Backstepping Controller Design for the Global Chaos Synchronization of Sprott's Jerk Systems

Sundarapandian Vaidyanathan, Babatunde A. Idowu and Ahmad Taher Azar

Abstract This research work investigates the global chaos synchronization of Sprott's jerk chaotic system using backstepping control method. Sprott's jerk system (1997) is algebraically the simplest dissipative chaotic system consisting of five terms and a quadratic nonlinearity. Sprott's chaotic system involves only five terms and one quadratic nonlinearity, while Rössler's chaotic system (1976) involves seven terms and one quadratic nonlinearity. This work first details the properties of the Sprott's jerk chaotic system. The phase portraits of the Sprott's jerk system are described. The Lyapunov exponents of the Sprott's jerk system are obtained as $L_1 = 0.0525$, $L_2 = 0$ and $L_3 = -2.0727$. The Lyapunov dimension of the Sprott's jerk system is obtained as $D_L = 2.0253$. Next, an active backstepping controller is designed for the global chaos synchronization of identical Sprott's jerk systems with known parameters. The backstepping control method is a recursive procedure that links the choice of a Lyapunov function with the design of a controller and guarantees global asymptotic stability of strict-feedback chaotic systems. Finally, an adaptive backstepping controller is designed for the global chaos synchronization of identical Sprott's jerk systems with unknown parameters. MATLAB simulations are provided to validate and demonstrate the effectiveness of the proposed active and adaptive chaos synchronization schemes for the Sprott's jerk systems.

Keywords Chaos · Backstepping control · Sprott's jerk system · Active control · Adaptive control

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1 Introduction

Chaos is a nonlinear behavior which is a strange random aggregate of responses to internal and external stimuli in dynamical systems. Chaos occurs, when the dynamical system is highly sensitive towards initial conditions. That is, chaotic systems starting off from very similar initial states can develop into radically divergent trajectories. Such sensitive dependence of chaotic systems is popularly known as the *butterfly effect* [1].

Mathematically, chaotic systems are classified as nonlinear dynamical systems which are sensitive to initial conditions, topologically mixing and with dense periodic orbits. The Lyapunov exponent is a measure of the divergence of phase points that are initially very close and can be used to quantify chaotic systems. A positive maximal Lyapunov exponent and phase space compactness are usually taken as defining conditions for a chaotic system.

Since the discovery of a 3-D chaotic system in 1963 by Lorenz to study convection in the atmosphere [27], there is a great deal of interest in the chaos literature in modelling and analysis of new chaotic systems. Some well-known paradigms of 3-D chaotic systems in the literature are [2, 3, 5, 6, 21, 25, 28, 38, 45, 50, 52, 58–60, 71, 72].

Recently, there is a great interest in finding elegant chaos, especially algebraically simple chaotic flows [45]. In 1997, Sprott discovered algebraically the simplest dissipative chaotic system consisting of only five terms and one quadratic nonlinearity [46]. Sprott's system (1997) has two terms fewer than the famous Rössler chaotic system [38], which consists of seven terms and one quadratic nonlinearity.

Sprott's simplest dissipative chaotic system (1997) is a simple jerk system consisting of five terms and one quadratic nonlinearity [46]. In this work, the phase portraits of the Sprott's jerk system are described. The Lyapunov exponents of the Sprott's jerk system are obtained as $L1 = 0.0525;$ $L2 = 0$ and $L3 = -2.0727$. The Lyapunov dimension of the Sprott's jerk system is obtained as $DL = 2.0253$. Since the maximal Lyapunov exponent (MLE) of the Sprott's jerk system is $L1 = 0.0525$, which is a small number, it follows that the Sprott's system exhibits mild chaos only.

Chaotic systems have several important applications in science and engineering. Some important applications can be mentioned as oscillators [18, 44], lasers [22, 67], chemical reactions [11, 33], cryptosystems [37, 55], secure communications [9, 29, 68], biology [8, 20], ecology [12, 48], robotics [30, 62], cardiology [34, 64], neural networks [15, 17], finance [13, 47], etc.

Synchronization of chaotic systems is a phenomenon that occurs when two or more chaotic systems are coupled or when a chaotic system drives another chaotic system [36, 51, 61].

Because of the butterfly effect which causes exponential divergence of the trajectories of two identical chaotic systems started with nearly the same initial conditions, the synchronization of chaotic systems is a challenging research problem in the chaos literature.

Major works on synchronization of chaotic systems deal with the complete synchronization (CS) which has the goal of using the output of the master system to control the slave system so that the output of the slave system tracks the output of the master system asymptotically.

