A Re-constructed Meta-Heuristic Algorithm for Robust Fleet Size and Mix Vehicle Routing Problem with Time Windows under Uncertain Demands

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Abstract. Recent work of the fleet size and mix vehicle routing problem with time windows mostly assumes that the input variables are deterministic. Practice in the real world, however, faces considerable uncertainty in the data. But recent research studies lack emphasis on this uncertainty. This paper focuses to contribute to a new challenging study by considering the customer demand as uncertain. This characteristic increases the difficulty for solving. The meta-heuristic algorithms are developed consisting a modification of a genetic algorithm and an adaptation of a greedy search hybridized with inter-route neighborhood search methods. Because this paper relates to uncertain customer demands, decision making is performed using the robust approach based on worst case scenarios. The final results are evaluated by using the extra cost and the unmet demand against the deterministic approach to balance the decision making.

Keywords: Robustness, Fleet size and mix vehicle routing problem, Uncertain demand, Meta-heuristic, Modified genetic algorithm, Adapted greedy search.

1 Introduction

The fleet size and mix vehicle routing problem (FSMVRP) is one of the specific problems of the classical vehicle routing problems (VRPs) in which heterogeneous fleets are composed. The heterogeneous fleets deal with real-world problems with more than the single kind of the vehicles. Moreover, the capacity of the vehicles is not the only factor used to consider the route assignment; the other variants such as time windows, split deliveries, etc. are constraints in practice. These extensions make the problems more complex and are much harder to solve than the classical VRP. This paper focuses on the time windows constraint, the FSMVRP is extended to be the fleet size and mix vehicle routing problem with time windows (FSMVRPTW).

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Real situations include very often uncertainty. The considerable input parameters, for example, customer demands, traveled times, etc., can vary during the period of time. The solutions for future are difficult to describe precisely if the uncertain characteristics of the concerned parameters are involved. The customer demands, for example, can be revised by some reasons such as an emergency shutdown of one of a customer's production lines which might reduce the actual demands. A vehicle that is planned to serve such a customer might have some remaining spaces. Therefore, the previous route assignments should be recalculated to yield the optimal solution. In this study, the customer demands are under the assumption of the uncertainty.

Many researchers put effort into developing heuristic/meta-heuristic algorithms instead of using exact methods to handle complex problems such as VRPs. The recent algorithms published have been constructed based on the iterated local search, greedy search, tabu search, ant colony optimization, genetic algorithm, scatter search, for instance. In this research work, the algorithm is proposed a re-constructed meta-heuristic algorithm for robust vehicle routing problem with time windows under uncertain demands.

Due to the demand characters are non-deterministic, the authors propose the robustness approach in robust decision making. The output can be represented as the average, the best, or the worst solutions. It depends upon the judgment of decision makers to investigate an appropriate way for individual situation. But this work emphasizes the fact that any unexpected situations which might occur in the real world future, that the decisions must be realized even in the worst case. Therefore, the worst case scenario approach is applied.

The further details of the problems, the literature review, the proposed methodologies, the computation results, and the conclusions are discussed in next sections.

2 Literature Review

This paper is the continuation of our previous work [1] that surveys the heterogeneous vehicle routing problems and has been constructed as an overview structure which puts a special emphasis on robustness approach. The heterogeneous fleet is classified into two major classes: the heterogeneous fleet vehicle routing problem (HFVRP) and the fleet size and mix vehicle routing problem (FSMVRP). The limitation on the vehicle numbers, i.e. limited and unlimited, is the borderline to divide the problems into both types [2,3]. The FSMVRP with the unlimited transportation devices and its additional variants are focused in this study.

In 1996, Osman and Salhi have presented the vehicle fleet size and mix problem under the assumption of all concerned input data such as customer demands, number of customers, traveled times, geographical locations, service times, etc. are known with certainty [4]. It is because the characteristics of the considerable input parameters are fixed values, the obtained solutions are unique. Years later, the FSMVRP and its extensions attract the academicians to attempt the difficult talks of developing the methodologies for achieving the optimal solutions, i.e. the total cost (fixed and variable costs) minimization.

