A Manufacturer-Retailer Supply Chain with Fuzzy Consumer Demand

A Vertical Cooperative Advertising and Pricing Model

Gerhard Aust

Abstract In this paper, we apply fuzzy set theory to a single-manufacturer single-retailer supply chain, where both players try to determine their optimal pricing and advertising decisions. The interaction between manufacturer and retailer is analyzed by means of a Stackelberg game. Moreover, a vertical cooperative advertising program is considered, which represents a financial agreement where the manufacturer offers to share a certain fraction of his retailer's advertising expenditures. Even though this topic gained substantial interest in recent years' operations research literature and studies reveal that results strongly depend on demand parameters, most analyses are limited to deterministic model formulations. Here, fuzzy set theory has the advantage that it is not only able to incorporate the uncertainty of demand parameters into analysis. Furthermore, it enables us to take into consideration the experience of decision makers, which is often not expressed numerically, but rather in vague linguistic terms.

Keywords Game theory · Fuzzy set theory · Supply chain management · Vertical cooperative advertising · Pricing

1 Introduction

Vertical cooperative advertising programs are financial agreements between manufacturers and their retailers on the sharing of advertising expenditures (cf. Crimmins 1984). In most cases, this financial assistance is offered by manufacturers, who thereby intend to increase the retailers' advertising in order to generate sales (cf. Somers et al. 1990). Reasons for this form of cooperation can be manifold: Besides cheaper access to local media or better knowledge of local markets, mainly the different effects of manufacturer's and retailer's advertising are mentioned.

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J. Dethloff et al. (eds.), *Logistics Management*, Lecture Notes in Logistics,
DOI 10.1007/978-3-319-13177-1_7

That means, manufacturers use their advertising campaigns primarily to build up brand image, while retailers' advertising aims on generating immediate sales (cf. Hutchins 1953; Young and Greyser 1983).

Therefore, vertical cooperative advertising programs are very common in practice. Empirical data clearly show an increasing trend, e.g., from $15 billion which were spent for such programs in the United States of America in 2000 up to $50 billion in 2008 (cf. He et al. 2012; Nagler 2006). However, Nagler's study reveals that manufacturers mostly set their participation rates to 50 % or 100 % instead of conducting an appropriate analysis on the optimum percentage.

This gap between importance and theoretical background in approaching cooperative advertising has motivated many researchers to study related questions, especially the determination of advertising expenditures and prices of the different echelons of a supply chain. Thereby, the findings of the different analyses reveal that results as optimal participation rate, prices, spending on advertising, or the profit split within the supply chain strongly depend on the underlying demand function as well as on the assumed parameters (see Aust and Buscher 2011).

However, as a result of uncertain consumer behavior, demand and advertising effectiveness parameters are often unknown in practice. Stochastic models based on probability distributions may be of avail in some cases, but they require extensive historical data, which is often not available to decision makers. At this point, the fuzzy set theory proposed by Zadeh (1965) may be a promising instrument, as it is able to incorporate the experience of decision makers, which is usually expressed in linguistic terms like *low*, *medium*, or *high* price sensitivity. Hence, our scope is to propose how fuzzy set theory can be applied to vertical cooperative advertising models.

The remainder of this article is organized as follows: In Sect. 2, we first introduce some basic concepts of fuzzy set theory. In the next section, we develop a mathematical model of a single-manufacturer single-retailer supply chain with fuzzy demand and advertising effectiveness parameters (3.1) and apply a Manufacturer Stackelberg game to that model (3.2). As a result, we derive closed-form solutions for the players' prices, advertising expenditures, and profits, which are further analyzed in Sect. 4. The paper is concluded with a short summary of the main findings and some open topics for future research.

2 Fuzzy Set Theory

In this work, we will only give a brief introduction into fuzzy set theory and calculation rules for fuzzy variables which are necessary for the following analysis. For a more formal introduction and the relevant definitions and axioms, we refer the reader to Zadeh (1965) and Nahmias (1978), or to the comprehensive books of Liu (2009, 2013). A more summarized but still formal discussion can be found in Zhou et al. (2008).

