit adjustment (FBA) is the benefit earned when excess cash is invested at a higher rate than the risk-free rate. Therefore, the funding value adjustment has two components

$$\mathrm{FVA} = \mathrm{FBA} + \mathrm{FCA},$$

where the FBA and the FCA terms have opposite signs.

Funding value adjustment arises because of two factors. Firstly, because banks cannot borrow at the risk-free rate any more and secondly because of collateralised trades. Figure 2 illustrates how a funding cost adjustment arises in terms of a trading floor set-up. Let us say for example a trader enters into a trade with a corporate client and at this point in time the trader is in-the-money on the trade. At the same time the trader enters into another trade with a hedge counterparty to hedge out the trader's exposure to the client. Because the trader is in-the-money on the client trade, the trader will be out-the-money on the hedge. Let us also assume that the trade with the hedge counterparty is collateralised; therefore, the trader is required to posted collateral to the hedge counterparty. The collateral posted by the trader will earn a collateral rate. If the client trade was not traded with a CSA (no collateral will be posted in the trade) and then the trader needs to fund the collateral requirement from the treasury of the bank. The trader cannot fund a short fall of cash from the treasury

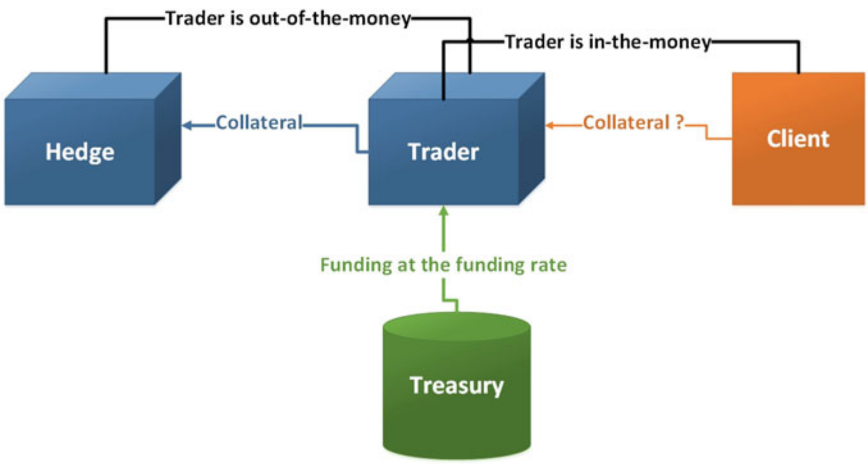

Fig. 2 A graphical illustration of funding cost adjustment

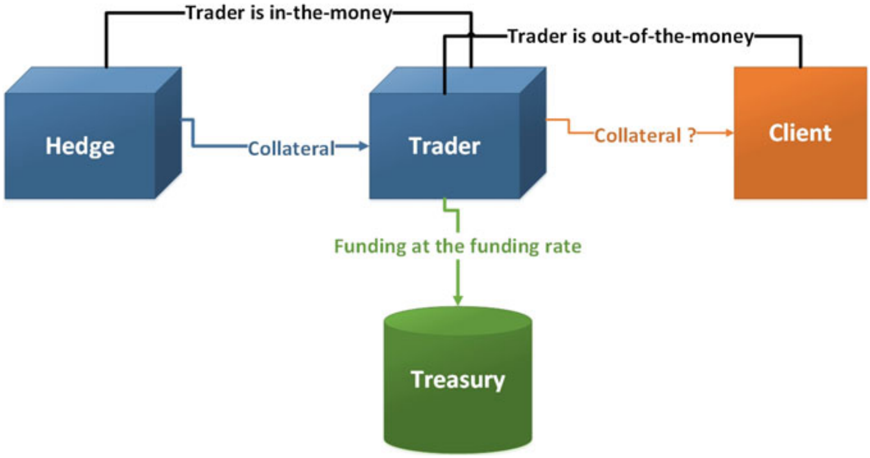

Fig. 3 A graphical ilustration of funding benefit adjustment

at the risk-free rate but at a higher funding rate. This excess cost of funding at the funding rate is exactly a funding cost adjustment. On the other hand, if the client trade is traded with a CSA, the client will be required to post collateral to the trader, and hence the trader can pass this collateral amount to the hedge counterparty. This situation results in no funding costs. The natural question at this point is: how does this scenario now differ from that prior to 2008? Before the crisis, the trader could fund from the treasury at a risk-free rate; hence, if the trader received collateral from the client or required funding from the treasury, this funding is at the risk-free rate. Therefore no funding cost adjustment would occur in this set-up.

