Multi-scroll Chaotic Oscillator Based on a First-Order Delay Differential Equation

Viet-Thanh Pham, Christos K. Volos and Sundarapandian Vaidyanathan

Abstract After the discovery of the well-known chaotic Lorenz's system, the study of chaos has received considerable attention due to its promising applications in a variety of fields, ranging from physics, economics, biology to engineering. Moreover, chaotic systems with multiple scrolls can exhibit more rich dynamics than the general chaotic ones with few attractors. This expansion of dynamics leads to multi-scroll chaotic oscillators showing better performance in several chaotic-based applications, such as secure communication, encrypting fingerprint image, controlling motion directions of autonomous mobile robots, or generating pseudo random numbers etc. As a result, investigating new chaotic oscillators with multiple scrolls has been become an attractive research direction of both theoretical and practical interest recently. Although numerous approaches for constructing multi-scroll attractors from conventional three-dimension chaotic systems have been reported intensively, there are few publications regarding the multi-scroll attractors from infinite dimensional time-delay systems. This work presents a new multi-scroll chaotic oscillator and its circuital design. This chaotic system is described by a first-order delay differential equation with piecewise linear function. It is shown through simulations that the proposed system can exhibit odd number of scrolls of chaotic attractors such as three-, five-, seven-, and nine-scroll attractors. In addition, the detailed implementation of the proposed multi-scroll oscillator using the electronic simulation package Multisim is also presented to show the feasibility of the oscillator. The Multisim results of the chaotic oscillator are well agree with the numerical simulation results. It is noting that the new multi-scroll chaotic circuit

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A.T. Azar and S. Vaidyanathan (eds.), *Chaos Modeling and Control Systems Design*,
Studies in Computational Intelligence 581, DOI 10.1007/978-3-319-13132-0_4

has been designed with simple common components, like resistors, capacitors, and operational amplifiers.

Keywords Chaos · Multi-scroll attractor · Time delay · Oscillator · Circuit · Multisim

1 Introduction

Chaotic systems have been studied for recent decades after the discovery of the first classical chaotic attractor in 1963 [1]. Despite the fact that there is not an universal definition of chaos, three remarkable characteristics of a chaotic system are: dynamical instability, topological mixing and dense periodic orbits [2]. Dynamical instability is known as the “butterfly effect”, which means that a small change in initial conditions of a system can create significant differences. In other word, this vital characteristic makes the system highly sensitive to initial conditions [1, 3]. Topologically mixing is refers as stretching and folding of the phase space, which means that the chaotic trajectory at the phase space will evolve in time so that each given area of this trajectory will eventually cover part of any particular region. Dense periodic orbits means that the trajectory can come arbitrarily close every possible asymptotic state. Therefore, chaotic systems have been widely applied in various practical fields [4, 5].

Interestingly, systems with multi-scroll attractors have become potential candidates for using in chaos-based applications because of their complex dynamics compared with conventional chaotic systems with few attractors. For example, fingerprint images were encrypted via a two-dimensional chaotic sequence achieved from multi-scroll chaotic attractors [6]. The entropy of a random number generator was improved by increasing the number of scrolls in generalized Jerk circuit [7]. In addition, encrypted audio and image information were transmitted when considering the synchronization of Chua’s circuits with multi-scroll attractor [8]. A parameter determination method for double-scroll chaotic systems was also applied to chaotic cryptanalysis [9]. As a result, there are numerous researches on generating multi-scroll chaotic attractor (see [10] and references cited there in).

The presence of time delay has been observed in various dynamical systems [11, 12]. In order to describe exactly their special features, delay differential equations (DDEs) have used because delay differential equations differ from ordinary differential equations in that the evolution of dependent variables at a certain time depend on their values at previous times. Infinite dimensional dynamics of time-delay systems could make complex phenomena [13, 14] which do not exist in original systems. As a result, chaotic oscillator can be constructed by using a time-delay system. In particular, the oscillators described by first-order DDEs can exhibit chaos [15]. These oscillators have attracted considerable attention due to their simplicities and feasibilities. It is worth noting that, multi-scroll attractors could be obtained from time-delay chaotic systems [16].