Let ζ and η be two independent and nonnegative fuzzy variables, $f(\cdot)$ a function, $\text{Pos}(\cdot)$ a possibility measure of a certain event, and φ a possibility value with $0 < \varphi \leq 1$. According to Liu (2009), we can define the φ-pessimistic value ζ_φ^L and the φ-optimistic value ζ_φ^U of fuzzy variable ζ as follows:

$$\zeta_\varphi^L = \inf\{r|\ \text{Pos}\{\zeta \leq r\} \geq \varphi\} \text{ and } \zeta_\varphi^U = \sup\{r|\ \text{Pos}\{\zeta \geq r\} \geq \varphi\}. \tag{1}$$

Hence, the φ-pessimistic value ζ_φ^L is the greatest lower bound that fuzzy variable ζ will reach with a possibility of φ, while the φ-optimistic value ζ_φ^U is the least upper bound that ζ will reach with a possibility of φ. ζ_φ^L and ζ_φ^U can now be used to calculate the expected value $E[\zeta]$ of ζ (cf. Liu and Liu 2003):

$$E[\zeta] = \frac{1}{2}\int_0^1 \left(\zeta_\varphi^L + \zeta_\varphi^U\right) \mathrm{d}\varphi. \tag{2}$$

Table 1 gives an overview of calculation rules for φ-optimistic and φ-pessimistic values as well as for related expected values, which will be used later on during calculus. Thereby, x and y denote normal real-valued numbers, which are also called *crisp numbers* within the context of fuzzy set theory (cf. Liu 2013).

After this consideration of general fuzzy variables, we turn our attention to triangular fuzzy variables, which are solely used in the following. These fuzzy variables are of the shape $\tilde{\zeta} = (x, y, z)$ and consist of three crisp numbers $x < y < z$. According to Eq. (1), the φ-pessimistic and φ-optimistic values of a triangular fuzzy variable are (cf. Zhao et al. 2012b):

$$\zeta_\varphi^L = y\varphi + x(1-\varphi) \text{ and } \zeta_\varphi^U = y\varphi + z(1-\varphi). \tag{3}$$

Table 1 Calculation rules for fuzzy variables

Operation	φ-pessimistic value	φ-optimistic value	Source
Scalar multiplication ($x > 0$)	$(x\zeta)_\varphi^L = x\zeta_\varphi^L$	$(x\zeta)_\varphi^U = x\zeta_\varphi^U$	Liu and Liu (2003)
Scalar multiplication ($x < 0$)	$(x\zeta)_\varphi^L = x\zeta_\varphi^U$	$(x\zeta)_\varphi^U = x\zeta_\varphi^L$	Liu and Liu (2003)
Addition	$(\zeta+\eta)_\varphi^L = \zeta_\varphi^L + \eta_\varphi^L$	$(\zeta+\eta)_\varphi^U = \zeta_\varphi^U + \eta_\varphi^U$	Liu and Liu (2003)
Multiplication	$(\zeta\cdot\eta)_\varphi^L = \zeta_\varphi^L \cdot \eta_\varphi^L$	$(\zeta\cdot\eta)_\varphi^U = \zeta_\varphi^U \cdot \eta_\varphi^U$	Zhao et al. (2006)
$f(\cdot)$ with $f'(\cdot) > 0$	$(f(\zeta))_\varphi^L = f(\zeta_\varphi^L)$	$(f(\zeta))_\varphi^U = f(\zeta_\varphi^U)$	Zhou et al. (2008)
$f(\cdot)$ with $f'(\cdot) < 0$	$(f(\zeta))_\varphi^L = f(\zeta_\varphi^U)$	$(f(\zeta))_\varphi^U = f(\zeta_\varphi^L)$	Zhou et al. (2008)
Expected value	$E[x\zeta + y\eta] = xE[\zeta] + yE[\eta]$		Liu and Liu (2003)