Pecora and Carroll pioneered the research on synchronization of chaotic systems with their seminal papers in 1990s [4, 32]. The active control method [26, 35, 49, 54, 57, 63] is commonly used when the system parameters are available for measurement and the adaptive control method [14, 24, 39–41, 65] is commonly used when some or all the system parameters are not available for measurement and estimates for unknown parameters of the systems.

Other important methods for chaos synchronization are the sampled-data feedback method [10, 23, 66, 69], time-delay feedback method [7, 16, 42, 43], backstepping method [31, 53, 56, 70], etc.

The backstepping control method is a recursive procedure that links the choice of a Lyapunov function with the design of a controller and guarantees global asymptotic stability of strict-feedback chaotic systems. The backstepping method is based on the mathematical model of the examined system, introducing new variables into it in a form depending on the state variables, controlling parameters and stabilizing functions. The use of backstepping method creates an additional nonlinearity and eliminates undesirable nonlinearities from the system.

This research work is organized as follows. Section 2 describes the Sprott's jerk system (1997). In this section, the phase portraits of the Sprott's chaotic system are also displayed using MATLAB. The Lyapunov exponents of the Sprott's system are also obtained in this system and the Lyapunov dimension of the Sprott's system is obtained using MATLAB. The dynamics of the Lyapunov exponents is plotted using MATLAB. Section 3 describes new results for the active backstepping controller design for the global chaos synchronization of identical Sprott's systems with known parameters. Section 4 describes new results for the adaptive backstepping controller design for the global chaos synchronization of identical Sprott's systems with unknown parameters. MATLAB simulations are shown to validate and illustrate all the main synchronization results derived for the Sprott's chaotic systems. Section 5 contains a summary of the main results derived in this research work.

2 Sprott's 3-D Jerk Chaotic System

This section describes the equations and phase portraits of the Sprott's jerk chaotic system (1997), which is algebraically the simplest dissipative chaotic system.

The Sprott chaotic system is described by the 3-D dynamics

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= x_3, \\ \dot{x}_3 &= -ax_1 + x_2^2 - bx_3 \end{aligned} \tag{1}$$

where x_1, x_2, x_3 are the states and a, b are constant, positive, parameters.

The system (1) is a five-term polynomial chaotic system with just one quadratic nonlinearity.

The system (1) depicts a strange chaotic attractor when the constant parameter values are taken as

$$a = 1, \quad b = 2.02 \tag{2}$$

For simulations, the initial values of the Sprott chaotic system (1) are taken as

$$x_1(0) = 4.0, \quad x_2(0) = 2.0, \quad x_3(0) = 0.5. \tag{3}$$

Figure 1 describes the strange chaotic attractor of the Sprott chaotic system (1) in 3-D view. The phase portrait of the Sprott chaotic system also indicates that the Sprott attractor is mildly chaotic. This will be also made clear the Lyapunov exponents of the Sprott attractor are calculated.

The Lyapunov exponents of the Sprott chaotic system (1) are numerically obtained as

