Stochastic Leaky Integrator Model for Interval Timing

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Abstract. Interval timing plays an important role in every aspect of our daily life. Intelligent behavior, adaptation and survival of mankind rely on proper judgment of temporal intervals. Since many decades, Pacemaker Accumulator Model (PAM) has been the most influential interval timing model in human time perception domain. It is purely a psychological model and many researchers from the neurobiology domain attempted to find the biological equivalents of the components of PAM. In this paper, we propose a computational model of interval timing based on spiking neurons which is biologically plausible yet preserving the simplicity and strength of psychological models. Preliminary results demonstrate that the computational model we proposed can mimic empirical data from psychological experiments.

Keywords: Cognitive modeling, Spiking neural networks, Time perception, Leaky integrate-and-fire, Scalar property.

1 Introduction

"Timing is everything: in making shots, in making love, in making dinner. Indeed, it is difficult to conceive of an action that doesn't require temporal control...In addition to coordinating complex sequences, timing serves a very basic function long recognized as a fundamental aspect of the learning process in animals: anticipation or prediction..." [8]. Time Perception refers to sense of time that involves experience and passage of time, temporal discrimination and time estimation. Time Perception is widely studied in Cognitive Science domains from varied perspectives. For example, psychologists study time perception as part of behavioral analysis [4] [28]; neuroscientists study time perception as part of finding information processing mechanisms in brain [12][10] and computer scientists study time perception as part of computational modeling and development of autonomous intelligent systems [15].

Multiple neural systems are responsible for processing multiple time scales in at least 10 orders of magnitude [17]. Buhusi and Meck [2] classify different orders

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of time into *circadian timing*, *interval timing* and *millisecond timing*. The circadian clock that keeps track of sleep-wake cycle, appetite and behavioral rhythms exists in suprachiasmatic nucleus (SCN) [12]. The millisecond timer responsible for speech processing and motor coordination exists in cerebellum [2]. The time scale in the range of seconds to minutes range is referred to as interval timing and involves conscious time estimation. The neural mechanisms of interval timing are not clearly identified and Ivry and Schlerf specify that some models are based on dedicated mechanisms and some models are based on intrinsic mechanisms [10]. Some psychologists categorize timing mechanisms into automatic and cognitively controlled mechanisms. The circadian timing and millisecond timing are labeled as automatic timing mechanisms and interval timing is labeled as cognitively controlled timing.

There are four standard psychophysical investigation methods for studying time perception: *verbal estimation*, *interval reproduction, interval production* and *interval comparison. Verbal estimation* refers to specifying the duration of a stimulus in time units (usually in seconds) ; *interval reproduction* refers to reproducing the duration of the presented stimulus by the repetition of same event or by other means. Some people refer to time reproduction task as *peak-interval procedure* [16], as responses from several trials peak at the criterion duration. *Interval production* refers to producing a timing task of the given duration; *interval comparison* refers to comparing different lengths of the same stimulus or comparing lengths of two different stimuli [28].

Interval timing exhibits an important characteristic namely *scalar property*. For both psychological and neural models of interval timing, accountability is achieved by *scalar property. Scalar property*, a specialized form of Weber's law, specifies that the time varying behavior of the subject stretches or scales in proportion to the length of the duration of the stimulus. In other words, it refers to the linear relationship between the standard deviation of the time estimate and mean of time estimate. Treisman [24] defines *scalar property* as the phenomenon wherein the error in time estimation increases with increased duration of the stimulus.

The cognitive and neural models of time perception are broadly categorized into dedicated and intrinsic models [10][27] as shown in Table 1 . Computational models are feasible for comprehensive understanding of mind and brain. In time perception realm, there is a lacunae for computational modeling. There are a very few computational models of interval timing [23] [1] [19]. The computational model by Taatgen et al. [23] is a procedural model and is built on ACT-R architecture. Addyman et al. [1] devised and simulated a computational model for interval timing in infants using a connectionist memory-decay approach. Oprisan and Buhusi [19] devised a mathematical model to analyze the characteristics of cosine oscillators and the role of noise in striatal beat frequency model.