The results of the literature review show that the FSMVRPs and the extensions lack attention in the research of non-deterministic variables. The summarization matrix is shown in Table 1. The second column describes the additional variants of the FSMVRPs, the abbreviations of TW, SD, MD, and dash stand for time windows, split deliveries, multiple depots, and classical problems, respectively. The next part of the matrix indicates the sets of the model parameters consist of customer demands (CD), number of customers (NC), traveled times (TT), geographical locations (GL), service times (ST), vehicle productivities (VP), and vehicle availability (VA). The characteristics of these input variables can be classified as certainty (C) and uncertainty (U). In this study, the decision approach as shown in the last column is categorized into 3 groups: deterministic (D), stochastic (S), and robustness (R). The publication works of the same group of the considerable input parameters studies are summarized in the shading rows. The names are sorted by the year of publication.

Table 1. Fleet size and mix vehicle routing problems (FSMVRPs) and the variants literature review summarization matrix

#	Authors	Additional Variants	Considerable input parameters							Decision approach
			CD	NC	TT	GL	ST	VP	VA	
1	[4,5]	-	C	C	C	C	C	-	-	D
2	[6,7,8,9,10,11,12,13]	-	C	C	-	C	-	-	-	D
3	[14]	-	C	C	C	C	-	-	-	D
4	[15,16,17,18,19,20]	TW	C	C	C	C	C	-	-	D
5	[21,22]	TWSD	C	C	C	C	C	-	-	D
6	[23]	MD	C	C	C	C	-	-	-	D
7	[24]	MD	C	C	-	C	-	-	-	D

Table 2. Robust and stochastic vehicle routing problems (RVRPs and SVRPs) and the related research studies literature review summary matrix

#	Authors	Problems	Considerable input parameters							Decision approach
			CD	NC	TT	GL	ST	VP	VA	
1	[25]	Robust Fleet Sizing - Transport Freight	U	C	C	-	-	-	-	R
2	[26]	RVRP	U	C	C	-	-	-	-	R
3	[27]	VRPTWST	C	C	U	-	C	-	-	S
4	[28]	Road Network	C	C	U	U	C	-	-	R
5	[29,30]	RVRP	U	C	C	C	-	-	-	R
6	[31]	RVRPTWSD	U	C	U	C	-	-	-	R
7	[32]	SVRP	U	C	-	C	-	-	-	S
8	[33]	RVRPTW	C	C	U	-	C	-	-	R
9	[34]	RVRP	U	C	-	C	-	-	-	R
10	[35]	SVRP	U	C	-	C	-	-	-	S

The stochastic or uncertainties of the input variables are employed in the other specific problems of VRPs but not for the FSMVRPs and its extensions. Table 2 demonstrates the survey summary in the same way as mentioned above in Table 1. But, the second column shows the names of the specific VPRs instead of the FSMVRP variants. The matrix of the robust and stochastic vehicle routing problems and the related research studies indicate that when the problems investigate insight in the real world by modeling one of the parameters as non-deterministic, the robust or stochastic decision making approaches are referred to handle the unknown futures.

It is well known that the VRPs have a complexity of NP-hard problems, so the FSMVRPs have as well. Therefore, most researchers put emphasis on developing heuristic/metaheuristic algorithms to produce (near)-optimal solutions. As this research is an extension of previous research that has concentrated on the literature review, the chronological order of the proposed methodologies can be concluded as shown in Table 3.

Table 3. Methodology approaches for the fleet size and mix vehicle routing problems (FSMVRPs) and the variants

