Colour Image Quantisation using KM and KHM Clustering Techniques with Outlier-Based Initialisation

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Abstract This chapter deals with some problems of using clustering techniques K-means (KM) and K-harmonic means (KHM) in colour image quantisation. A lot of attention has been paid to initialisation procedures, because they strongly affect the results of the quantisation. Classical versions of KM and KHM start with randomly selected centres. Authors are more interested in using deterministic initialisations based on the distribution of image pixels in the colour space. In addition to two previously proposed initialisations (DC and SD), here is considered a new outlier-based initialisation. It is based on the modified Mirkin's algorithm (MM) and places the cluster centres in peripheral (outlier) colours of pixels cloud. New approach takes into account small clusters, sometimes representing colours important for proper perception of quantised image. Pixel clustering was created in the *RGB*, *YCbCr* and *CIELAB* colour spaces. Finally, resulting quantised images were evaluated by means of average colour differences in *RGB* (*PSNR*) and *CIELAB*(ΔE) colour spaces and additionally by the loss of colourfulness (ΔM).

1 Introduction

True colour images acquired by a camera contain only a small subset of all possible 16.7 million colours. Therefore, it makes sense to further reduce the number of colours in the image. Nowadays, the colour image quantisation (CIQ) is an important auxiliary operation in the field of colour image processing and is very useful in image compression, image pre-segmentation, image watermarking and content-based image retrieval (CBIR). These algorithms are also still used to present the true colour images on devices with limited number of colours. CIQ reduces significantly the

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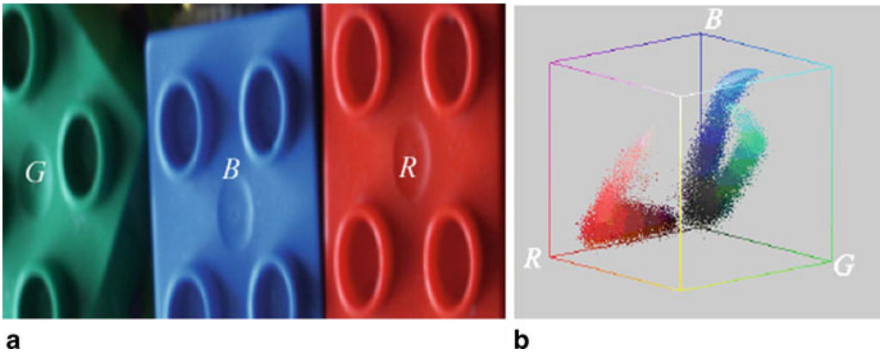


Fig. 1 Simple colour image and its clusters in *RGB* colour space

number of colours in the image to the specially selected set of representative colours (colour palette). Colour palette generation is the most important step in any CIQ method. Proper choice of the colour palette helps minimize the colour difference between the original image and the quantised image.

There exist two main classes of CIQ techniques: splitting techniques and clustering techniques [1]. The splitting techniques divide the colour space into smaller disjoined subspaces and then a colour palette is built by choosing representative colours from these subspaces. Good examples of such techniques are the Median Cut [8], Octree [5] and Wu's [16] algorithms. For example, the Median Cut method first locates a tightest box in *RGB* colour space, that encloses all image colours. Then, the box is cut on the longest side and two subboxes are formed. As a result of a such cut both subboxes should contain the same number of colours and from here comes the name of the method. Next, a subbox with longest side is cut. This process continues until the total number of subboxes is smaller than the number of colours in the palette chosen for the quantised image. All colours in one subbox are represented by their mean value.

On the other hand, the clustering techniques are the optimization tasks that minimize the quantisation errors by minimization the sums of distances between the cluster centres and cluster points. One of the most popular clustering techniques is the K-means (KM) technique [10] and its existing modifications e.g. K-harmonic means (KHM) technique [19]. The clustering has a long tradition of use to quantize colour images [18]. It can be easily to see that each of the dominant colours in natural image corresponds to a separate fragment of pixels cloud in the colour space, which can be called a cluster (Fig. 1). As a generally statement, it may be found that the splitting techniques are faster than the clustering techniques but they have larger quantisation errors.

The results of many clustering techniques depend on method of determination of initial cluster centres, used colour space, applied colour metric etc. Such sensitivity to initialisation is an important disadvantage of these clustering techniques. A random selection of the initial centres, used in classical KM version, is not able to achieve repeatable results in colour image quantisation. Therefore, in our previous paper [3] we attempted to use two new heuristic methods of initialisation. The first method, which is an arbitrary one, is based on uniform partitioning of diagonal of *RGB* cube (DC) into k segments. Gray levels in the middle of segments are used as initial centres. If an image is clustered into k clusters, k initial cluster centres are located on the gray level axis. The second method, which is an adaptive one, uses a size of pixel cloud of a colour image and the method has been marked as SD. First, the mean value and standard deviation (SD) for each *RGB* component of all image pixels are calculated. Then, around the point of mean colour (a pixel cloud centre) a rectangular cuboid with sides equal $2\sigma_R$, $2\sigma_G$ and $2\sigma_B$ is constructed. We assume that it lies within the *RGB* cube. Next, the main diagonal of the cuboid is divided into k equal segments. The centres of these segments are used as initial cluster centres. Initial cluster centres in KM can also come directly from splitting algorithms e.g. from MC or Wu's algorithms and such combined approach (MC+KM, Wu+KM) was proposed few years ago [14]. Experiments have shown that Wu+KM technique offers a slightly better performance than MC+KM and KM initialised by SD approach.

Appropriate initialisation provides the high quality clustering achieved by running small number of iterations and avoids the formation of empty clusters, which sometimes occurs in the case of DC initialisation. The result of empty clusters is a reduction in the number of colours in quantised image. Removing empty clusters needs changing the cluster centres or splitting a newly created cluster. Good initialisation for the KM technique, used in colour quantisation, is still looked for by many researchers [2].