We propose Stochastic Leaky Integrator Model(SLIM) which is based on leaky integrate-and-fire spiking neuron model and the idea is inspired by PAM. Our model, SLIM is a hybrid model that combines the features of dedicated and intrinsic models of timing. It combines a biologically plausible model of neuron

Table 1. Models of Time Perception

Dedicated Models	Intrinsic Models
– In dedicated models, a neural structure or components for handling temporal information could be a specialized one or a distributed network of different brain regions or components. – Cerebellum, basal ganglia, supplementary motor area and right prefrontal cortex are the examples of specialized dedicated mechanisms proposed in various neural models of time perception. – PAM is the seminal cognitive model that assumes a dedicated clock component to deal with time – Other examples of dedicated models are Beat Frequency model by Miall [18] and Connectionist Model proposed by Church and Broadbent [5]	– In intrinsic models, timing is inherent and is part of sensory information processing. – State dependent network models and energy readout models come under this category. – Other examples are Memory-decay model proposed by Staddon [22], Dual Klepsydra model by Wackermann and Ehm [26], Population clocks proposed by Buonomano and Laje [3], Spectral timing model by Grossberg and Schmajuk [9].

activation with counting mechanism in order to handle interval timing as cortical neurons fire in the range of milliseconds. SLIM is a simple, biologically plausible computational model for interval timing. Section 2 presents an overview of spiking neural networks and delineates the feasibility of leaky integrate-and-fire neuron model as the computational substrate of interval timing.

2 Spiking Neural Networks

A neuron is assumed to be a dynamic element that generates pulses or spikes whenever its excitation reaches a threshold value. Biological neural systems use spike timing for information processing. The generated sequence of spikes contains the information and it gets transmitted to the next neuron [7]. Spiking neurons are the computational units in brain. The presence of output spikes from the neurons and/or the timing of the spiking neurons is assumed to be the information transmission mechanism in spiking neural networks. Though artificial neural networks are proved to be powerful problem solving tools in the domains such as bioinformatics, pattern recognition, robotics etc., they suffer in processing large amounts of data and adaptation to the dynamic environment [20]. Maass [14] quotes spiking neural networks as third generation of artificial neural networks, the first generation of networks being threshold logic units and the second generation of neural networks being sigmoidal units (see Table 2).

Table 2. Classification of Neural Networks according to [14]

Generation	Computational Unit	Working Principle	Architecture/ Model
First Generation	Threshold Gates (McCulloch-Pitts neurons)	Digital output	Perceptrons, Hopfield Network, Boltzmann machine
Second Generation	Sigmoidal Units	Real valued outputs which are interpreted as firing rates of natural neurons	Multilayer perceptrons, Radial Basis Function networks
Third Generation	Spiking Neurons	Spike Timing	Hodgkin-Huxley model, Integrate-and-Fire model, Spike response model

2.1 Spiking Neuron Models

The most influential spiking neuron models are: Hodgkin-Huxley Model, Integrate-and-Fire Model and Spike Response Model. Hodgkin-Huxley Model is a traditional conductance-based model of spiking neurons. When compared to Integrate-and-Fire model and Spike response model, it is quite complex to analyze mathematically. Spike Response model could be demonstrated as a generalized version of Integrate-and-Fire model. Leaky integrate-and-fire model is a variant of integrate-and-fire model wherein , a neuron is modeled as a leaky integrator of its input. The neurons in integrate-and-fire model could be stimulated by some external current or by the input from presynaptic neurons.

We focus on leaky integrate-and-fire model as the nature of leaky integration of spikes resembles the accumulation of pulses in accumulator of PAM. And, when the membrane potential reaches a threshold, an output spike is generated and the no.of spikes that gets generated is in proportion to the length of the stimulus.Eq. 1 describes a simple resistor-capacitor (RC) circuit where the neuron is modeled as a leaky integrator with the input I(t).

$$\tau_m \frac{d\nu}{dt} = -\nu(t) + RI(t) \tag{1}$$

The membrane potentials of the leaky integrate-and-fire neurons are calculated depending on the input type. Eqs.2, 3, 4 describe the computation of membrane potential with the inputs of constant current, time dependent stimulus and presynaptic currents respectively.