$$L_1 = 0.0525, \quad L_2 = 0, \quad L_3 = -2.0727 \tag{4}$$

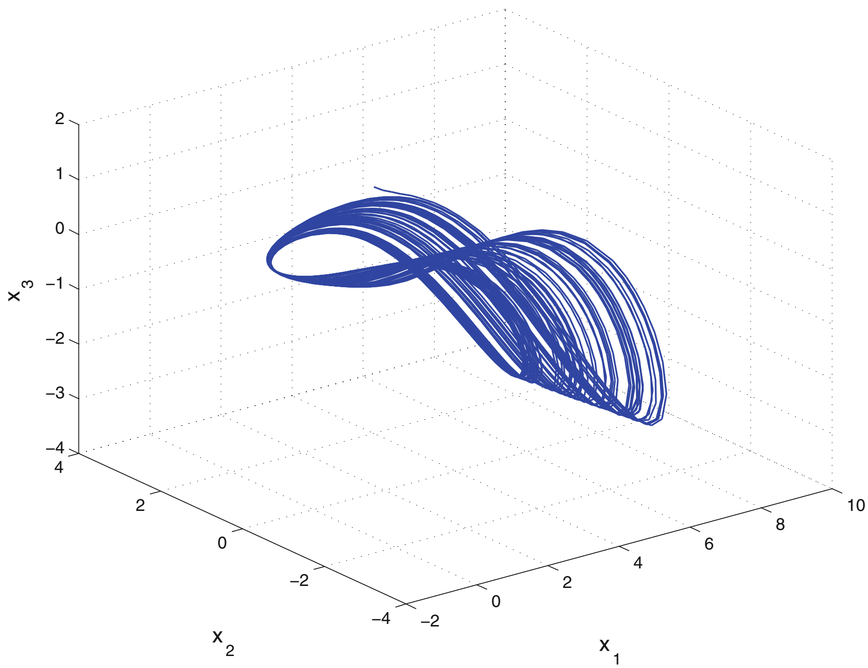

Fig. 1 Strange attractor of the Sprott chaotic system (1997) in $\mathbb{R}^3$

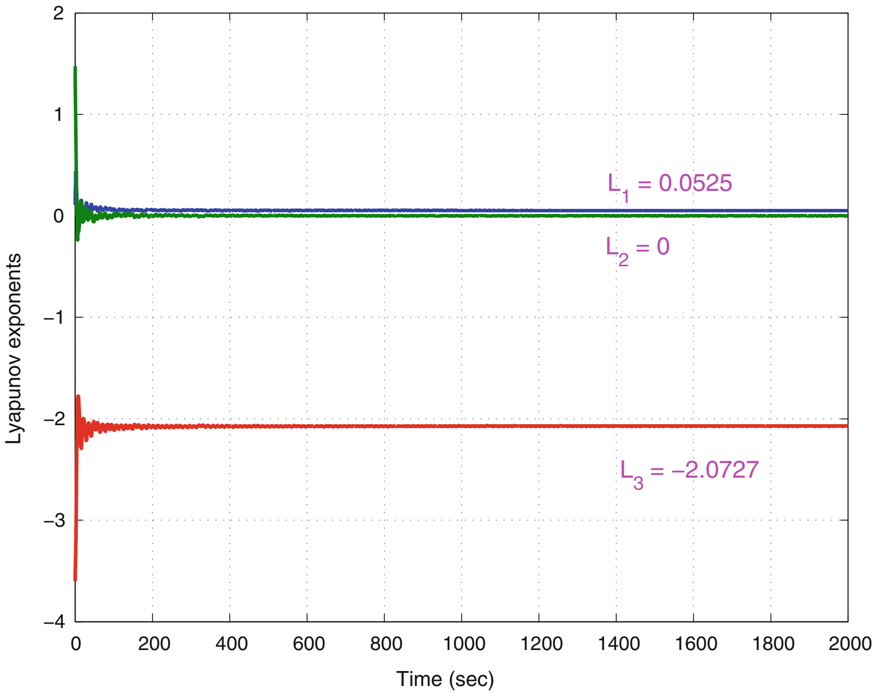

Fig. 2 Dynamics of the Lyapunov exponents of the Sprott chaotic system (1997)

Since $L_1 + L_2 + L_3 = -2.0202 < 0$, the system (1) is dissipative.
Also, the Lyapunov dimension of the system (1) is obtained as

$$D_L = 2 + \frac{L_1 + L_2}{|L_3|} = 2.0523 \tag{5}$$

Figure 2 depicts the dynamics of the Lyapunov exponents of the novel chaotic system (1). From this figure, it is seen that the maximal Lyapunov exponent of the novel chaotic system (1) is $L_1 = 0.0525$, which is a very small value. Thus, the Sprott chaotic system (1) exhibits only mild chaotic properties.

3 Active Backstepping Design for the Synchronization of Identical Sprott Jerk Chaotic Systems

This section derives new results for the active backstepping design for the global chaos synchronization of the identical Sprott jerk systems with known parameters. The main result of this section is proved using Lyapunov stability theory.

The master system is described by the Sprott chaotic system

$$\begin{aligned}\dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= -ax_1 + x_2^2 - bx_3\end{aligned} \tag{6}$$

where x_1, x_2, x_3 are state variables and a, b are positive, constant, parameters.

The slave system is described by the controlled Sprott chaotic system

$$\begin{aligned}\dot{y}_1 &= y_2 \\ \dot{y}_2 &= y_3 \\ \dot{y}_3 &= -ay_1 + y_2^2 - by_3 + u\end{aligned} \tag{7}$$

where y_1, y_2, y_3 are state variables and $u(t)$ is the active backstepping controller to be designed.