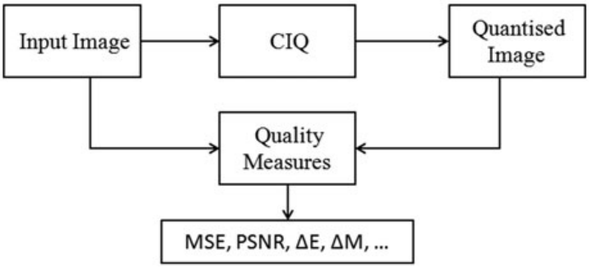
The KHM is based on harmonic means, instead of arithmetic means and additionally uses fuzzy membership of pixels to clusters and dynamic weight functions, what means different influence an individual pixel on calculating new values of centres in each iteration. KHM is robust to initialisation and creates non-empty clusters. A disadvantage of KHM in relation to KM is greater computational complexity, resulting in a longer computation time.

The clustering process can be conducted not only in the *RGB* colour space, but also in other colour spaces. Here a special role is played by recommended in 1976 the *CIELAB* colour space [17]. It is a perceptually uniform colour space which approximately expresses a way of human colour perception. The Euclidean distance in this space is approximately equal to the perceptual colour difference. This should be of great importance in the process of clustering. Unfortunately, the transform from *RGB* to *CIELAB* is complicated and nonlinear.

The *YCbCr* colour space is applied in CIQ task among other used colour spaces. Its advantage, in comparison to *CIELAB* colour space, is a linearity of transformation from *RGB* space, which results in faster calculation of the *YCbCr* components. Although the colour difference in the *YCbCr* space less corresponds to the human colour perception than the colour difference calculated in *CIELAB*, however makes it better than the Euclidean distance calculated in *RGB* space. The *YCbCr* components can be received from the following transformation [9]:

$$Y = 0.257R + 0.504G + 0.098B + 16 \tag{1}$$

Fig. 2 The general idea of CIQ quality measure



$$Cb = -0.148R - 0.291G + 0.439B + 128 \tag{2}$$

$$Cr = 0.439R - 0.368G - 0.071B + 128 \tag{3}$$

Therefore, later in this chapter, are tested the CIQ methods in the following colour spaces: basic *RGB*, *YCbCr* and perceptually uniform *CIELAB*.

This chapter is organized as follows. In Sect. 2, we present two typical and one untypical quality measures used for CIQ quality evaluation. The results of experimental tests for determination of factors influencing the quantisation errors are described in Sect. 3. The idea of the new proposed initialisation method (modified Mirkin's algorithm) is illustrated on several images in Sect. 4. Section 5 shows on a larger set of images a usefulness of a new MM initialisation in quantisation process, which is oriented toward image segmentation. Finally we conclude the chapter in Sect. 6.

2 CIQ Quality Measures

The colour quantisation error depends on the number of colours in palette (e.g. 256, 64, 16, 8, 4 colours): the smaller number of colours in palette, then larger is the quantisation error. Objective CIQ quality measures (Fig. 2) are very important in the evaluation process of different colour quantizers.

Commonly used most popular measure is the Mean Squared Error (*MSE*) defined by:

$$MSE = \frac{1}{3MN} \sum_{i=1}^{M} \sum_{j=1}^{N} \left[(R_{ij} - R^*_{ij})^2 + (G_{ij} - G^*_{ij})^2 + (B_{ij} - B^*_{ij})^2 \right] \tag{4}$$

where M and N are the image dimensions in pixels, R_{ij}, G_{ij}, B_{ij} are the colour components of the pixel at location (i, j) in the original image and R^*_{ij}, G^*_{ij}, B^*_{ij} are the colour components of the pixel in quantised image. The smaller the *MSE* value, the better is the quantised image. Other error measure applied to evaluation of quantisation is Peak Signal-to-Noise Ratio (*PSNR*), good correlated with *MSE* value and expressed in decibel scale:

$$PSNR = 20\log_{10} \frac{255}{\sqrt{MSE}} \tag{5}$$

Unfortunately, these both measures that come from the signal processing field are poorly correlated with subjective visual quality of an image. The quantisation error can be treated as a colour error that should be determined in a perceptually uniform colour space. Therefore, an average colour difference in *CIELAB* colour space (ΔE) is sometimes applied as a quantisation error:

$$\Delta E = \frac{1}{MN} \sum_{i=1}^{M} \sum_{j=1}^{N} \sqrt{(L_{ij} - L^*_{ij})^2 + (a_{ij} - a^*_{ij})^2 + (b_{ij} - b^*_{ij})^2} \tag{6}$$

where: L_{ij}, a_{ij}, b_{ij} are the colour components of the pixel at location (i, j) in the original image and L^*_{ij}, a^*_{ij}, b^*_{ij} are the *CIELAB* colour components of the pixel in the quantised image. Also the loss of image colourfulness due to colour quantisation can be used as an additional tool for evaluation of quantisation error [13].

$$\Delta M = \left| M_{orig} - M_{quant} \right| \tag{7}$$

where: M_{orig} - colourfulness of the original image, M_{quant} - colourfulness of the quantised image. Formulas for computing of image colourfulness are simple and good correlate with the perceptual colourfulness of the image [6]:

$$M = \sqrt{\sigma^2_{rg} + \sigma^2_{yb}} + 0.3 \times \sqrt{\mu^2_{rg} + \mu^2_{yb}} \tag{8}$$

where σ_{rg}, σ_{yb} are the standard deviations and μ_{rg}, μ_{yb} are the mean values of opponent colour components of the image pixels. The opponent components are approximated by following simplified equations:

$$rg = R - G \tag{9}$$

$$yb = 0.5(R + G) - B \tag{10}$$

where *rg* - red-green opponency and *yb* - yellow-blue opponency.