1. Stimulation by a constant input current I(t):

$$\nu(t) = RI[1 - \exp\left(-\frac{t}{\tau_m}\right)] \tag{2}$$

2. Stimulation by a time-varying input current:

$$\nu(t) = \nu_r \exp\left(-\frac{t - t_0}{\tau_m}\right) + \frac{R}{\tau_m}\int_0^{t-t_0} \exp\left(-\frac{s}{\tau_m}\right) I(t-s)ds \tag{3}$$

3. Stimulation by synaptic currents:

$$I_i(t) = \sum_j w_{ij} \sum_f \alpha(t - t_j^{(f)}) \tag{4}$$

For devising the model of interval timing, we considered the case of 'stimulation by synaptic currents', as the sensory stimulus passes through the cortical neurons and cortical neurons would act as presynaptic neurons for leaky integrate-and-fire neuron.

3 The Model

After an extensive review of literature on time perception, we identified that there is a lacunae of computational models of time perception. To fill this gap in literature, we attempted to devise a computational model that is biologically plausible based on spiking neurons. In addition, the model should satisfy the criterion of being lucid and concise for explaining the mechanisms involved in interval timing like PAM. This led to devising a hybrid model possessing the properties of dedicated and intrinsic models of time perception. The model is called Stochastic Leaky Integrator Model (SLIM) because it depicts the stochastic firing nature of cortical neurons and leaky integration of the integrate-and-fire neuron. The schematic representation of SLIM is presented in Fig. 2.

In PAM (Fig. 1), which is based on Scalar Expectancy Theory (SET), a Poisson *pacemaker* continuously emits pulses. The *switch* is closed on the onset of a stimulus and the pulses get accumulated in *accumulator*. During training, the contents of the *accumulator* are stored in *reference memory*. During testing, the contents of the *accumulator* are stored in *working memory* and a response is generated as an outcome of the *ratio comparison* between contents of *working memory* and *reference memory*. When there is a sensory stimulus, the cortical neurons start firing stochastically at irregular intervals [6]. So, it is convenient to maintain the number of spikes/pulses generated by the cortical neurons during the presence of stimulus rather than keeping track of spiking times of the neurons over a time course. Our model SLIM exactly utilizes this property for interval time estimation. According to SLIM, when there is a stimulus, cortical neurons start firing and generate spikes. To mimic the random firing nature of cortical neurons, we used a uniformly distributed random number generator to determine the number of neurons that fire at any instance of time. During the presence of stimulus, the potential of the leaky integrate-and-fire (LIF) neuron increases in proportion to the spiking of presynaptic neurons. When the potential reaches a threshold, the LIF neuron generates an output pulse and counter keeps track of these pulses. After reaching threshold, potential is set to a resting value and the integration of potentials starts again at LIF neuron. This accumulation continues until the stimulus is presented and after that the contents of counter are shifted to memory and a response is generated(see Algorithm1).