By means of Eq. (2), we can derive the following expression for the expected value of a triangular fuzzy variable:

$$E[\tilde{\zeta}] = \frac{x + 2y + z}{4}. \tag{4}$$

3 A Manufacturer-Retailer Supply Chain Model with Fuzzy Consumer Demand

The first mathematical model on cooperative advertising in a manufacturer-retailer supply chain was proposed by Berger (1972). In the following, many different extensions have been published, prevalently with game-theoretic analyses. We refer the reader to a recent review of Xie and Zhang (2011), where relevant articles are summarized. Although one can realize an increased interest in this field in recent years, there are only few stochastic approaches (see, e.g., Chen 2011; He et al. 2011; Tsao and Sheen 2012), while most authors consider deterministic models. However, to the best of our knowledge, no application of fuzzy set theory to a cooperative advertising model yet exists. Therefore, we take on a deterministic model formulation recently published by Aust and Buscher (2012), which is simplified in order to ensure mathematical tractability, and transform the parameters of the demand function as well as the advertising effectiveness into fuzzy parameters. Similar approaches of applying fuzzy set theory to supply chain models, which are not related to cooperative advertising, can be found in, e.g., Zhou et al. (2008), who consider fuzzy demand and manufacturing cost in a two-echelon pricing game. This model is further expanded to a manufacturer-duopoly (see Zhao et al. 2012b) or a retailer-duopoly (see Zhao et al. 2012a).

3.1 Model Formulation

We consider a single-manufacturer single-retailer supply chain, which is illustrated in Fig. 1. This supply chain sells one product to the customer market, which demands a quantity D of the product. For each unit, customers pay a retail price p to the retailer, who, for his part, pays a wholesale price w to the manufacturer. The consumer demand $D(p, a)$ depends both on the retail price p and on the retailer's advertising expenditures a. Please note that we do not consider manufacturer's advertising expenditures in order to simplify our analysis (see, e.g., Karray and Zaccour 2006; Yang et al. 2013; though, the distinction between manufacturer and retailer advertising is a common assumption, which can be found in, e.g., SeyedEsfahani et al. 2011; Xie and Wei 2009). However, the manufacturer has the possibility to participate in his retailer's advertising expenditures by means of a cooperative advertising program. Here, we assume that the manufacturer decides on a participation rate t, with $0 \leq t < 1$ (see Table 2 for a listing of symbols used in this article).

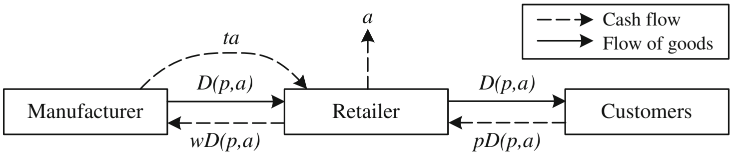

Fig. 1 Manufacturer-retailer supply chain

Table 2 List of symbols

Variables		Parameters	
m	Manufacturer's margin	$\tilde{\alpha}$	Base demand
w	Retailer's margin	$\tilde{\beta}$	Price sensitivity
a	Retailer's advertising expenditure	$\tilde{k}_r$	Advertising effectiveness
t	Advertising participation rate		
Π	Profit	$h(\cdot)$	Price demand function
D	Demand quantity	$g(\cdot)$	Advertising demand function

With respect to the cash flows in Fig. 1, we can set up the profit functions of the manufacturer (Π_m) and the retailer (Π_r):

$$\Pi_m = wD(p,a) - ta \tag{5}$$

$$\Pi_r = mD(p,a) - (1-t)a. \tag{6}$$

Here, m denotes the retailer's margin, which can be calculated via $m = p - w$. As stated above, customer demand depends both on retail price p and advertising expenditures a. Thereby, one can distinguish a price-induced demand component $g(p)$ and an advertising-induced demand component $h(a)$. Following Kunter (2012) and Yan (2010), we assume a linear price demand function

$$g(p) = \tilde{\alpha} - \tilde{\beta}p, \tag{7}$$

where $\tilde{\alpha}$ and $\tilde{\beta}$ are fuzzy parameters. In detail, $\tilde{\alpha}$ describes the initial base demand, i.e., the customer demand which occurs for $p = 0$, while $\tilde{\beta}$ can be interpreted as customers' price sensitivity. In order to ensure a non-negative demand quantity, we set $\text{Pos}(\{\tilde{\alpha} - \tilde{\beta}p < 0\}) = 0$.