In this work a novel multi-scroll oscillator, which is modelled by a first-order delay differential equation, is proposed. This work is organized as follows. After presenting the related works in Sect. 2, the mathematical model of the new proposed oscillator is introduced in Sect. 3. Then, the circuital realization of a three-scroll oscillator is described and illustrated in Sect. 4. In Sect. 5, the obtained results are represented. Discussions relating to the implementation and the potential applications of the new time-delayed oscillator are given in Sect. 6. Finally, some conclusion remarks are drawn in the last Section.

2 Related Works

Chua's circuit, an invention of L.O. Chua [17], is one of the most well-known nonlinear electronic circuits This simple electronic circuit satisfies three criteria to exhibit chaotic behavior due to the fact that Chua's circuit includes three energy storage elements (two capacitors and an inductor), a locally active resistor and a nonlinear element (Chua's diode with a piecewise-linear characteristic). Despite of its simple structure, Chua's circuit can display complex dynamics. Therefore, Chua's circuit is considered as a platform for the study of nonlinear phenomena, like chaos [18].

It is interesting that Chua's circuit can display the classical double scroll. Based on this known circuit, Suykens and Vandewalle investigated a new family of n-double scroll attractors by modifying the characteristic of the nonlinear resistor with additional break points [19]. The major breakthrough of Suykens and Vandewalle has promoted a considerable number of studies on nonlinear systems with multi-scroll attractors. Diverse noticeable design approaches to generate multi-scroll attractors have been reported in the literature, i.e. quasi-linear function approach, nonlinear modulating function approach, step function approach, hysteresis series approach, saturated function series approach, or step series switching approach etc. [10, 20–24]. Recently, generating the multi-scroll attractors from infinite dimensional time-delay systems have been a focal topic of interest because even one delay differential equation is enough to generate multi-scroll chaos [7, 16, 25, 26].

While a large number of studies [27–30] focused mainly on time-delay oscillators with mono- and double-scroll attractors, there are a few reported oscillators can display multi-scroll attractors [7, 16]. In fact, designed multi-scroll oscillators have built mainly based on suitable nonlinear functions. Some typical nonlinear functions are summarized as follows.

A simple time-delay systems was introduced by Sprott [31] with a sinusoidal nonlinearity

$$F(x) = \sin(x). \tag{1}$$

This system displayed 6-scroll chaotic attractor. Here, the sine function can be implemented by using a trigonometric function integrated circuit AD639. Wang [26] considered the activation function took a reflection symmetric piecewise linear function of the form

$$F(x) = Ax + 0.5(A - B)[(|x + m| - |x - m|) - (|x + n| - |x - n|)], \quad (2)$$

with $A = 4.3$, $B = -5.8$, $m = 1.1$ and $n = 3.3$. By adjusting the value of only a single parameter, four-scroll attractor was obtained. This nonlinear function could be built conveniently by operational amplifiers and constant voltage sources. A piecewise linear function with a threshold controller [25] was applied in the following form

$$F(x) = AF^* - Bx, \quad (3)$$

where

$$F^* = \begin{cases} -x^* & x < -x^* \\ x & -x^* \le x \le x^* \\ x^* & x > x^*, \end{cases} \quad (4)$$

where $A = 5.2$, $B = 3.5$, and $x^* = 0.7$ is the controllable threshold value. The nonlinearity was implemented by using only two diodes and few operational amplifiers. By adding more number of threshold values, two double-scroll attractors could be produced. In addition, Yalcin [7] proposed a nonlinearity based on a hard limited function

$$F(x) = \sum_{i=1}^{M_x} g_{(-2i+1)/2}(x) + \sum_{i=1}^{N_x} g_{(-2i+1)/2}(x), \quad (5)$$

where

$$g_\theta(\zeta) = \begin{cases} 1 & \zeta \ge \theta, \ \theta > 0 \\ 0 & \zeta < \theta, \ \theta > 0 \\ 0 & \zeta \ge \theta, \ \theta < 0 \\ -1 & \zeta < \theta, \ \theta < 0. \end{cases} \quad (6)$$