It should be noted here that a common drawback of all these quality measures based on colour similarity is that they compare the images by using pixel to pixel comparison, without taking into account an impact of neighbouring pixels on the perception of colour of considered pixel. The additional factors, defining the quality of quantised image can include the edge similarity and structural similarity [7]. In this paper a new quality measure as a combination of all three similarities was proposed. However, in many cases the human visual system is the best final judge of quality of quantised image (subjective quality measures).

3 Preliminary Experimental Tests

A set of five natural images has been randomly chosen from Berkeley's image database [11] and presented in Fig. 3 in order of their number of unique colours.

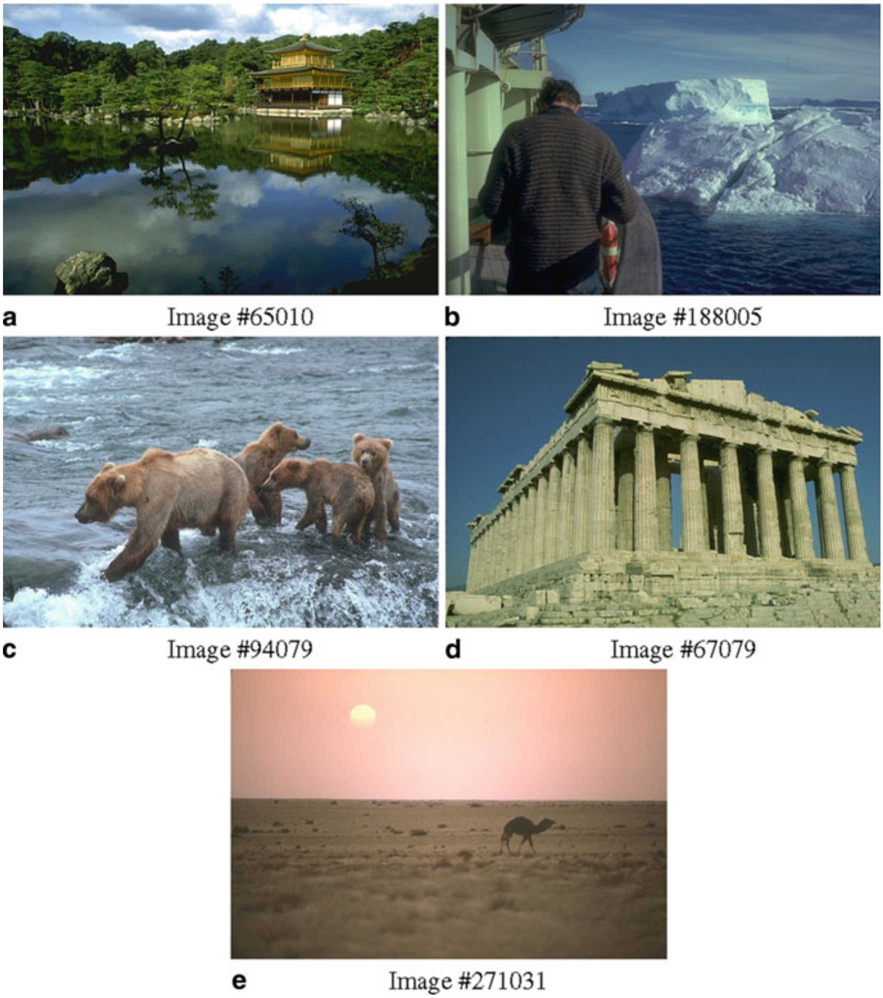


Fig. 3 A set of five test images from Berkeley image database

All these images were acquired at the same spatial resolution, i.e. 481×321 pixels. First tests were conducted to show that the larger unique number of colours in the original image, the larger also quantisation errors for a given size of the palette (here eight colours). The number of iterations in used clustering techniques was equal to 15 and the quantisation was realised by KM and KHM techniques in the *RGB* colour space. The data in Table 1 shows the error values for KM technique with two different initialisations: DC and SD. Similarly, Table 2 contains error values calculated for more efficient KHM technique. It should be noted that in both cases with the decreasing numbers of unique colours in images in Fig. 3 generally decreases the values of

Table 1 Quantisation results of the KM technique ($k = 8$)

Image	Colours	$PSNR$ (dB)		ΔE		ΔM	
		DC	SD	DC	SD	DC	SD
#65010	49404	24.8	25.0	11.3	9.8	9.9	7.0
#188005	31797	26.4	26.4	8.6	8.5	9.3	8.2
#94079	31225	27.1	27.6	6.1	5.3	9.1	4.6
#67079	22217	27.9	29.4	5.0	4.3	1.8	1.0
#271031	7652	30.1	32.1	3.7	3.2	1.2	1.5

Table 2 Quantisation results of the KHM technique ($k = 8$)

Image	Colours	$PSNR$ (dB)		ΔE		ΔM	
		DC	SD	DC	SD	DC	SD
#65010	49404	24.9	24.9	11.1	10.8	11.1	10.8
#188005	31797	26.8	26.5	8.1	9.0	8.1	9.0
#94079	31225	27.2	27.2	6.0	5.8	6.0	5.8
#67079	22217	28.2	29.0	4.9	4.4	4.9	4.4
#271031	7652	32.1	32.1	3.2	3.2	3.2	3.2

Table 3 Quantisation results of the splitting techniques ($k = 8$)

Image	Colours	$PSNR$ (dB)		ΔE		ΔM	
		MC	Wu's	MC	Wu's	MC	Wu's
#65010	49404	22.3	24.7	11.2	10.0	4.1	9.4
#188005	31797	25.7	26.1	8.6	8.9	6.9	10.4
#94079	31225	26.4	27.1	6.2	5.7	7.4	7.5
#67079	22217	28.1	28.6	5.2	4.7	1.1	1.6
#271031	7652	30.3	32.0	3.4	3.2	1.3	1.2

quantisation errors, i.e. increases *PSNR* and decrease ΔE and ΔM. A similar effect also occurs for two tested splitting algorithms: MC and Wu's (see Table 3).