Concerning advertising demand, we apply a square root function, which corresponds to the widely spread advertising saturation effect (see, e.g., Kim and Staelin 1999; Zhang and Xie 2012):

$$h(a) = \tilde{k}_r\sqrt{a}. \tag{8}$$

The fuzzy variable $\tilde{k}_{\mathrm{r}}$ determines the effectiveness of advertising expenditures. We assume that advertising affects demand like a multiplicator (cf. Thompson and Teng 1984). With this multiplicative relationship between price and advertising demand, we can now formulate the extensive total demand function as well as the profit functions of both players:

$$D(p,a) = g(p)h(a) = (\tilde{\alpha} - \tilde{\beta}p)\tilde{k}_{\mathrm{r}}\sqrt{a}, \tag{9}$$

$$\Pi_{\mathrm{m}}(w,t) = w\left[\tilde{\alpha} - \tilde{\beta}(w+m)\right]\tilde{k}_{\mathrm{r}}\sqrt{a} - ta \tag{10}$$

$$\Pi_{\mathrm{r}}(m,a) = m\left[\tilde{\alpha} - \tilde{\beta}(w+m)\right]\tilde{k}_{\mathrm{r}}\sqrt{a} - (1-t)a. \tag{11}$$

3.2 A Manufacturer Stackelberg Equilibrium

For our analysis of the supply chain interaction, we use a Stackelberg game, where the manufacturer obtains the channel leadership, while the retailer acts as a follower. That means the manufacturer has perfect knowledge of the retailer's reaction on his own decision and is therefore able to take this reaction into consideration when determining wholesale price and cooperative advertising participation rate. Mathematically, we first have to calculate the retailer's response functions by solving the following optimization problem:

$$\begin{array}{ll} \max\limits_{m,a} & E[\Pi_{\mathrm{r}}(m,a)] \\ \text{s.t.} & \mathrm{Pos}(\{\tilde{\alpha} - \tilde{\beta}(w+m) < 0\}) = 0 \\ & m, a > 0. \end{array} \tag{12}$$

Please note that we assume that both players try to maximize their expected profits $E[\Pi_{\mathrm{r}}]$ and $E[\Pi_{\mathrm{m}}]$. Another possible objective could also be the φ-optimistic values $\Pi^{U}_{\mathrm{r}\varphi}$ and $\Pi^{U}_{\mathrm{m}\varphi}$, respectively, which can be seen as the maximum profits the players could realize with at least possibility φ. In contrast, the φ-pessimistic values $\Pi^{L}_{\mathrm{r}\varphi}$ and $\Pi^{L}_{\mathrm{m}\varphi}$, respectively, stand for the minimum profits the players could achieve with at least possibility φ (see Zhou et al. 2008). Therefore, we first have to determine the expected profit function of the retailer:

$$\begin{aligned}
E[\Pi_{\mathrm{r}}] &= E\Big[m[\tilde{\alpha} - \tilde{\beta}(m+w)]\tilde{k}_{\mathrm{r}}\sqrt{a} - (1-t)a\Big] \\
&= \frac{1}{2}\int_0^1 \bigg[\Big(m[\tilde{\alpha} - \tilde{\beta}(m+w)]\tilde{k}_{\mathrm{r}}\sqrt{a} - (1-t)a\Big)_{\varphi}^{U} \\
&\quad + \Big(m[\tilde{\alpha} - \tilde{\beta}(m+w)]\tilde{k}_{\mathrm{r}}\sqrt{a} - (1-t)a\Big)_{\varphi}^{L}\bigg]\,\mathrm{d}\varphi \\
&= \frac{1}{2}\int_0^1 \bigg[m\Big(\tilde{\alpha} - \tilde{\beta}(m+w)\Big)_{\varphi}^{U}\big(\tilde{k}_{\mathrm{r}}\sqrt{a}\big)_{\varphi}^{U} - (1-t)a \\
&\quad + m\Big(\tilde{\alpha} - \tilde{\beta}(m+w)\Big)_{\varphi}^{L}\big(\tilde{k}_{\mathrm{r}}\sqrt{a}\big)_{\varphi}^{L} - (1-t)a\bigg]\,\mathrm{d}\varphi \\
&= \frac{m\sqrt{a}}{2}\int_0^1 \Big[[\tilde{\alpha}_{\varphi}^{\mathrm{U}} - \tilde{\beta}_{\varphi}^{\mathrm{L}}(w+m)]\tilde{k}_{\mathrm{r}\varphi}^{\mathrm{U}} + [\tilde{\alpha}_{\varphi}^{\mathrm{L}} - \tilde{\beta}_{\varphi}^{\mathrm{U}}(w+m)]\tilde{k}_{\mathrm{r}\varphi}^{\mathrm{L}}\Big]\,\mathrm{d}\varphi - (1-t)a \\
&= \frac{m\sqrt{a}}{2}\int_0^1 \Big[\tilde{\alpha}_{\varphi}^{\mathrm{U}}\tilde{k}_{\mathrm{r}\varphi}^{\mathrm{U}} - \tilde{\beta}_{\varphi}^{\mathrm{L}}\tilde{k}_{\mathrm{r}\varphi}^{\mathrm{U}}(w+m) + \tilde{\alpha}_{\varphi}^{\mathrm{L}}\tilde{k}_{\mathrm{r}\varphi}^{\mathrm{L}} - \tilde{\beta}_{\varphi}^{\mathrm{U}}\tilde{k}_{\mathrm{r}\varphi}^{\mathrm{L}}(w+m)\Big]\,\mathrm{d}\varphi - (1-t)a \\
&= m\sqrt{a}\bigg[E[\tilde{\alpha}\tilde{k}_{\mathrm{r}}] - \frac{w+m}{2}\int_0^1 \Big(\tilde{\beta}_{\varphi}^{\mathrm{L}}\tilde{k}_{\mathrm{r}\varphi}^{\mathrm{U}} + \tilde{\beta}_{\varphi}^{\mathrm{U}}\tilde{k}_{\mathrm{r}\varphi}^{\mathrm{L}}\Big)\,\mathrm{d}\varphi\bigg] - (1-t)a. \\
&= m\sqrt{a}\big[E[\tilde{\alpha}\tilde{k}_{\mathrm{r}}] - \Psi(w+m)\big] - (1-t)a,
\end{aligned} \tag{13}$$

with Ψ being defined as follows:

$$\Psi = \frac{1}{2}\int_0^1 \Big(\tilde{\beta}_{\varphi}^{\mathrm{L}}\tilde{k}_{\mathrm{r}\varphi}^{\mathrm{U}} + \tilde{\beta}_{\varphi}^{\mathrm{U}}\tilde{k}_{\mathrm{r}\varphi}^{\mathrm{L}}\Big)\,\mathrm{d}\varphi. \tag{14}$$

In order to determine the retailer's response functions, we have to calculate the first order partial derivatives with respect to m and a:

$$\frac{\partial E[\Pi_{\mathrm{r}}]}{\partial m} = \sqrt{a}\big[E[\tilde{\alpha}\tilde{k}_{\mathrm{r}}] - \Psi(w+m)\big] - \Psi m\sqrt{a} \tag{15}$$

$$\frac{\partial E[\Pi_{\mathrm{r}}]}{\partial a} = \frac{m}{2\sqrt{a}}\big[E[\tilde{\alpha}\tilde{k}_{\mathrm{r}}] - \Psi(w+m)\big] - (1-t). \tag{16}$$

Setting Eqs. (15) and (16) to zero and eliminating m from $a(m, w, t)$ leads to:

$$m(w) = \frac{E[\tilde{\alpha}\tilde{k}_{\mathrm{r}}] - \Psi w}{2\Psi} \tag{17}$$

$$a(w,t) = \frac{\big(E[\tilde{\alpha}\tilde{k}_{\mathrm{r}}] - \Psi w\big)^4}{64\Psi^2(1-t)^2}. \tag{18}$$

Thereafter, we now consider the manufacturer's decision problem given by:

$$\begin{array}{ll} \max\limits_{w,t} & E[\Pi_{\mathrm{m}}(w,t)] \\ \text{s.t.} & m = (E[\tilde{\alpha}\tilde{k}_{\mathrm{r}}] - \Psi w)/2\Psi \\ & a = \left(E[\tilde{\alpha}\tilde{k}_{\mathrm{r}}] - \Psi w\right)^4/64\Psi^2(1-t)^2 \\ & \mathrm{Pos}(\{\tilde{\alpha} - \tilde{\beta}(w+m) < 0\}) = 0 \\ & w > 0, 0 \le t < 1. \end{array} \tag{19}$$