The system exhibited n-scroll chaotic attractor for suitable values of M_x and N_x. For example, three-, four-, five-, and six-scroll attractors obtained when $\{M_x = 1, N_x = 1\}$, $\{M_x = 1, N_x = 2\}$, $\{M_x = 0, N_x = 4\}$, and $\{M_x = 1, N_x = 4\}$, respectively. Nonlinear block comprised voltage comparators whose total number depending on the number of scrolls. Moreover, Kilinc [16] represented a oscillator employed the nonlinearity as a function of hysteresis series

$$F(x) = \sum_{i=1}^{N}(h(x-2i)-i) + \sum_{j=0}^{M}(h(x+2j)+j), \tag{7}$$

where $h(x)$ is the basic hysteresis function

$$h(x) = \begin{cases} -1 & x < 0.5 \\ 1, & x > -0.5 \end{cases} \tag{8}$$

Similar to the previous system (5), by choosing the appropriate values of M and N, $N + M + 2$ scrolls could be generated. For instance, the three-, four-, and five-scroll attractors could be observed when $\{N = 0, M = 1\}$, $\{N = 0, M = 2\}$, and $\{N = 0, M = 3\}$, respectively. Hysteresis comparators were realized employing positive feedback around the classical operational amplifiers. In order to generate N-double scroll delayed chaotic attractors Wang [32] studied a piecewise-linear activation function of the form

$$F_{2N}(x) = m_N x + \frac{1}{2}\sum_{i=1}^{N}(m_{i-1}-m_i)(|x+c_i| - |x-c_i|), \tag{9}$$

where N represents number of double scroll. Here m_i and c_i are the parameters which present the slopes and values in the abscissa corresponding inflexion of the piecewise-linear activation function, respectively [32]. For example, a four-double scroll chaotic attractor could be obtained when $N = 4$, $m_0 = m_2 = m_4 = 6.5$, $m_1 = m_3 = 8.5$, $c_1 = 0.8$, $c_2 = 2.4$, $c_3 = 4$, and $c_4 = 5.6$. The piecewise-linear activation function (9) could be easily implemented by a combination of operational amplifiers, resistors, and dc voltage sources. It is easy to see that novel time-delayed systems could be proposed by discovering appropriate nonlinear functions.

3 Mathematical Model of New Multi-Scroll Oscillators

The first-order DDE describing the proposed oscillator can be written in the following form

$$\frac{dx}{dt} = -ax(t) - bF_{2N+1}(x_\tau), \tag{10}$$

where a and b are positive parameters, x is a dynamical variable, $x_\tau = x(t-\tau)$ with τ is a constant time delay. $F_{2N+1}(x_\tau)$ is a piecewise linear function, which is given as

$$F_{2N+1}(x_\tau) = x_\tau - \sum_{k=1}^{N}[sgn(x_\tau + (2k-1)) + sgn(x_\tau - (2k-1))], \tag{11}$$

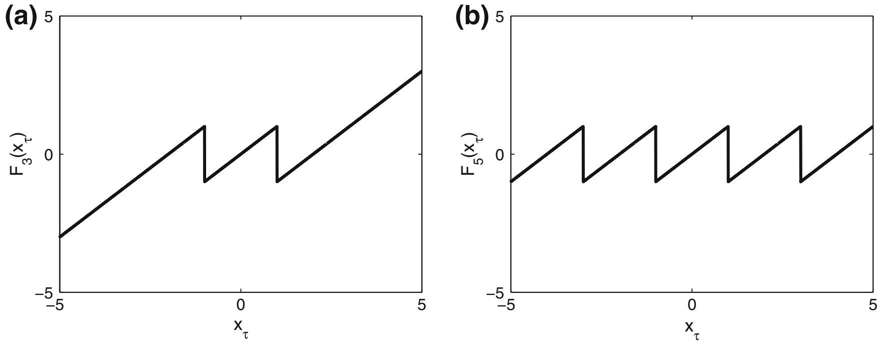

Fig. 1 The piecewise linear function $F_{2N+1}(x_\tau)$: **a** $F_3(x_\tau)$ when $N = 1$, **b** $F_5(x_\tau)$ when $N = 2$

where N and k are positive integers, while the signum function $sgn(.)$ is defined by

$$sgn(x_\tau) = \begin{cases} -1 & x_\tau < 0 \\ 0 & x_\tau = 0 \\ 1 & x_\tau > 0. \end{cases} \tag{12}$$