In this way we confirmed a quite obvious hypothesis about the impact of the number of unique colours in the image on the quantisation error.

4 Idea of Outlier-Based Initialisation

Both DC and SD initialisations generate the starting centres of clusters located close to gray line. In the case of KM these locations of centres largely determine the final colours of the quantised image. There exist colour images for which the KM technique with earlier presented initialisations (DC, SD) does not give good results, particularly when the size of colour palette is small (e.g. four or eight colours).

Good example of such image is shown in Fig. 4a. This image is not very colourful, but it contains 138 877 unique colours! Colour pixels, as in other images, are generally grouped along diagonal of the *RGB* cube. Small red part of pixel cloud represents a red letter lying in the middle of the image (see Fig. 4b). The formation of the separate red cluster can be very important for CIQ application in image segmentation.

Unfortunately, the colour quantisation into 4 colours by KM and KHM techniques with initialisations DC and SD does not permit to obtain the red letter in quantised image (see Fig. 4c, d). Therefore we looked for a better method of initialisation for our clustering techniques and we have found an intelligent initialisation of KM proposed by Mirkin [12]. In this method the initialisation of KM is based on so-called Anomalous Pattern (AP) clusters, which are the most distant from the centre of cloud of points. Such outliers (peripheral points of the cloud) are the most important in this initialisation. This algorithm is general in nature and can be used in many different pattern recognition tasks.

Mirkin's algorithm consists of the following steps:

1. Find the centre of cloud of points in *RGB* colour space and mark it as C.
2. Find a furthest point away from centre C and mark it as $Cout$.
3. Perform the KM clustering into two clusters based on appointed previously centres: C and $Cout$ and just the centre $Cout$ is repositioned after each iteration.
4. Add the *RGB* components of $Cout$ to the list of stored centres.
5. Remove all points belonging to the cluster with centre $Cout$.
6. Check that there are still points in the cloud. If so, go back to the pt.2.
7. Sort obtained clusters by size (the number of elements) and select k largest clusters. Their centres are final starting centres for KM clustering.

Modification of Mirkin's (MM) algorithm proposed below is based on two important changes in relation to the original Mirkin's algorithm. First, the initial centres $Cout$ are used as starting centres instead of the final centres, which, in original algorithm, are found after clustering into two clusters. Second, clusters are not sorted according to size in the final step of the algorithm. In this way, the MM initialisation locates the starting cluster centres in outlier points (colours) i.e. points, which are furthest from the centre of pixels cloud. This allows to take into account the small clusters, which represent the colours of small, but perceptually essential regions [4, 15].

Modified Mirkin's algorithm looks as follows:

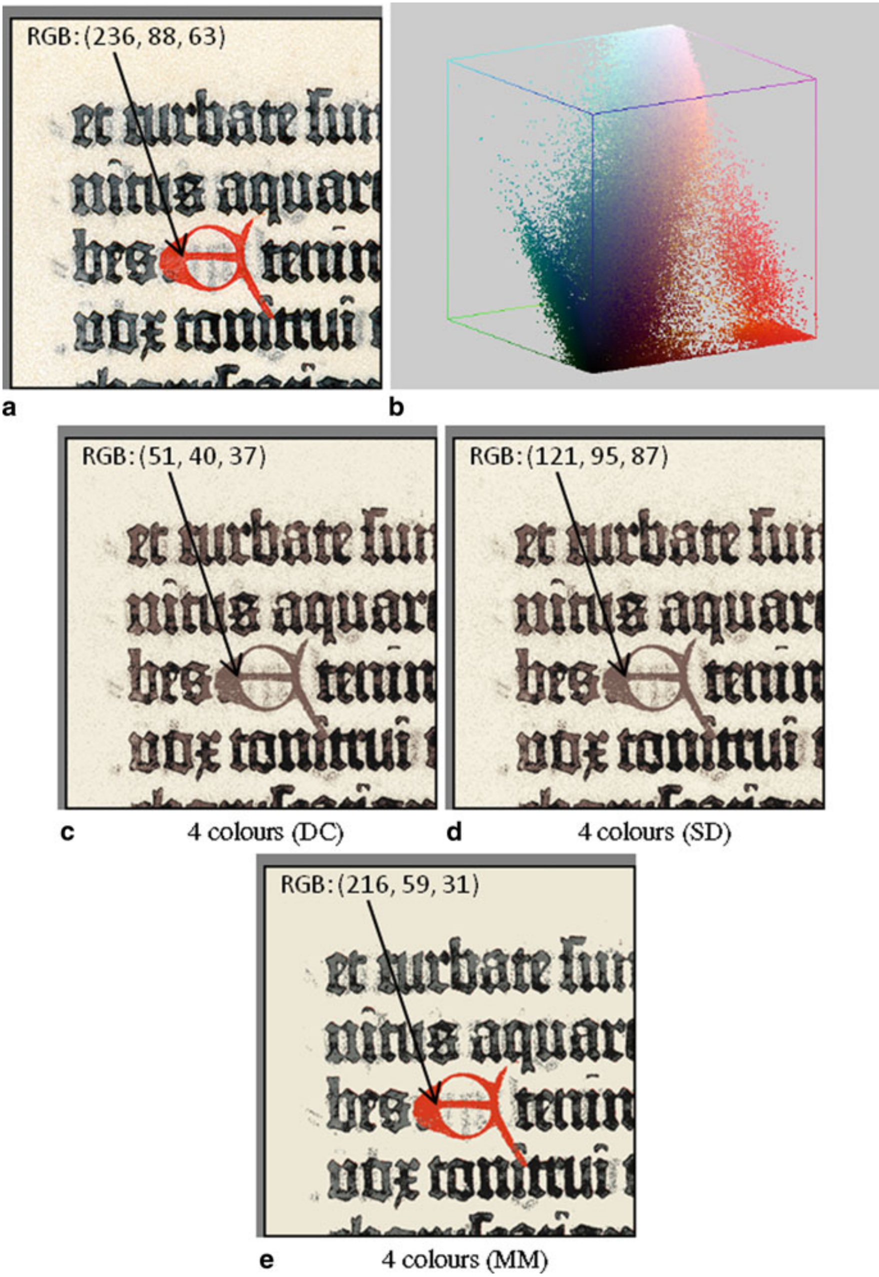


Fig. 4 Results of colour quantisation: **a** original image, **b** colour gamut, **c** KM with DC (4 colours), **d** KM with SD (4 colours), **e** KM with MM (4 colours)

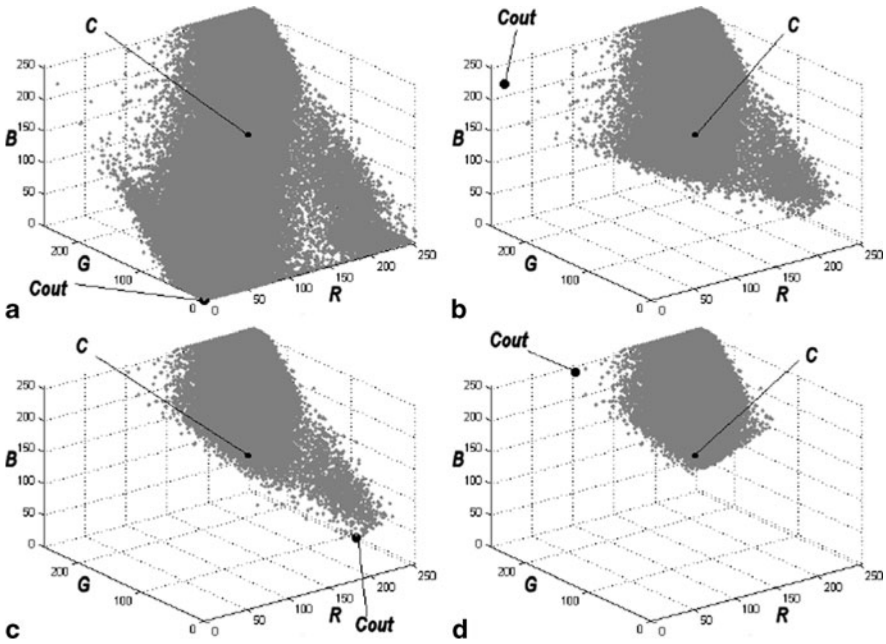


Fig. 5 Outlier cluster centres found during MM initialisation

1. Find the centre of cloud of points in *RGB* colour space and mark it as C.
2. Find a furthest point away from centre C and mark it as $Cout$.
3. Add the *RGB* components of $Cout$ to the list of stored centres.
4. Perform the KM clustering into two clusters based on appointed previously centres: C and $Cout$ and just the centre $Cout$ is repositioned after each iteration.
5. Remove all points belonging to the cluster with centre $Cout$.
6. Check that there are still points in the cloud. If so, go back to the pt.2.
7. Select the first k clusters determined by this algorithm. Their centres are final starting centres for KM clustering.

The MM initialisation permits to get the red letter in the image during the quantisation into 4 colours (see Fig. 4e). For all the considered initialisations we calculated a colour error for the red letter in quantised image: $\Delta E(\text{DC}) = 77$, $\Delta E(\text{SD}) = 60$ and $\Delta E(\text{MM}) = 11$. These results demonstrate the superiority of MM initialisation over other tested initialisations.

Figure 5 illustrates subsequent eliminations of outlier clusters from the cloud of points presented as 3D scatter plot and helps to understand the algorithm. Here, the

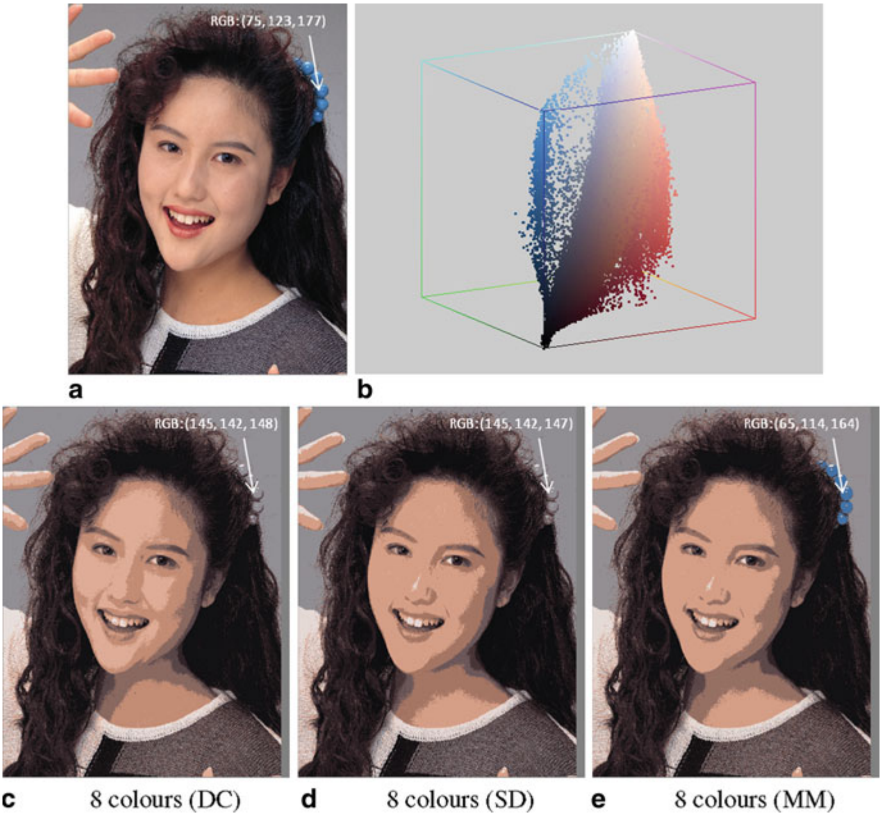


Fig. 6 Results of colour quantisation **a** original image, **b** colour gamut for original image, **c** KM with DC initialisation, **d** KM with SD initialisation, **e** KM with MM initialisation

third step of MM (see Fig. 5c) has particular importance, because a centre of red cluster is detected.