Year	Authors	Problem	Algorithm
1996	[4]	FSMVRP	Route perturbation procedure and tabu search
1997	[23]	FSMVRPMD	A multi-level (p-level) composite heuristic
1999	[6]	FSMVRP	Generalized insertion and unstringing/ stringing, tabu search using sweep procedure, adaptive memory procedure
2002	[5]	FSMVRP	Sweep-based algorithm approach and suborders of petals selection
2002	[7]	FSMVRP	Tabu search (TS) mixed with reactive TS concepts, variable neighbor-hoods, data-memory structures, and hashing functions
2002	[15]	FSMVRPTW	Adapted combine savings, adapted optimistic opportunity savings, adapted realistic opportunity savings
2007	[16]	FSMVRPTW	Insertion-based parallel approach and a meta-heuristic procedure that adopts the ruin and recreate paradigm for current solution improvement
2007	[17]	FSMVRPTW	Scatter search approach
2008	[18]	FSMVRPTW	Multi-restart deterministic annealing with 3 phases algorithm
2009	[8]	FSMVRP	Tabu search and the generalized insertion and neighborhood reductions
2009	[9]	FSMVRP	Genetic algorithm (GA) applied local search mutation
2009	[10]	FSMVRP	GA hybridized with a local search and distance measure in solution space
2009	[19]	FSMVRPTW	Three-phase hybridized meta-heuristic
2009	[21]	FSMVRPTWSD	Scatter search approach
2009	[24]	FSMVRP and FSMVRPMD	Exact algorithm based on the set partitioning formulation using 3 types of bounding procedures
2010	[20]	FSMVRPTW	Adaptive memory programming solution approach, semi-parallel con-struction heuristic, and tabu search
2011	[12]	FSMVRP	Iterated local and a set partitioning formulation
2011	[14]	FSMVRP	Hybridized heuristic based on iterated local search useing a variable neighborhood descent procedure, with a random neighborhood ordering
2012	[13]	Fleet composition	9-step meta-heuristic based on evolutionary algorithms and local search
2012	[36]	Fleet composition	A ring radial topology continuous model to define vehicle zones and types
2013	[22]	FSMVRPTWSD	Scatter search approach

3 Problem Description

The problem description is divided into two major parts: the fleet size and mix vehicle routing problem with time windows (FSMVRPTW), and the robust fleet size and mix vehicle routing problem with time windows (RFSMVRPTW).

3.1 Fleet Size and Mix Vehicle Routing Problem with Time Windows

The fleet size and mix vehicle routing problem with time windows is an extension of the fleet size and mix vehicle routing problem (FSMVRP) that is a specific problem of the classic vehicle routing problems (VRPs). The FSMVRPTW can be formed on the directed graph $G = (N, A)$. N represents the node set consisting of the customers and a depot. $N = \{0, 1, \ldots, n+1\}$. The depot is denoted by two nodes of $\{0\}$ and $\{n+1\}$, the remaining set of nodes $C = \{1, 2, \ldots, n\}$ is a given customer set. A is the arc set that design the routes.

In this paper, the FSMVRPTW is classified into two points of view: customers and fleet size and mix vehicles. The problem formulations are based on the models as proposed by several researchers [16],[17],[18],[20]. The customers are represented by the node set, where $C = \{1, 2, \ldots, n\}$, a location of each individual customer scatters around a depot represented by the graph $G = \{N, A\}$. Each customer must be visited by exactly one vehicle. In this FSMVRPTW, the time windows are determined by the customer i denoted as the earliest and latest arrival time, $[e_i, l_i]$ and e_i is less than or equal to l_i. The time windows are supposed to be of the hard type in this paper. It means that the customers do not allow any services to violate the time windows constraint. The truck that arrives at the destination too early has to wait until the earliest time permission is opened [15]. In this research, the demand of the customers, d_i is stochastic or uncertain and is modeled as shown in the next topic.

The fleet size and mix vehicle routing problem is approached at the strategic level, the problem is initiated from the assumption that there are unlimited number of available vehicles [3]. The fleet is heterogeneous with K different types of vehicles, such type $K = \{1, 2, \ldots, k\}$, are composed in order to serve all customers' uncertain demand d_i, that is particular for this paper. Each vehicle type k is an element of K, and has a capacity q^k and $q^1 < q^2 < \ldots < q^K$. Alike other general capacitated VRPs, each truck can carry a maximum of its capacity. Repoussis and Tarantilis [20] present two addition formulations that eliminate all possible infeasible sub-tour. The accumulated carried weight of each vehicle k, a_i^k is computed and to make sure that the truck k will not service the customer j if its total weight moment at the customer j, in the route from i to j, is exceeded the maximum capacity of the truck.