In this work the parameters of system (10) are determined as: $a = 0.1$, $b = 15$ and $\tau = 0.15$. The piecewise linear function $F_{2N+1}(x_\tau)$ has the saw-tooth shape. Moreover the break points can vary when increasing the value of N, i.e. the piecewise linear functions corresponding to $N = 1$ and $N = 2$ are drawn in Fig. 1. According to the mentioned effect methodology [10], multi-scroll attractors can be generated by adding some additional break points into the piecewise linear functions.In the other word, multi-scroll attractor can be created by changing the value of N. The number of scrolls generated from the system (10) is equal to $2N + 1$. Figure 2 shows the attractors obtained from the system (10) with different values of N. Obviously, the proposed chaotic oscillator can generate odd number of scrolls of chaotic attractors such as three-, five-, seven, and nine-scroll attractors.

4 Circuitry Design of Multi-Scroll Chaotic Oscillator

The simple physical approach for investigating dynamics of a chaotic system is design the electronic circuit which emulates the system [33, 34]. There are some obvious advantages of this approach. Firstly this approach avoids the uncertainties arise from systematic and statistical errors in numerical simulations [35], for example the discretization and round-off errors in the numerical procedures or finite-time approximation of a quantity that is properly described by an infinite-time integral. Secondly signals generated from chaotic electronic oscillators can be displayed on the oscilloscope and observed quickly, comparing to the long

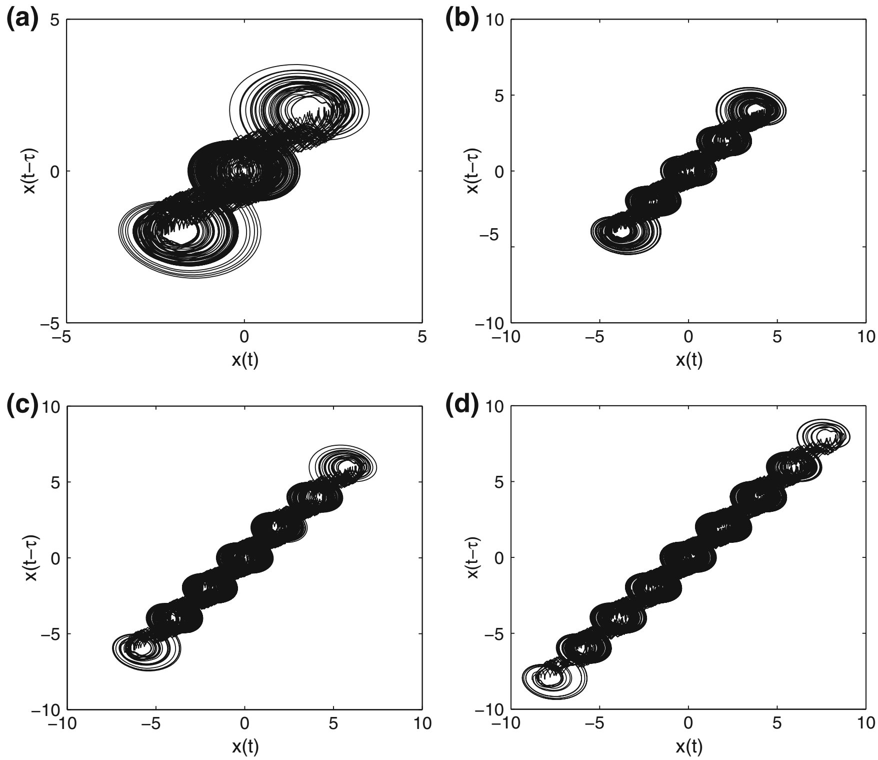

Fig. 2 Multi-scroll attractors obtained from the new chaotic oscillator (10) when changing the value of N: **a** three-scroll attractor ($N = 1$), **b** five-scroll attractor ($N = 2$), **c** seven-scroll attractor ($N = 3$), and **d** nine-scroll attractor ($N = 4$)

computer-based simulation time. In fact, the experimental bifurcation diagram can be also obtained conveniently by changing the value of circuital components. As a result, a wide range of dynamical behavior of the circuit can be compared with the numerical simulations of the corresponding theoretical model. Moreover circuital realization of chaotic system plays an important role in practical applications, i.e. secure communications [36, 37], random generator [38, 39], image encryption [40] or path planning for autonomous robots [18, 37].