The manufacturer's expected profit function $E[\Pi_{\mathrm{m}}(w,t)]$ can be determined analogously to the retailer's expected profit given in Eq. (13). Hence, we derive:

$$E[\Pi_{\mathrm{m}}] = w\sqrt{a}\left(E[\tilde{\alpha}\tilde{k}_{\mathrm{r}}] - \Psi(w+m)\right) - ta, \tag{20}$$

with Ψ being defined identical to Eq. (14). Inserting m and a from Eqs. (17) and (18) into $E[\Pi_{\mathrm{m}}]$ yields:

$$E[\Pi_{\mathrm{m}}] = \frac{\left(E[\tilde{\alpha}\tilde{k}_{\mathrm{r}}] - \Psi w\right)^3\left(4\Psi w - 3\Psi wt - E[\tilde{\alpha}\tilde{k}_{\mathrm{r}}]t\right)}{64\Psi^2(1-t)^2}. \tag{21}$$

Similar to the retailer's problem, one has to set the partial first order derivatives to zero. If we first consider the manufacturer's participation rate t, we get:

$$\frac{\partial E[\Pi_{\mathrm{m}}]}{\partial t} = \frac{\left(E[\tilde{\alpha}\tilde{k}_{\mathrm{r}}] - \Psi w\right)^3\left[\left(-3\Psi w - E[\tilde{\alpha}\tilde{k}_{\mathrm{r}}]\right)(1-t) + 8\Psi w - 6\Psi wt - 2E[\tilde{\alpha}\tilde{k}_{\mathrm{r}}]t\right]}{64\Psi^2(1-t)^3}. \tag{22}$$

From $\partial E[\Pi_{\mathrm{m}}]/\partial t = 0$ one can derive:

$$t(w) = \frac{-E[\tilde{\alpha}\tilde{k}_{\mathrm{r}}] + 5\Psi w}{E[\tilde{\alpha}\tilde{k}_{\mathrm{r}}] + 3\Psi w}. \tag{23}$$

Please note that this expression can take negative values for $w < E[\tilde{\alpha}\tilde{k}_{\mathrm{r}}]/5\Psi$, which would violate the domain of definition given in Sect. 3.1. Therefore, it is necessary to check if the obtained solution for w complies with the condition $w > E[\tilde{\alpha}\tilde{k}_{\mathrm{r}}]/5\Psi$; otherwise, we have to set $t = 0$. Setting the partial first order derivative with respect to w,

$$\begin{aligned} \frac{\partial E[\Pi_{\mathrm{m}}]}{\partial w} = & \frac{\left(E[\tilde{\alpha}\tilde{k}_{\mathrm{r}}] - \Psi w\right)^2}{64\Psi^2(1-t)^2} \\ & \cdot \left[-3\Psi\left(4\Psi w - 3\Psi wt - E[\tilde{\alpha}\tilde{k}_{\mathrm{r}}]t\right) + \left(E[\tilde{\alpha}\tilde{k}_{\mathrm{r}}] - \Psi w\right)(4\Psi - 3\Psi t)\right], \end{aligned} \tag{24}$$

Table 3 Manufacturer Stackelberg equilibrium

	Margins	Advertising	Profits
Retailer	$m = \frac{E[\tilde{\alpha}\tilde{k}_r]}{3\Psi}$	$a = \frac{E^4[\tilde{\alpha}\tilde{k}_r]}{144\Psi^2}$	$\Pi_r = \frac{E^4[\tilde{\alpha}\tilde{k}_r]}{216\Psi^2}$
Manufacturer	$w = \frac{E[\tilde{\alpha}\tilde{k}_r]}{3\Psi}$	$t = \frac{1}{3}$	$\Pi_m = \frac{E^4[\tilde{\alpha}\tilde{k}_r]}{144\Psi^2}$
With $\Psi = \frac{1}{2}\int_0^1 \left(\tilde{\beta}^L_\varphi \tilde{k}^U_{r\varphi} + \tilde{\beta}^U_\varphi \tilde{k}^L_{r\varphi}\right) \mathrm{d}\varphi$			