Recall the graph $G = (N, A)$, there is only one depot, it is the centralized node of the graph. The depot is represented by two nodes of $N = \{0\}$ and $\{n+1\}$. Every arc, called route of the vehicle fleet, must be started at the depot, linked to the other assigned customer(s), and ended a loop by returning to the depot. Once the first customer in a path is visited, a vehicle has to leave from that place, continues to the next one until the tasks are completed. The depot is also determined the time interval restriction, denoted by $[e_0, l_0] = [e_{n+1}, l_{n+1}]$. As described in the above part of customer definition, when a truck k arrives at customer i, it is allowed to begin the unloading services, denoted by service beginning time y_i^k, within the time windows of such customer. Two possible events of the truck arrival time are considered, 1) too early arrival and 2) in due time arrival, it is noted that time windows are hard, too late arrival is not permitted. In the case of a truck arriving early, the useless activity is considered as waiting time. Anyhow, both cases can be modeled the moment at which service begins at customer *j*, by setting y_j^k = max{0, e_j - (y_i^k+st_i+t_{ij})}. The feasible schedule for each vehicle route can be guaranteed by forces y_i^k to zero whenever customer *i* is not visited by vehicle *k* [20] and be imposed a minimum time for beginning the customer service *j* in a determined route with no subtours guarantee [17]. The further trick linearizes the formulation by using the big M-method, that may be replaced by max{l_i+st_i+t_{ij}–e_j, 0} for all (*i*, *j*) are the elements in *A* and for all *k* are the members of *K* [18].

In each route, two types of cost are considered for the total transportation cost, a fixed acquisition cost and a variable cost. The fixed acquisition cost, f^k is for a vehicle of type *k* where $f^1 < f^2 < \ldots < f^K$. The variable cost, c_{ij} is the cost of traveling from customer *i* to *j*, (*i*, *j*) is a set of arc *A*. The travel distance dt_{ij} and the travel time t_{ij} are given and can be obtained by joining a pair of node. The symmetry and deterministic properties are imposed for both parameters. Further, a unit of distance is assumed to be equaled to one and has the same unit of the travel time, t_{ij} [15]. Because the time windows are stated of being constrained in this case, assume the variable cost equals to the total time spending along the determined route. The total time spending is computed by considering three types of usage time consisting of 1) traveling time between a pair of nodes (t_{ij}), 2) service times (st_i) that the truck spends for performing a loading or unloading activity at each customer site and 3) waiting time (w_i) that can occur only if the truck arrives the customer *i* before the permitted earliest time. In the general problem, not yet the robustness case, the objective function of the FSMVRPTW may consider three components, 1) fixed acquisition cost, 2) variable traveling cost and 3) waiting time and/or service time consideration as the ‚en route‘ cost. In this paper, the FSMVRPTW is the total summation of the fixed cost obtained from vehicle fleet composition acquisition and the sum of total times spending including waiting times, both components are demonstrated in the first and second term of the FSMVRPTW objective function (equation 1), respectively.

$$Z_{FSMVRPTW} = \min \sum_{k\in K} f_k \sum_{j\in N} x_{0j}^{k} + \sum_{k\in K} (y_{n+1,k} - y_{0,k}) \tag{1}$$

3.2 Robust Fleet Size and Mix Vehicle Routing Problem with Time Windows (RFSMVRPTW)

In this paper, the term “*robustness*” refers to the solution robustness in which the obtained solution remains *close* to optimal for all scenarios. The specific definition of the *robustness* is applied the definitions as definited by Kouvelis and Yu [37], Manisri *et al.* [33], and Moghaddam *et al.* [30] as following:

Definition 1: A scenario *s* is a set of customer demands realizations,*Ud*. A whole system *S* is a combination of individual scenario in which $s_1 \cup s_2 \cup \ldots \cup s_n \in S$, $\forall s \in S$.

Definition 2: A scenario s_i is a representation of a system in which the customers’ demands are uncertain by the impact of individual customer’s behavior based on risk aversion ($\beta^s \alpha^s$). The permutation percentage, β^s, of each customer is randomly generated, assuming as uncertainty represented by the uniform distribution of [β_l^s,β_u^s], and the normal distribution of $N(\mu, \sigma)$. The β_l^s and β_u^s are the lower and upper bounds of an avoidance of the risk. The symbols μ and σ are the mean and the variance of the risk. The customer demands of each scenario are either lower or higher than the expected values. The independent random value, α^s, of -1 and 1 are assumed as generated randomly to indicate the direction of the uncertain demand that makes the value of the demand becomes lower or upper the expected value.