Hence, in this Section, the proposed multi-scroll chaotic oscillator will be realized by using a general circuital diagram as illustrated in Fig. 3. There are three main blocks: the integrator (which includes the two resistors R_1, R_2, a capacitor C_0 and an operational amplifier U_1), the piecewise linear block and the time-delay block. In the design and simulation, a lossless transmission line is utilized as a time-delay block [41]. In practice the time-delay block can be implemented by either a network of T-type LCL filters [15, 42] or a series of Bessel filters in cascade [43].

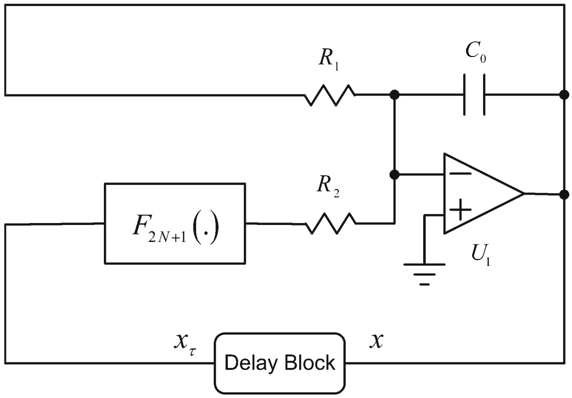

Fig. 3 Circuit diagram of the multi-scroll oscillator. The values of components are chosen as R_1 = 150 kΩ, R_2 = 1 kΩ, C_0 = 0.2 μF

Here the power supplies are ±15 V. From the Sect. 3, it is obvious that the state variables of the multi-scroll chaotic system (10) are in a range between −10 and +10 V. So it does not require rescale the state variables.

The state variable x of the mathematical model (10) is the voltage across the capacitor C_0. The circuit equation has the following form

$$\frac{dV_{C_0}(t)}{dt} = -\frac{1}{R_1 C_0} V_{C_0}(t) - \frac{1}{R_2 C_0} F_{2N+1}\left(V_{C_0}\left(t - T_{delay}\right)\right), \qquad (13)$$

where $R_1 = R_0/a$ and $R_2 = R_0/b$ with R_0 = 15 kΩ.

The value of the time-delay implemented in delay block is T_{delay} = 450 μs, so the dimensionless delay τ is calculated as

$$\tau = \frac{T_{delay}}{R_0 C_0} = 0.15. \qquad (14)$$

For the sake of simplicity, only the implementation of the piecewise function $F_3(x_\tau)$ is introduced in this Section as an example (see Fig. 3). However the realization of other functions $F_{2N+1}(x_\tau)$, i.e. $F_5(x_\tau)$ can be implemented in the same way. It is worth noting that we only use common electronics components to realize the function $F_3(x_\tau)$. Hence, the nonlinearity block can be realized in an easy and compact way. Selected components are summarized in Table 1.

Table 1 Values of selected circuital components in Figs. 2 and 3

Circuit's components	Selected values
R	10 kΩ
R_1	150 kΩ
R_2	1 kΩ
R_3	10 kΩ
R_4, R_5	142.5 kΩ
R_6, R_7, R_8, R_9	150 kΩ
C_0	0.2 μF

5 Obtained Results

The designed circuit is implemented by using Multisim. It is an electronic schematic capture and simulation program of National Instruments (NI). Multisim provides an advanced, industry-standard SPICE simulation environment and is used widely in the world. The whole circuit is realized by only simple off-the-shelf circuital components like resistors, a capacitor and operational amplifiers. In this work, operational amplifiers TL084 have been used. The simulation of the circuit of Fig. 4, which realizes the function in Multisim reveals the piecewise linear nature of this function (Fig. 5a). It is clear that the circuit can produce three-scroll chaotic attractor (Fig. 5b). Figure 5 also shows a good qualitative agreement between the numerical simulation in Sect. 3 and the Multisim results of the designed circuit.

Furthermore, in order to illustrate the feasibility, practicality and flexibility of the introduced model (10), obtained phase portraits with Multisim of another circuit with piecewise linear function $F_5(x_\tau)$ are also presented in Fig. 6, which are consistent with the numerical results in Figs. 1b and 2b. The results in Figs. 5 and 6 confirm that different multi-scroll attractors can be generated when changing the piecewise linear function (11).