to zero yields an expression for w, which solely depends on participation rate t:

$$w(t) = \frac{E[\tilde{\alpha}\tilde{k}_r]}{\Psi(4 - 3t)}. \tag{25}$$

We can now solve the system of equations given by Eqs. (17), (18), (23), and (25) in order to obtain closed-form solutions of the Manufacturer Stackelberg equilibrium. The results as well as the corresponding profits are given in Table 3. It is easy to see that the calculated wholesale price $w = E[\tilde{\alpha}\tilde{k}_r]/3\Psi$ always complies with the condition which follows from Eq. (25). Hence, the participation rate $t = 1/3$, which results independent of model parameters, is feasible.

4 Numerical Studies

This section provides numerical examples of the previously obtained results (see Table 3). As described above, one advantage of fuzzy set theory is the ability to include the experience of decision makers, which is mostly verbalized by linguistic expressions like 'customers are *very sensitive*, *sensitive*, or *less sensitive* to changes in prices', which are rather vague than clearly assignable to a single (crisp) value. Therefore, we use triangular fuzzy variables of the form $\zeta = (x, y, z)$, which do not only describe one single number, but rather a range of possible values.

Hence, the first step is to determine appropriate triangular fuzzy variables, which correctly represent the decision makers' experience and estimation. One possible way can be found in Cheng (2004), who proposes a group opinion aggregation model based on a grading process. However, for the sake of simplicity, we arbitrarily choose triangular fuzzy variables for the parameters $\tilde{\alpha}$, $\tilde{\beta}$, and $\tilde{k}_r$ in this paper, which can be found in Table 4.

Let us now assume a medium base demand $\tilde{\alpha}$ (about 20), a sensitive price sensitivity $\tilde{\beta}$ (about 1.25), and a low advertising effectiveness $\tilde{k}_r$ (about 2). By means of Eq. (3), we can calculate the φ-pessimistic and φ-optimistic values:

Table 4 Allocation of linguistic expressions to triangular fuzzy variables

	Linguistic expression	Triangular fuzzy variable
Base demand $\tilde{\alpha}$	Low (about 10)	(5, 10, 15)
	Medium (about 20)	(15, 20, 25)
	High (about 30)	(25, 30, 35)
Price sensitivity $\tilde{\beta}$	Very sensitive (about 1.75)	(1.5, 1.75, 2)
	Sensitive (about 1.25)	(1, 1.25, 1.5)
	Less sensitive (about 0.75)	(0.5, 0.75, 1)
Advertising effectiveness $\tilde{k}_r$	Low (about 0.2)	(0.1, 0.2, 0.3)
	Medium (about 0.3)	(0.2, 0.3, 0.4)
	High (about 0.4)	(0.3, 0.4, 0.5)

$\tilde{\alpha}^{L}_{\varphi} = 15 + 5\varphi$	$\tilde{\beta}^{L}_{\varphi} = 1 + 0.25\varphi$	$\tilde{k}^{L}_{r\varphi} = 0.1 + 0.1\varphi$
$\tilde{\alpha}^{U}_{\varphi} = 25 - 5\varphi$	$\tilde{\beta}^{U}_{\varphi} = 1.5 - 0.25\varphi$	$\tilde{k}^{U}_{r\varphi} = 0.3 - 0.1\varphi.$

These values are inserted into Eq. (2) in order to determine $E[\tilde{\alpha}\tilde{k}_r]$,

$$
\begin{aligned}
E[\tilde{\alpha}\tilde{k}_r] &= \frac{1}{2}\int_0^1 \left(\tilde{\alpha}^{L}_{\varphi}\tilde{k}^{L}_{r\varphi} + \tilde{\alpha}^{U}_{\varphi}\tilde{k}^{U}_{r\varphi}\right) d\varphi \\
&= \frac{1}{2}\int_0^1 [(15 + 5\varphi)(1 + \varphi) + (25 - 5\varphi)(3 - \varphi)]\, d\varphi \\
&= 5\int_0^1 \left(\varphi^2 - 2\varphi + 9\right) d\varphi = 41.67.
\end{aligned}
$$

Analogously, we can calculate $\Psi = 0.24$.