Definition 3: By the definitions 1 and 2, a scenario is a set of uncertain customers' demands Ud, modeled as $Ud_i = (1+\beta^s\alpha^s)d_0$ where i = 1, 2, …, n; d_0 is an expected demand of customer i, and $s \in S$.

A mathematical formulation for the RFSMVRPTW belongs to the FSMVRPTW but the customer demand (d_i) is replaced by the set of uncertain demand (Ud) model as modified [30]. The original model assigns one single fixed value of the percentage deviation of the risk averse for all customers and for each scenario, but in this paper the risk averse depends upon each customer's behavior which is represented by a random value with the uniform and normal distribution. The uncertain demand is possible to be lower or higher than the expected demand (d_0) that depends on the independent random variable and has a value between [-1,1].

As presented in the robust handbook of Kouvelis and Yu [37], this concept is applied in some research such as [31] and [33]. In this research, the robust decision making framework is adapted the concept of Kouvelis and Yu [37] but the final result is evaluated against the the deterministic approach [26]. Thus it can balance between the expensive cost when a robust approach is applied and the unmet need when the deterministic approach has to suffer if the worst case happens.

Even this research assumption considers the uncertain input variables, the robust discrete optimization is suggested by using the minimax criterion to reduce the complexity of the problems. The minimax criterion is one of the worst case approaches. The criterion aims to evaluate the highest level of cost taken across all possible future input data scenarios to be as low as possible, as a result that the outcome can protect the worst that might happen [37].

Referring to Kouvelis and Yu [37], let X be the set of the decision variables and D^s denotes the instance of the input data that corresponds to scenario s. The notation F^s stands for the set of all feasible decisions when the scenario s is realized. The function $f(X, D^s)$ is used for evaluating the robustness quality of the decision $X \in F^s$. Then, the optimal single scenario decision X^{s*} for the input data instance D^s is the solution to a deterministic optimization problem and it satisfies Equation 2.

$$z^s = f(X^{s*}, D^s) = \min_{X \in F^s} f(X, D^s) \quad (2)$$

The proactive robustness approach is focused to benefit in long run planning by hedging against all scenarios. The absolute robustness is one of the proactive robustness approaches that is applied for RFSMVRPTW. The absolute robust decision X_A is defined as the one that minimizes the maximum total cost, among all feasible decisions over all realizable input data scenarios. The absolute robust decisions are of a conservative nature, as they are based on the anticipation that the worst might happen. One way to motivate such a criterion is for competitive situations where the parameters of the decision model are affected by competitors' actions. The main uncertainty to this RFSMVRPTW concentrates on the customer demands. The demands vary over a pre-specified planning horizon. Thus, the fleet and routing designs are decided over a long period of time for reducing the impact on the system effectiveness. It means that the solutions obtained from the decision making are good enough for a variety of future operating scenarios and this is referred to the term *robustness* [27].

Recall $s \in S$ be the input data scenario index and S be the set of all possible scenarios. The objective function of RFSMVRPTW is to minimize the maximum total cost of FSMVRPTW (Equation 1) is in placed by the absolute robustness as shown in Equation 3. The robust objective function is subject to the constraints as common used in the fleet size and mix vehicle routing problems. The comprehensive meaning of the robust optimization solution i.e. solution of the total transportation cost is good for all possible data uncertainty and hedge against the worst case.

$$Z_A(\text{RFSMVRPTW}) = \min_{X,Y}\max_{s\in S}\left(\sum_{k\in K} f_k \sum_{j\in N} x_{0j}^{k} + \sum_{k\in K}(y_{n+1,k} - y_{0,k})\right) \quad (3)$$

4 Solution Approach

Resulting from the reviews, the heuristic and metaheuristic algorithms can be either renewed or innovated to solve the RFSMVRPTW under uncertainty of an input parameter such as travel time, demand, etc. In this paper, the uncertain characteristics of the customer demands are focused, so the modification of randomized search heuristics based on genetic algorithms is suggested. The heuristics and me-tageuristic is reconstructed by performing three major phases. The first phase is to build an initial solution, and pass to the next phase for improvement. The robustness is generated in the last phase based on worst case scenarios.