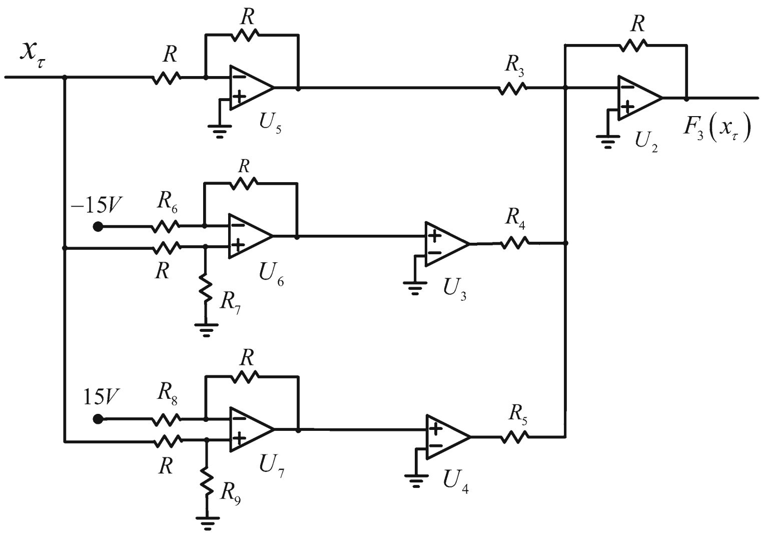

Fig. 4 Circuit diagram of piecewise linear function $F_3(x_\tau)$. The circuit parameters are set as $R_3 = R = 10$ kΩ, $R_4 = R_5 = 142.5$ kΩ and $R_6 = R_7 = R_8 = R_9 = 150$ kΩ

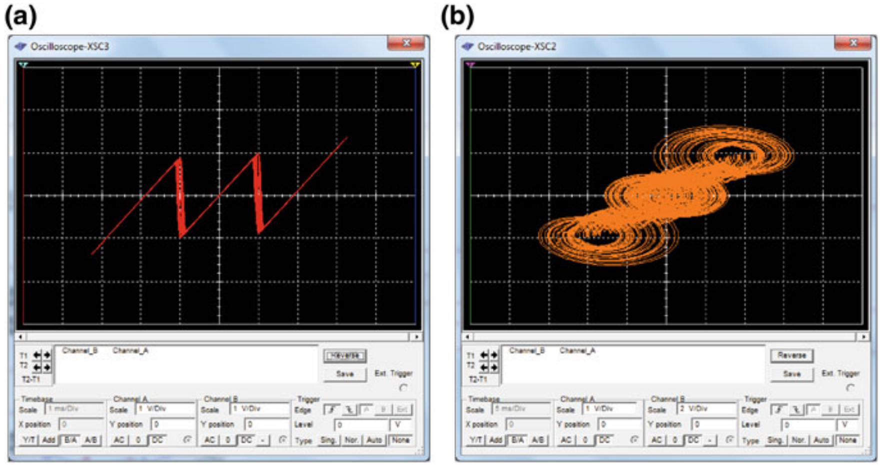

Fig. 5 Simulation results with Multisim of the circuit in Fig. 3 with piecewise linear function $F_3(x_\tau)$: **a** measured characteristic curve of the piecewise linear function $F_3(x_\tau)$, **b** three-scroll attractor presented in $x(t) - x(t - \tau)$ plane