Another example of usefulness of MM initialisation is the quantisation of the image shown in Fig. 6. Particular attention should be paid to the blue beads, which are perceptually important region in the original image. The image is quantised into 8 colours. The colour quantisation by KM technique with initialisations DC and SD generates the images without blue pixels in quantised image; the beads are gray (see *RGB* values in Fig. 6c, d). This problem is solved by using the MM initialisation, as shown in Fig. 6e. We calculated appropriate colour errors for the blue beads: $\Delta E(\mathrm{DC}) = 32$, $\Delta E(\mathrm{SD}) = 33$ and $\Delta E(\mathrm{MM}) = 4$. Definitely the smallest error again achieved the MM initialisation.

Similar experiments were also carried out with other images. Their visual evaluation confirmed the advantages of the MM initialisation. Despite the limited palette, each quantised image contained the perceptually significant colours. On the other hand, generally accepted image quality measures for quantised images do not give clear results (see Table 4). Only ΔM, the loss of image colourfulness, that is strongly related to colour perception, shows the advantage of MM initialisation.

Table 4 Quantisation results of the KM with MM initialisation

Image	$PSNR$ (dB)			ΔE			ΔM		
	DC	SD	MM	DC	SD	MM	DC	SD	MM
Fig. 4	20.7	20.7	20.4	13.4	13.4	12.9	32.0	31.4	14.2
Fig. 6	26.5	26.2	25.9	6.0	5.7	6.1	4.7	3.9	3.6

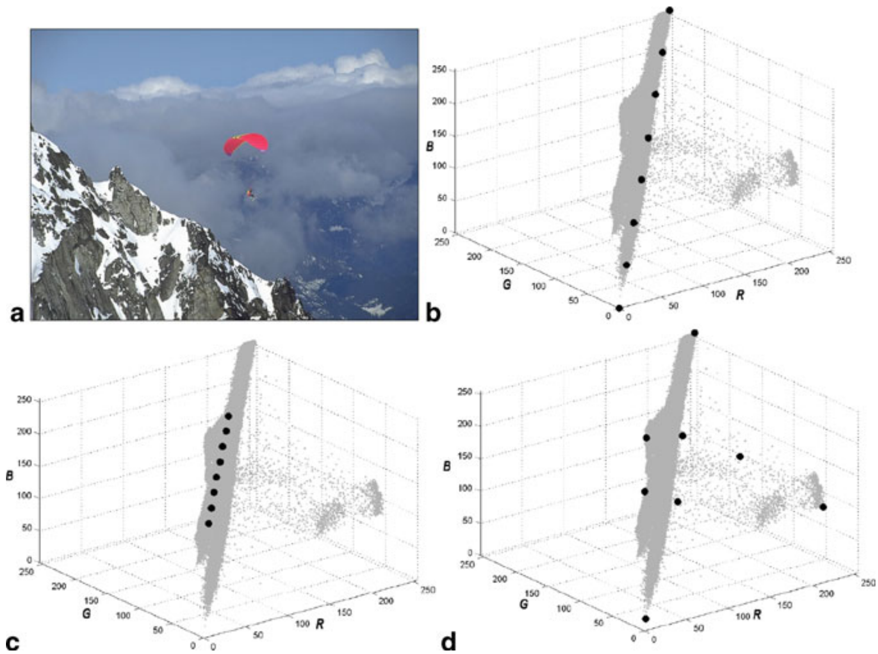


Fig. 7 An original image, **b** Centres found in DC initialisation, **c** Centres found in SD initialisation, **d** Centres found in MM initialisation. All clusterings for 8 clusters

5 Further Tests of New Initialisation

In the first group of tests were determined the positions of starting centres of clusters for three compared initialisations: DC, SD and MM. The colour pixels in pixels cloud of natural images are generally grouped along diagonal of the *RGB* cube. Black dots plotted on a pixels cloud present the location of these centres. All clusterings presented in this section were achieved after 30 iterations of KM technique.

5.1 Distributions of Clustering Starting Centres

The first test image (Fig. 7a) contains perceptually important red area lying in the middle of the image and showing a paraglider, which can be seen in Fig. 7b, c, d as a small part of pixels cloud directed to the red colour. In the case of DC and SD