Even the customer demands in this paper are considered as uncertainty, this complexity is reduced by converting the uncertainty to deterministic scenario-based approach. A single scenario is a representation of a set of input data uncertainty to the decision model, and all total assumed cases represent a whole system. The number of realizable scenarios over a pre-specified planning horizon normally depends upon a person who takes charge of the strategic planning task or a person who gets involved in managing the customers' demands information. The scope of this research does not involve finding the potential number of scenarios, so several numbers of scenarios will be assumed to be the representations of realistic situations. The first and second phases are processes until all scenarios are completely solved. All results are passed through the last phase for finding the robustness solution.

4.1 Phase I: Initial Solution Construction

The re-constructed meta-huristic based on modified genetic algorithm (mGA) which adapts the algorithm [38] is used to construct an initial solution of a giant tour. Two kinds of operations in genetic algorithms are induced: crossover and mutation. Kirk [38] developed the GA process using a one-point crossover with an order-based operator. The one-point crossover operator randomly selects one crossover point and then copies everything before this point from the first parent and then everything after the crossover point copy from the second parent. The mutation process uses a frame-shift based on flip operation. The pair of genes substitution is applied by a swap operation. By believing that the best solution can heal the weak cells, the original program has been modified by memorizing the best solution. After the solutions have been generated for several generations, the best solution will take place either a randomized solution or the worse solution. According to this assumption, the next generation result will be improved by then.

4.2 Phase II: Solution Improvement

Step 1: Route Insertion Based TSP Ordered Hybridized Shift Operation
In this phase, the single route of the TSP is converted into multiple routes of the VRP. A single customer node inserts to each route by TSP ordered but the algorithm is designed to check the lower and upper bounds of the time intervals. If the ordered node that is inserted into the route violates the time windows constraint, the next city will be considered. The algorithm is programmed by applying the shift procedure. Meanwhile, the algorithm considers the result of a single iteration and uses the best result obtained from each shift operation to form the next arc of the current route. The tour is constructing continuously until time windows constraint is violated. The other routes are created until all customers are completely assigned by a single vehicle.

Step 2: Route Merging
It is according to one component of the objective function is to achieve the minimization of the 'en route' time travelled. The waiting time is the critical key that impacts the objective. It means that the customer who has the least open time window with least waiting time should be prioritized to serve first. Thus, the procedure is designed to perform ascending the order of the waiting times once the random tours are merged. After having combined the routes and sorted the waiting times, a greedy search is activated. The greedy algorithms build up a solution piece by piece, always choosing the next piece that offers the most obvious and immediate benefit [39].

Step 3: Fleet Sizing and Mixing
In this step, the number of scenarios is determined. The input variable data sets are generated according to the uncertain demand model as described by Definition 3. The process is executed in a one by one scenario. The total uncertain customer demand of each assigned tour is calculated. Vehicle matching is performed by selecting the best fit between the total demands and the vehicle type in which the remaining spaces after loaded is as less as possible. The total transportation cost composes the fixed vehicle cost and the total en route time travelled is performed the calculation after the fleet size selection is done. The process is continued and is terminated after the predetermined number of iterations is reached.

Step 4: Inter-route Neighborhood Search Methods
The neighborhood search procedures using the concept of the inter-route moves based on relocation, exchange, and cross is applied. The major scheme is to randomize the

sets of a total number of routes, the route orders, and the customer nodes for performing the inter-route moves. The procedure is designed either for a pair-route and a set-route of the randomized sets as mentioned previously. If an infeasible solution is obtained, then steps 1-3 are recalled for regenerating a feasible result. The best solution is memorized and will be replaced by the current solution if its outcome is better. The program is terminated when the determined iteration number is reached.

4.3 Phase III – Robustness Decision Making

The proactive robustness is concentrated in this paper by assuming the planning is decided over a long period of time for reducing the impact by the demand uncertainty on the system. The decision making in this research is supposed to perform before the fact and using the expected demands of the original fleet size and mix vehicle routing problem represent the actual realized data. From phase I and II, the scenarios of the input uncertain demands of the customers are created. The previous process seeks for a solution of each run for each scenario in which the total transportation cost is minimized. The robust solution is evaluated using three criteria: absolute difference robustness criteria, relative difference robustness criteria, and variable or deviation robustness criteria. That is to select the maximum solution among all decisions of each scenario and to perform the calculation among Equation 3. The worst-case implementation is proposed to find the solution which is hedges against the worst of all possible scenarios.