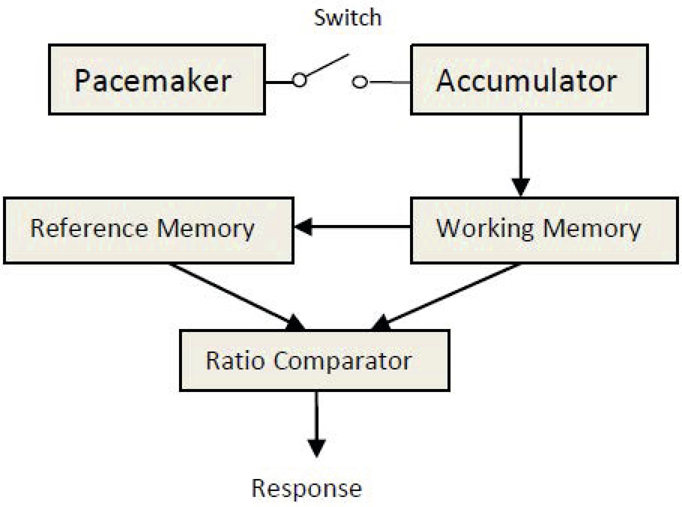

Fig. 1. Pacemaker Accumulator Model

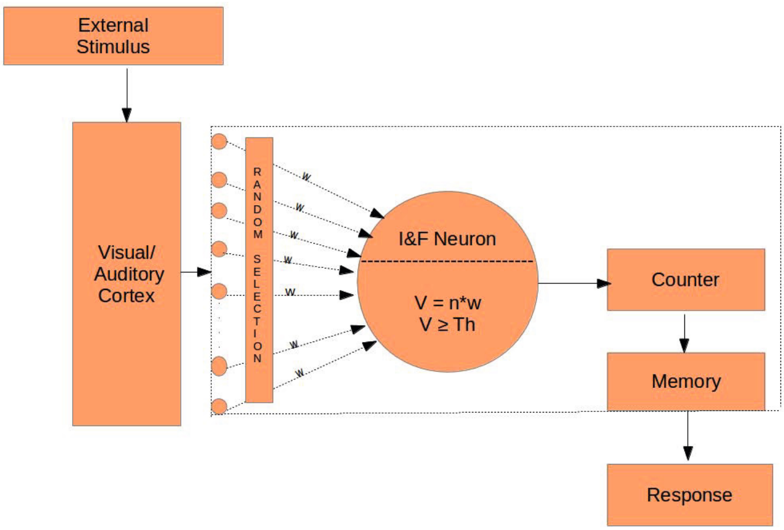

Fig. 2. Stochastic Leaky Integrator Model for Interval Timing

Algorithm 1. Functioning of SLIM

Network Architecture & Initializations:
No. of cortical neurons, M= 100
No. of Leaky integrate-and-fire neurons: 1
Threshold, Th= 50
Resting potential, R= 0.5
Presynaptic weight of each neuron, W= 0.75
Potential, V= 0
Counter, S= 0

while STIMULUS **do**
  When there is an external stimulus, the cortical neurons start firing
  The no. of cortical neurons that fire at a moment are selected by a random number generator;
  N= Random(M)
  The potential of leaky integrate-and-fire neuron is calculated as the weighted sum of firing neurons.
$V = N_* W$
  if $V \geq Th$ **then**
    LIF neuron generates a spike and counter is incremented by one
    $S = S + 1$
    Potential, V is set to resting potential R
    $V = R$
  else
    $I = N_* W$
    $V = V + I$
  end if
end while
Shift the values of Counter to Memory;
$Mem = S$
return S

4 Results and Discussion

We experimented to model the results of time reproduction task given in [21] for 8s, 12s and 21s through SLIM. Time duration is plotted on x-axis and the percentage of response rate is plotted on y-axis. We simulated the model with 100 presynaptic neurons and the number of neurons that get excited at an instance of time is chosen by a uniformly distributed random number generation function to demonstrate more realistic the firing of neurons. Fig. 3 shows the result of the simulations with *fixed threshold*. To mimic the results of behavioral tests done by Rakitin et al. [21], we also ran the simulations for 80 trials and the outcomes of these trials are plotted in Fig. 3.

Though there is a significant variance between results of Rakitin et al. experiments and the results of simulations of SLIM, we welcome the stochastic firing nature of cortical neurons as it is the mundane nature of biological neurons. Another source of variance is *fixed threshold*. We considered a threshold value of