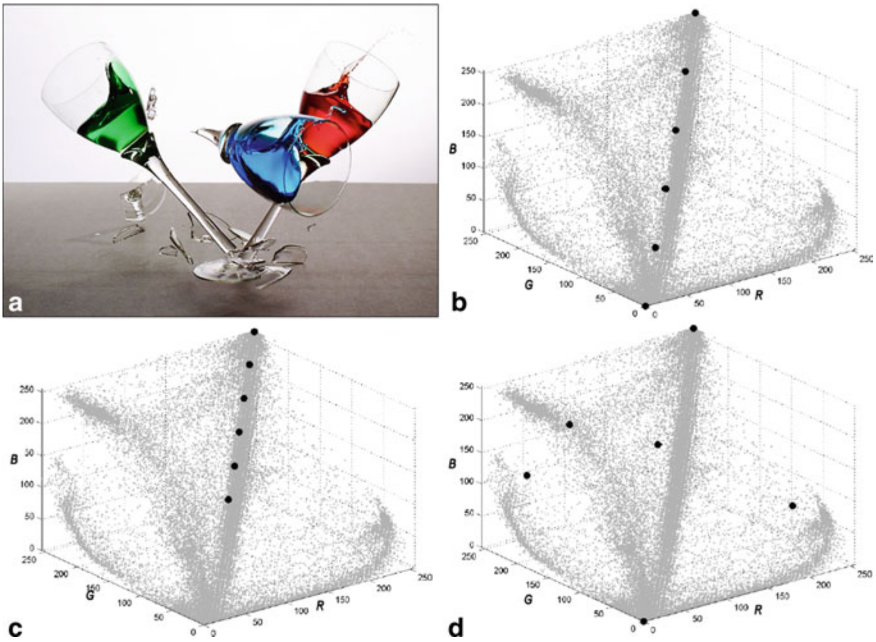


Fig. 8 An original image, **b** Centres found in DC initialisation, **c** Centres found in SD initialisation, **d** Centres found in MM initialisation. All clusterings for 8 clusters

initialisation all eight initial centres are located on the diagonal of the *RGB* cube, only the MM initialisation (Fig. 7d) generates two peripherally located centres, one of which is contained in a cluster of red pixels. This gives a chance to get a good CIQ result using the KM technique with MM initialisation.

The second test image (Fig. 8a), quantised into 6 colours, creates a specific pixels cloud (Fig. 8 b, c, d) in the *RGB* space with three branches for three colours R, G and B. Only the MM initialisation puts the initial centres in these sectors, which are important for further clustering (Fig. 8d).

The third considered test image (Fig. 9a) presents a book cover and contains six colour characters with distinct chromatic colours and it is characterized by more complex pixels cloud (Fig. 9b, c, d). Again, only the MM initialisation (Fig. 9d) settles a part of centres outside of the main pixels cloud, which gives a opportunity to obtain a good CIQ result.

5.2 CIQ for Salient Region Detection

The second part of tests serves to compare the quality of images quantised with different initialisations. These tests were performed in the *RGB* colour space and two

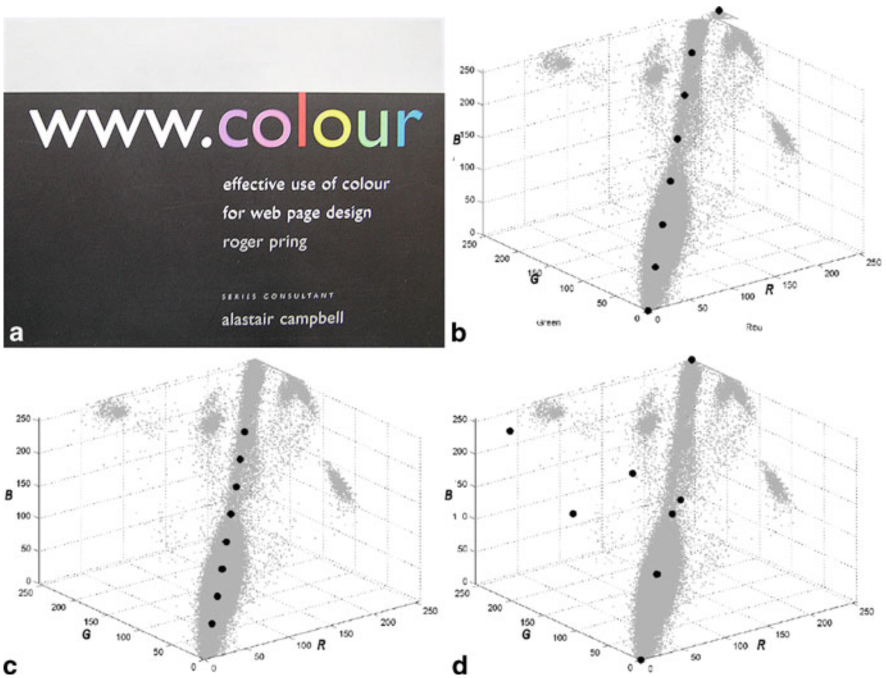


Fig. 9 An original image, **b** Centres found in DC initialisation, **c** Centres found in SD initialisation, **d** Centres found in MM initialisation. All clusterings for 8 clusters

additional colour spaces: *YCbCr* (linear transformation of *RGB* space) and perceptually uniform *CIELAB* colour space (non-linear transformation of *RGB* space). In addition to the subjective visual assessment a loss of image colourfulness ΔM was used, and the other typical quality measures described in Sect. 2 were rejected. Their nature makes that the colours of the perceptually significant regions with small areas do not play a noticeable role.

Figure 10 shows quantised versions of the original image presented in Fig. 7a. The visual assessment indicates a dominance of the MM initialisation since regardless of the type of colour space a reddish paraglider remains in quantised image. Particular attention is paid the quantisation in the *CIELAB* colour space, where the loss of image colourfulness ΔM regardless of initialisation is smallest.

Figure 11 shows quantised versions of the original image presented in Fig. 8a. Original image contains the three chromatic colours only, so it is easy to visually assess the results of quantisation. These three chromatic colours remained in the quantised images in four of nine cases only. There are three images quantised after MM initialisation and one image quantised in *CIELAB* space after SD initialisation. The original image is not a natural image. Perhaps that is why the relation between the results is here not so clear.