In this research, the evaluation of the robust solution results uses the extra cost comparison and the unmet demand indicators [26]. The extra cost performance measurement indicator (ratio x) quantifies the relative extra cost of the robust with respect to the cost of the deterministic. It means that once the robust approach is selected, there are the additional costs caused by the worst case based consideration. If the deterministic approach is purposed, this extra cost is not suffered. The extra cost ratio is calculated using the Equation 4.

$$\text{Extra cost ratio, } x = (Z_{RFSMVRPTW} - Z_{FSMVRPTW}) / Z_{RFSMVRPTW} \tag{4}$$

$$\text{Unmet demand ratio, } u = \max Ud / \textstyle\sum_{i \in C} d_i^0 \tag{5}$$

The unmet demand performance measurement indicator is used to show the effect if the deterministic optimization is applied to the uncertain data problems. The unmet demand denoted by the ratio u is a performance indicator used to measure the demand when facing with the worst case. The deterministic approach is chosen to solve the problem under the expected demand, but the demands are vary in the real situations. The unmet demand indicator quantifies the relative maximum unsatisfied demand (max Ud) of the uncertainty based scenarios with respect to the total expected demands of the deterministic problem. The unmet demand ratio is calculated using the Equation 5. It is because the robust approach based worst case scenarios is designed to protect against the worst of all possible scenarios, thus the unmet demand of the robust decision making is equaled to zero.

Two proposed robust performance indicators: extra cost and unmet demand, are used for balancing between the expensive cost when a robust approach is applied and the unsatisfied demands when the deterministic approach has to suffer once the worst case happens. The additional solution performance measurement is compared with some benchmark problem sets. Due to the non-existence of recent published papers on the RFSMVRPTW, the solutions resulted from some FSMVRPTW research works, for example, [18] and [20] are used to examine the competitive performance obtained from the proposed methodology.

5 Computational Results

The proposed methology of the modification of a genetic algorithm and the adaptation of a greedy search hybridized with inter-route neighborhood search methods has been programmed in MATLAB and has ran on an Intel(R) Core(TM) i5-3337U CPU@1.80GHz 8.00GB-RAM.

In this paper, the first trial of the re-constuctued meta-heuristic algorithm for RSFMVRPTW is tested on the data set of the first problem, i.e. R101 with 100-customer, of the well-known benchmark problem sets generated by Solomon. The geographical data are randomly generated in R101, a short scheduling horizon and only a few customers per route allowing are the characteristics of this problem as stated by Solomon [40]. The cost structure of Liu and Shen [42], as referred in [18],[20] is the benchmark problem using for the experiment. The performance of the algorithm is compared with three previous outputs of the other authors, i.e. (A) the best known solution that have collected from the survey [20], (B) the result obtained by the adaptive memory programming [20], and (C) the multirestart deterministic annealing meta-heuristic [18].

On the real business, the scenarios are determined by the decision makers who are authorized in the decision making of the indiviual problem. In this paper, the algorithm has been operated to handle three scenarios. The first scenario (Scen-1) uses the expected demands using the original given demands of the benchmark problem set. The second scenario (Scen-2) assumes that the demands are patterned as the uniform distribution of the risk averse ($\beta^s \alpha^s$) within [-1,1] interval are used for generating β^s, further about the α^s, only the upper interval of 100 is determined. The last scenario (Scen-3) has the same characteristic of β^s, but α^s is represented as the normal distribution using the the concept of the weekly demand as proposed in [41] where the expected demand is converted based on weekly i.e. the monthly expected and the variance of the demand is divided by 4.33 (52 weeks per year/12 months) and the square root of 4.33, respectively. The results are determined as the mean and variance, respectively. It is because 1) the algorithm has been programmed based on random permutation selection in Phase I and Phase II, and 2) Scenario 2 and Scenario 3 assume that the customer demands are not certain, the output is different for each run. Thus, each scenario is repeated the execution for ten runs.