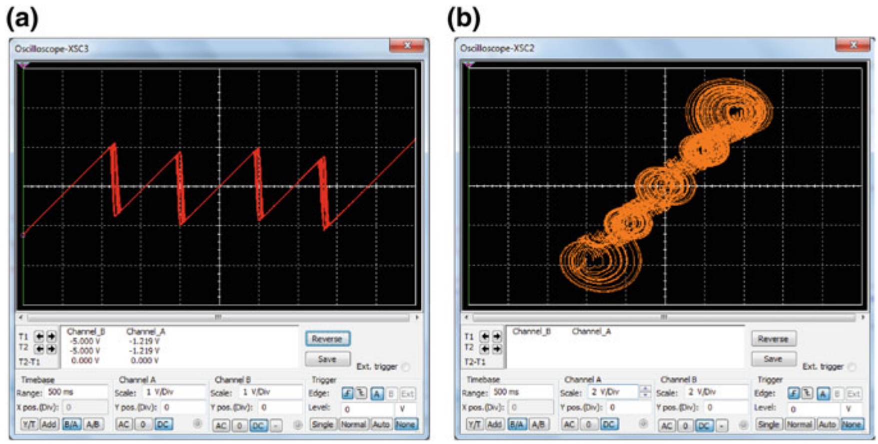

Fig. 6 Simulation results with Multisim of the designed circuit with piecewise linear function $F_5(x_\tau)$ (for the sake of simplicity, detailed design of this function is not reported in this work): **a** measured characteristic curve of the piecewise linear function $F_5(x_\tau)$, **b** five-scroll attractor displayed in $x(t) - x(t - \tau)$ plane

6 Discussion

Dynamical systems with the presence of time delays have been observed in diverse fields such as engineering [11, 13], neutral system [44], physics [14] or biology [45, 46] and so on. The presence of time delay is unavoidable because of the limit calculation speed, memory effects, finite transmission velocity etc. Hence time-delay systems have been described more accurately practical models, for example a single vehicle induced by traffic light and speedup [47], broadband bandpass electro-optic oscillator [48], road traffic [49], or food web systems [50].

On one hand, delay can be utilized to stabilize nonlinear systems. Time-delay feedback control [51] is a highly effective control method which has applied in different systems [13, 14, 52]. The most advantage of this method is that it does not require the prior knowledge about the model. Different to the conventional feedback control, the feedback signal is proportional to the difference of output signal and its delayed version. Time-delay feedback control can stabilize the chaotic behavior to one of unstable fixed points or unstable periodic orbits embedded within chaotic attractor. There are two control parameters: the feedback gain and the feedback time delay. It is worth noticing that the feedback time-delay is often different from the intrinsic delay of time-delay system. Two parameters have been selected by trial-and-error procedures or Lyapunov stability analysis approaches. In particular, for stabilizing of the unstable periodic orbit, the controller time delay has to be chosen as an integer multiple of the period of the desired unstable periodic orbit.

On the other hand, delay could lead to complex behaviors which do not exist in original systems. For example, the systems described by first order delay differential equations (DDE) can exhibit complex phenomena, like chaos [15]. Such systems have attracted more attention because of their complex chaotic attractors as well as their feasibilities, reliability and practicality. For these reasons, there are many efforts to design chaotic time-delay systems, which can generate multiple scrolls. Although some such systems can be found in the literature [7, 16], their nonlinear functions are often extremely complicated to realize in electronic circuits. Therefore two advanced features of the proposed system (10) can be taken into account: the simplicity and the application potentiality. In the former feature, the implementation of the whole system uses only common off-the-shelf circuital components. In other words, the complexity of the implementation stage is reduced significantly. In addition, different numbers of scroll attractors can be obtained easily by changing the nonlinear function (11). Moreover, from the view point of on-chip realization, the proposed system is a suitable candidate for a integrated circuit chaotic generator. Because of its richness of dynamics, the introduced multi-scroll chaotic oscillator is more appropriate to chaos-based applications, especially chaos-based secure communication [53, 54]. Obviously, the new oscillator (10) contains higher dimensional chaotic behavior of delay differential equation and the ability to create multi-scroll chaotic attractors. Hence, if it is involved in chaos-based secure communication architecture, the eavesdropper cannot reconstruct the chaotic attractor and retrieve the hidden message [55–57].

7 Conclusion

In this work, a mathematical model of a multi-scroll oscillator has proposed. Although the new oscillator is described by only a DDE, it can exhibit complex behaviors. By changing the piecewise linear function of the oscillator, three-, five-, seven-, and nine-scroll attractors have been observed. The proposed oscillator has also implemented with analog electronic circuit using common electronic components. The Multisim results of the circuit agree well with the numerical calculations. Because the new oscillator generates complex multi-scroll chaotic attractor, it can be used for secure communication applications or a random bit generator. However, when being applied in secure communications, synchronization property of the new oscillator should be considered carefully in future works. Furthermore, another future direction of research is how to obtain even numbers of scrolls from this oscillator.

Acknowledgments This research is funded by Vietnam National Foundation for Science and Technology Development (NAFOSTED) under grant number 102.99-2013.06.

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