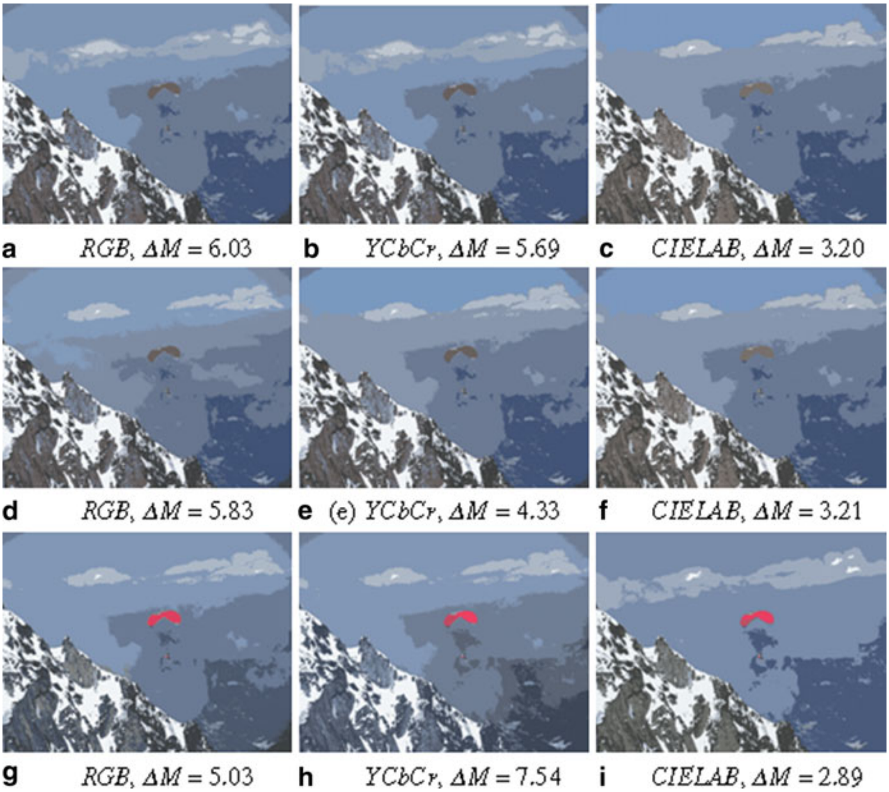


Fig. 10 KM results for the image from Fig. 7a, **a**, **b**, **c**, results with DC initialisation, **d**, **e**, **f** results with SD initialisation, **g**, **h**, **i** results with MM initialisation ($k = 8$)

Figure 12 shows quantised versions of the original image in Fig. 9a. The original image contains six characters with distinct chromatic colours, making easy a visually assessment of the quantised images. The caption below Fig. 12 includes a number of chromatic colours recognized by the observer. You can notice that the maximal number of chromatic colours obtained after CIQ is 4 and it has been achieved only in case of MM initialisation in the *YCbCr* and *CIELAB* spaces. These results occur simultaneously with the least loss of image colourfulness ΔM.

6 Conclusions

In this chapter we showed for two different CIQ techniques that the number of unique colours in the natural image significantly influences on the value of quantisation error. But the main contribution of the work is a new alternative way for initialisation of KM, which provides better CIQ results. This approach based on detection and elimination

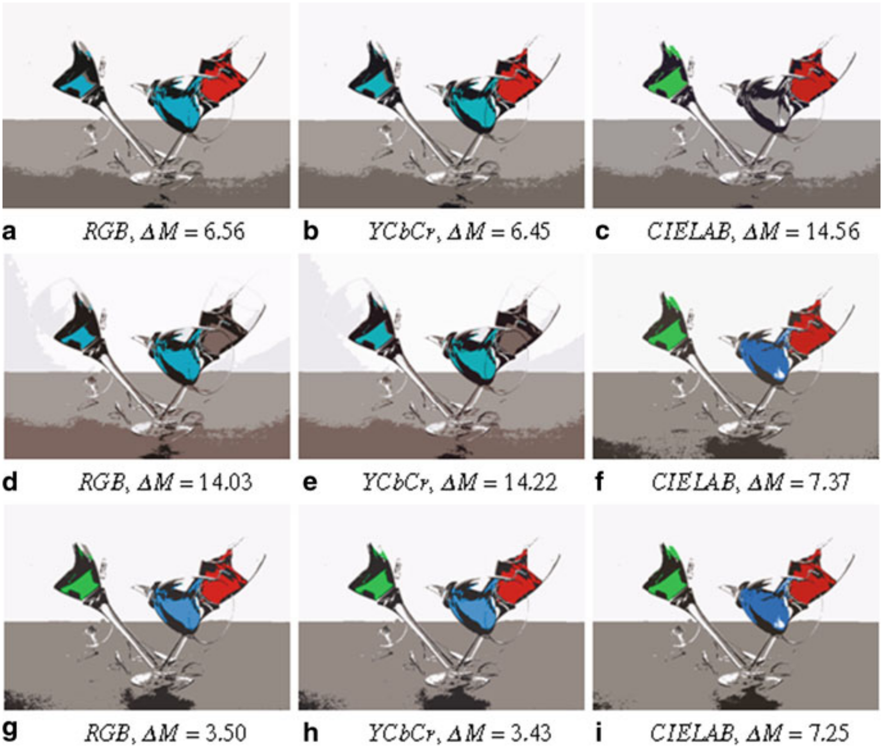


Fig. 11 KM results for the image from Fig. 8a, **a**, **b**, **c** results with DC initialisation, **d**, **e**, **f** results with SD initialisation, **g**, **h**, **i** results with MM initialisation ($k = 6$)

outlier clusters, named here MM, does not lose the perceptually important colour regions of the original image. Additionally, the usefulness of quality measure called the loss of colourfulness to CIQ assessment has been confirmed.

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Fig. 12 KM results for the image from Fig. 9a, **a**, **b**, **c** results with DC initialisation, **d**, **e**, **f** results with SD initialisation, **g**, **h**, **i** results with MM initialisation ($k = 8$)

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