The computation results demonstrate in a cost form (*1,000) of {DC;FC;TC} where the abbrevations in the bracket denote the distance cost (DC), fleet cost (FC), and total cost (TC) of all three scenarios, respectively. The average outcomes of each scenratios are as following: Scen-1: {4.01;2.74;6.75}, Scen-2: {4.01;2.77;6.78}, and Scen-3: {4.09;2.71;6.80}. The total cost deviation of three scenarios are eqaul to 0.08, 0.27, and 0.10 for Scen-1, Scen-2, and Scen-3, respectively. The Scen-1, Scen-2, and Scen-3 have the average CPU run time (RT) as eqaul to 78.125, 78.996, and 90.507, respectively. Recall Formulation 3, the summarization of the maximum total cost of FSMVRPTW among all three scenatios for all 15 runs can be listed as following: {6.71;6.93;6.79;6.87;6.85;6.86;7.28;7.10;6.85;6.77}, it results to obtain the minimize the maximum total cost, i.e. $Z_{RFSMVRPTW}$, equals to 6,706.35. This result is called the robustness solution. Such solution of the total transportation cost is good for all possible data uncertainty and hedge against the worst case.

The second report shows the evaluation of the robust solutions used the extra cost comparison and the unmet demand indicators [26]. The robust solution is obtained from the whole scenarios, i.e. $Z_{RFSMVRPTW}$ = 6,706.35 and the deterministic solution is based on Scen-1, i.e. $Z_{FSMVRPTW}$ = 6,621.83. The extra cost ratio (x) equals to 0.0126, it means that when the robustness solution is selected, this extra cost percentage is on top of the normal case. The maximum uncertain demand (maxUd) as equals to 1,628 is selected from the whole runs that the program has generated based on the uncertain demand model. The total value of the expected demands (totalD_o), i.e. 1,458, is the base case based on the data set as determined in the benchmark problem. The result of unmet demand ratio (u) is equal to 1.1166. It indicates that if the deterministic solution is chosen, it has to plan for sufferring the unknown demands with such extra ratio of the expected demand that might be occurred in some periods of time.

The last performance indicators show the evaluation of the percent improvement among the previous outputs of the other authors. The letters, A,B, and C represent the best known solutions that have been collected from the survey [20], the result obtained from the adaptive memory programming [20], the multirestart deterministic annealing mera-heuristic [18], respectively. The comparison between the deterministic and deterministic solution of ten experiments for each scenario shows the improvement of 4.41%, 4.54%, and 1.95% in average when compare with A, B, and C, respectively. Further, the result comparisons of the robust solution against the deterministic solution indicate that the outcomes of 4.96%, 5.10%, and 2.52% are improved in average when compare with A, B, and C, respectively.

6 Conclusions and Discussions

In this paper, the re-constructed meta-heuristic aglorithm based on the modification of the genetic algorithm and the adaptation of the greedy search hybridized with inter-route neighborhood search methods are proposed to solve the robust fleet size and mix vehicle routing problem with time windows under uncertain demands. The results indicate that by applying this technique, the average total cost of each scenario is not quite different. The deviation of the solutions of Scenario 1 (base case) and Scenario 3 (demands with normal distribution) are less than Scenario 2 (demands with uniform distribution). The robust decision making is performed based on the worst case scenario, the minimum of the maximum over the whole scenario is evaluation as the robustness solution in this experiment. The solution is preferred to select for hedging against the worst of all possible scenarios that might occur in the unknown future if the uncertain situtation(s) is involved in the problem. In this paper the robustness performance is evaluated by using the extra cost and the unmet demand, both criterions are for the decision maker to consider when the robust approach is implemented. The final results are compared to three benchmark solutions. In the base case (scenario-1), the total cost is reduced compared among all three previous known. When the robustness approch is implemented, the solutions are still significant in the competitive performance.

As mentioned in the previous section, this is the first experiment of this proposed algorithm. Thus, in order to illustrate that the proposed technique is more efficient and competitive, the main recommendation is that the addition future researches should be conducted as following: 1) the other problem sets of the well-known benchmark problem have to be tested, 2) the experiment of this developed methodologies should be performed and implemented in the real business cases 3) the number of trial runs have to be re-considered, and 4) the other meta-heuristic algorithms such as fuzzy logic, differential evolution, ant corony optimization, particle swarm optimization, articificial neural network, hybrid evolution swarm with local search methods, etc. should be cosidered for the comparative studies.

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