Figure 3 illustrates how a funding cost adjustment arises in terms of a trading floor set-up. In this case, the trader is out-of-the-money on the client trade and in-the-money on the hedge. Now the hedge counterparty is required to post collateral with the trader. If no CSA is placed between the client and the trader, then the trader can place these funds with the treasury and earn a rate better than the collateral rate. This extra benefit is known as a funding benefit adjustment. If the trader is required to post collateral to the client, then there is no resulting benefit.

In this example, we assume that rehypothecation is possible. Rehypothecation is the practice by banks and brokers of using, for their own purposes, assets that have been posted as collateral by their clients.

5 The FVA Debate

The inclusion of FVA in pricing financial instruments is a controversial issue. Hull and White [19, 20] argue against it. They argue that the funding costs and benefits realised in a trade, violate the idea of risk-neutral pricing and should not be included in the pricing of the derivative. Inclusion of FVA in the price of a derivative trade violates the law of one price in the market because the two counterparties may price a trade and obtain a different outcome.

There are views that are different from those of Hull and White. Laughton and Vaisbrot [25] suggest that in practice the market is not complete and the uniqueness of prices and the law of one price will not hold (see Harrison and Pliska [17]). They state that applying the so called FVA to the risk-neutral value, is justified. Banks with a lower funding rate will be more competitive on trades that require funding. This is fully consistent with the current situation in the markets, as theory should aim to be. In summary, they believe that the beautiful and elegant theory of BSM is not applicable and needs to be rethought because of the theory's unrealistic assumptions, especially post the credit crisis.

Castagna [9] also disagrees with Hull and White. In the BSM model there is only one interest rate and that is the risk-free interest rate. Castagna suggests if one considers a framework where more than one interest rate exists, such as a risk-free rate and a funding rate, then one could still produce a replicating portfolio which perfectly replicates the derivative. If a bank can only invest at the risk-free rate and fund at a higher funding rate, then it is well known that this will not impediment the replication of the derivative (see Bergman [2], Rubinstein and Cox [28] and Hunzinger and Labuschagne [23]). This will lead to a different prices for the buy side and the sell side; however, a closed form solution will still exist. Castagna suggests that models need to be amended in order to be more useful to traders. They should remove the assumption of the ability to borrow at the risk-free rate to finance trades.

Inclusion of both FBA and DVA in the price could also lead to double counting: FVA references the firms own funding spread (which is the difference between the funding rate of the bank and the risk-free rate) in both terms, FCA and FBA. The funding spread is based on the credit rating of the firm. The counterparty's credit spread (which is the difference between the yield on a firm's credit risky bond and the yield of a risk-free bond) is referenced in the CVA term and the firm's own credit spread in the DVA term. A change in the credit rating of the bank leads to a change in the price of the derivative. Since the DVA and FBA terms have the same sign, the change in the price is reflected twice if both the DVA and FBA terms are included in the valuation.

6 The BSM Model

In order to discuss extensions to the BSM model which follow from the discussions above, we include a summary of the BSM model and its assumptions for the convenience of the reader.

The interest rate assumptions of the BSM model are:

- The BSM model is a model with a single interest rate.
- This interest rate is the risk-free rate r.

The extensions to BSM model are concerned with amendments to these interest rate assumptions. The extended BSM model assumes multiple interest rates.

Other assumptions of the BSM model include:

- Stock prices follow geometric Brownian motion.
- Short selling is permitted.
- There are no taxes and transaction costs.
- No dividends on the underlying (although the BSM model can be adjusted to include dividends).
- No arbitrage opportunities exist.
- Continous trading of securities.