The resulting prices, advertising expenditures, and expected profits of manufacturer and retailer in a Manufacturer Stackelberg equilibrium, which derive from inserting $E[\tilde{\alpha}\tilde{k}_r]$ and Ψ into the expressions given in Table 3, are listed in Table 5, together with the results of the crisp case. Here, we can see that both players set the same margins m and w. However, the manufacturer can realize a higher profit than his retailer, which can be explained by the participation rate t = 1/3: The whole supply chain invests a = 35.84 into advertising (fuzzy case), whereof the manufacturer bears one-third, while two-thirds remain in the retailers responsibility. Even if this is only one certain set of parameters, these findings can be generalized to

Table 5 Numerical example with medium $\tilde{\alpha}$, sensitive $\tilde{\beta}$, and low $\tilde{k}_r$ (see Table 4)

	m^*	w^*	a^*	t^*	$E[\Pi_r]$	$E[\Pi_m]$
Fuzzy parameters	5.75	5.75	35.84	0.33	23.89	35.84
Crisp parameters	5.33	5.33	28.44	0.33	18.96	28.44

Table 6 Variation of fuzziness of $\tilde{\alpha}$, with sensitive $\tilde{\beta}$ and low $\tilde{k}_r$ (see Table 4)

$\tilde{\alpha}$	m^*	w^*	a^*	t^*	$E[\Pi_r]$	$E[\Pi_m]$
(10, 20, 30)	5.98	5.98	41.93	0.33	27.95	41.93
(12.5, 20, 27.5)	5.86	5.86	38.79	0.33	25.86	38.79
(15, 20, 25)	5.75	5.75	35.84	0.33	23.89	35.84
(17.5, 20, 22.5)	5.63	5.63	33.06	0.33	22.04	33.06
(20, 20, 20)	5.33	5.33	28.44	0.33	18.96	28.44

some extent, as it is visible from Table 3: Retailer's and manufacturer's margin are always identical ($m = w$) according to this model, and also the inequality $E[\Pi_m] > E[\Pi_r]$ holds for any parameters. Furthermore, the participation rate $t = 1/3$ is constant, as it does not depend on any parameter of the model.

Therefore, we turn our attention to the comparison of fuzzy and crisp case. Here, we can see that, besides the constant participation rate t, each variable assumes higher values, and that both players can expect higher profits under a fuzzy customer demand. The variation of the fuzziness of the market base $\tilde{\alpha}$ in Table 6 shows similar results. The higher the fuzziness of the market base, the higher the players set margins and advertising expenditures, which lead to higher expected profits. This is consistent with previous research on pricing models without advertising (see, e.g., Zhao et al. 2012b).

5 Conclusion

In this article, we analyzed a single-manufacturer single-retailer supply chain with fuzzy consumer demand, which is sensitive to prices and advertising. In order to increase the retailer's advertising expenditures, the manufacturer has the possibility to participate in his retailer's advertising costs by means of a vertical cooperative advertising program. To the best of our knowledge, this is the first application of fuzzy set theory to a cooperative advertising model. In contrast to deterministic approaches, which require detailed information about customer behavior and market characteristics, we are able to include experience of decision makers into our model, as fuzzy set theory allows us to transform linguistic expressions (e.g., *high* or *low* base demand), into triangular fuzzy variables. Through our numerical examples, we furthermore derive that a higher fuzziness of parameters, i.e., a bigger range of values the parameter may take, leads to higher expected profits, while the participation rate should be constantly set to one-third, independent of the market demand parameters.

However, this is only a first approach of applying fuzzy set theory to cooperative advertising in a supply chain and, therefore, underlies certain limitations: First, in order to reduce mathematical complexity, we had not only to restrict the price demand function to a linear shape instead of the more general form previously published; furthermore, we were only able to consider advertising of the retailer, while it is common in research to integrate also the manufacturer's decision on advertising into analysis. Besides this, future research should also consider different membership functions of fuzzy variables instead of the triangular one.

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