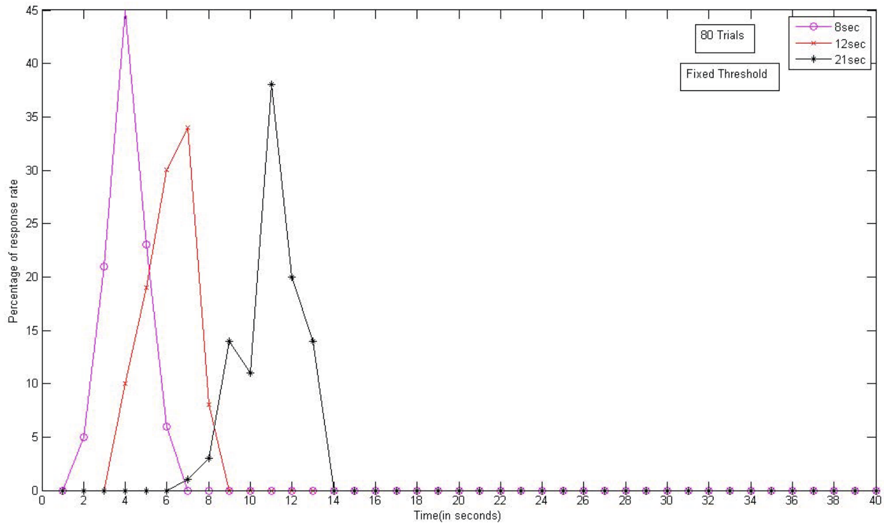

Fig. 3. Performance of SLIM with fixed threshold

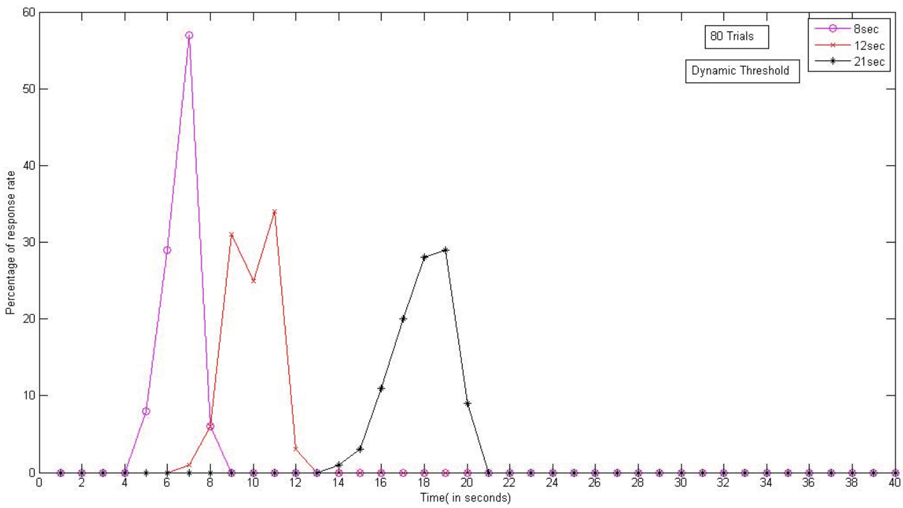

Fig. 4. Performance of SLIM with dynamic threshold

50 as it is a quite reasonable assumption that atleast 50% neurons should fire in order to generate an output spike.

To reduce variance between SLIM results and Rakitin et al. results, we attempted to work with *dynamic threshold*. Initially the threshold of the simulations start with 50 and later the threshold is reduced by the inhibitory input at previous instance. The results of the simulations are much improved by inducing *dynamic threshold* which is again a common feature of biological neurons [11][13] and are shown in Fig. 4. As we focused much on studying the behavior of stochastic leaky integrator as a computational substrate of interval timing that is also biologically plausible, at present we did not work with training the model. We hope that if the model is trained with a reasonable number of trials, the results of the simulations of SLIM would closely resemble the results given by Rakitin et al.

5 Conclusion and Future Work

The model proposed in this paper, SLIM is simple yet computationally efficient. Further, it integrates the features of dedicated and intrinsic models of time perception and these features are represented by spiking cortical neurons and the counting mechanism respectively.

As the model is based on spiking neurons it is evident that the model is biologically plausible [25]. To make the model more robust and compatible with psychological models of time perception, the results from behavioral experiments by Rakitin et al. were used to assess the performance of SLIM. Initially the model was tested for 8s, 12s and 21s time durations with fixed threshold of leaky integrate-and-fire neuron. To improve the results, dynamic threshold that varies from iteration to iteration and whose value depends on inhibitory input of previous iteration is considered. Though the results of simulations using dynamic threshold are much better than the results of simulations using fixed threshold, the results indicated that there is no close resemblance to the results of Rakitin et al. experiments. This may be due to lack of training for the model.

Future line of work should focus on incorporating training for the model. It is assumed that by incorporating training, the model would be congruent with psychological models. Further it is intended to study the feasibility of spiking neuron populations for interval timing. At present, we did not focus on time course of spiking neurons, and in future we intend to explore it to study the relation between time course and interval timing.

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