Given the described assumptions, we present the BSM partial differential equation (PDE). Let T denote the fixed time of maturity of a derivative contract and σ as the volatility of the underlying security, in this case a stock price. The Black-Scholes-Merton PDE is given by

$$\frac{\partial f}{\partial t} + rS\frac{\partial f}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} = rf$$

where f, is the price of a derivative which is contingent on the stock price S_t and time $t \in [0, T]$. For a European call and put option with strike K, the BSM PDE has solution

$$V_t = \alpha \left(S_0 N\left(\alpha d_1\right) - K e^{-r(T-t)} N\left(\alpha d_2\right) \right)$$

where

$$d_1 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r + \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{(T-t)}}$$

and

$$d_2 = d_1 - \sigma\sqrt{T-t},$$

where $\alpha = 1$ for a call option and $\alpha = -1$ for a put option. $N(x)$ is the cumulative distribution function of the standard normal distribution.

In practice a continuous-time model, such as the BSM model, is difficult to implement and is usually discretised to facilitate implementation. The Cox, Ross and Rubinstein (CRR) discrete-time model is a discretisation of the BSM model. Details of the CRR model can be found in Cox et al. [10] or Steland [30].

6.1 The BSM Model Which Includes Collateral and Funding Costs

Standard pricing theory excludes the intricacies of the collateralisation of the market. The posting of collateral in a derivative trade changes the traditional way in which a derivative is priced.

Piterbarg [27] extends the BSM continuous-time model to include collateral in a derivative trade and shows how the posting of collateral in a derivative trade affects the price.

In the Piterbarg model, the price of a collateralised derivative trade is given by

$$\text{risky price} \;=\; \text{risk-free price} \;+\; \text{FVA}.$$

The risk-free price is the BSM-price of a derivative that includes no credit risk and funding costs. This price is calculated by discounting all expected cash flows at the risk-free rate. The risky price is defined as the risk-free price plus any adjustments. Piterbag's paper won him the Quant of the Year Award in 2011.

It is possible to extend the CRR model to include dividends and collateral. Moreover, by using ideas along the lines of those in Hunzinger and Labuschagne [23], it can be shown that discretising Piterbarg's model (which is the BSM model that includes collateral and dividends) coincides with the aforementioned model. This is achieved by showing that Piterbarg's PDE, which represents the value of a collateralised derivative trade, can be represented as an expectation via the Feynman-Kac theorem.

6.2 The BSM Model Which Includes CVA, DVA and FCA

Currently, there are three unified frameworks which incorporate funding costs, collateral and credit risk into a derivative trade. These frameworks are proposed by

1. Piterbarg (see [27]), Burgard and Kjaer (see [6, 7]).
2. Brigo et al. (see [5]).
3. Crépey (see [11, 12]).

We take a closer look at the Burgard and Kjaer framework. The model proposed by Burgard and Kjaer [6] gives the price of a derivative trade by

$$\text{risky price} = \text{risk-free price} + \text{CVA} + \text{DVA} + \text{FCA},$$

where the CVA and DVA terms have opposite signs. The risky price is given by the risk-free price plus the adjustments for CVA, DVA and FCA.

It is possible to extend the CRR model to include CVA, DVA and FCA. Moreover, it can be shown that discretising the Burgard and Kjaer model (which is the BSM model that includes CVA, DVA and FCA) coincides with the aforementioned model. This is achieved by showing that Burgard and Kjaer's PDE can be represented as an expectation via the Feynman-Kac theorem. The details may be found in Hunzinger and Labuschagne [23].

Burgard and Kjaer [7] extends these two models discussed in Sects. 4 and 5 to create a general framework to price a credit risky derivative that is collateralised. This general framework, which is in the form of a PDE, reduces to the models presented in the previous two subsections if certain assumptions are made, the details of which are contained in Burgard and Kjaer [7].

7 Conclusion

The adaptions of the BSM model required post the 2008 credit crisis are hotly debated amongst academics and practitioners. There is an intense controversy in the financial quantitative industry regarding inclusion of FVA when pricing financial instruments, as it could be argued that inclusion of FVA violates the law of one price.

The 2008 crisis has had a massive effect on derivative pricing and has plagued our markets with uncertainty. There is no general consensus about how to price a derivative trade after the 2008 credit crisis. This uncertainty has presented regulators, practitioners and academics with new challenges around financial markets. If all the market participants share ideas, then these challenges could possibly be overcome.

The debate continuous.

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