The synchronization error between the master system (6) and the slave system (7) is defined as

$$\begin{aligned}e_1(t) &= y_1(t) - x_1(t) \\ e_2(t) &= y_2(t) - x_2(t) \\ e_3(t) &= y_3(t) - x_3(t)\end{aligned} \tag{8}$$

The error dynamics is obtained as

$$\begin{aligned}\dot{e}_1 &= e_2 \\ \dot{e}_2 &= e_3 \\ \dot{e}_3 &= -ae_1 + y_2^2 - x_2^2 - be_3 + u\end{aligned} \tag{9}$$

Next, we shall prove the main result of this section.

Theorem 1 *The identical Sprott jerk chaotic systems* (6) *and* (7) *are globally and exponentially synchronized for all initial conditions* $x(0), y(0) \in \mathbb{R}^3$ *by the active controller*

$$u(t) = (a-3)e_1 - 5e_2 + (b-3)e_3 - y_2^2 + x_2^2 \tag{10}$$

where the parameters a and b are known.

Proof We prove this main result by using Lyapunov stability theory Khalil. First, we define a Lyapunov function candidate

$$V_1(z_1) = \frac{1}{2}z_1^2 \tag{11}$$

where

$$z_1 = e_1 \tag{12}$$

Differentiating V_1 along the dynamics (9), we get

$$\dot{V}_1 = z_1\dot{z}_1 = e_1e_2 = -z_1^2 + z_1(e_1 + e_2) \tag{13}$$

Next, we define

$$z_2 = e_1 + e_2 \tag{14}$$

Then the Eq. (13) can be simplified as

$$\dot{V}_1 = -z_1^2 + z_1z_2 \tag{15}$$

Secondly, we choose the Lyapunov function candidate as

$$V_2(z_1, z_2) = V_1(z_1) + \frac{1}{2}z_2^2 = \frac{1}{2}(z_1^2 + z_2^2) \tag{16}$$

Differentiating V_2 along the dynamics (9), we get

$$\dot{V}_2 = \dot{V}_1 + z_2\dot{z}_2 = -z_1^2 - z_2^2 + z_2(2e_1 + 2e_2 + e_3) \tag{17}$$

Next, we define

$$z_3 = 2e_1 + 2e_2 + e_3 \tag{18}$$

Then the Eq. (17) can be simplified as

$$\dot{V}_2 = -z_1^2 - z_2^2 + z_2z_3 \tag{19}$$

Finally, we choose the Lyapunov function candidate as

$$V(z_1, z_2, z_3) = V_2(z_1, z_2) + \frac{1}{2}z_3^2 = \frac{1}{2}(z_1^2 + z_2^2 + z_3^2) \tag{20}$$

By definition, V is a quadratic, positive-definite function on $\mathbb{R}^3$. Differentiating V along the dynamics (9), we get

$$\dot{V} = -z_1^2 - z_2^2 - z_3^2 + z_3\left[(3-a)e_1 + 5e_2 + (3-b)e_2 + y_2^2 - x_2^2 + u\right] \tag{21}$$

Substituting the active controller (10) into (21), we obtain

$$\dot{V} = -z_1^2 - z_2^2 - z_3^2 \tag{22}$$

which is a quadratic, negative-definite function on $\mathbb{R}^3$.

Thus, by Lyapunov stability theory [19], $z_i(t) \to 0 \ (i = 1, 2, 3)$ as $t \to \infty$ exponentially for all initial conditions $z(0) \in \mathbb{R}^3$.

Hence, it is immediate that $e_i(t) \to 0$ ($i = 1, 2, 3$) as $t \to \infty$ exponentially for all initial conditions $e(0) \in \mathbb{R}^3$.

Hence, the identical Sprott jerk chaotic systems (6) and (7) are globally and exponentially synchronized for all initial conditions $x(0), y(0) \in \mathbb{R}^3$.

This completes the proof. □

For numerical simulations, the fourth-order classical Runge-Kutta method with initial step $h = 10^{-8}$ has been used to solve the two systems of differential Eqs. (6) and (7) with the active backstepping controller defined by (10).

The parameter values are taken as in the chaotic case, i.e.

$$a = 1, \quad b = 2.02 \tag{23}$$

The initial values of the master system are taken as

$$x_1(0) = 2.7, \quad x_2(0) = -3.8, \quad x_3(0) = 0.9 \tag{24}$$

The initial values of the slave system are taken as

$$y_1(0) = -1.5, \quad y_2(0) = 3.1, \quad y_3(0) = 1.7 \tag{25}$$

Figure 3 shows the complete synchronization of the states $x_1(t)$ and $y_1(t)$. From Fig. 3, it is seen that the states $x_1(t)$ and $y_1(t)$ are synchronized in 7 S. This shows the efficiency of the active backstepping controller defined by (10).

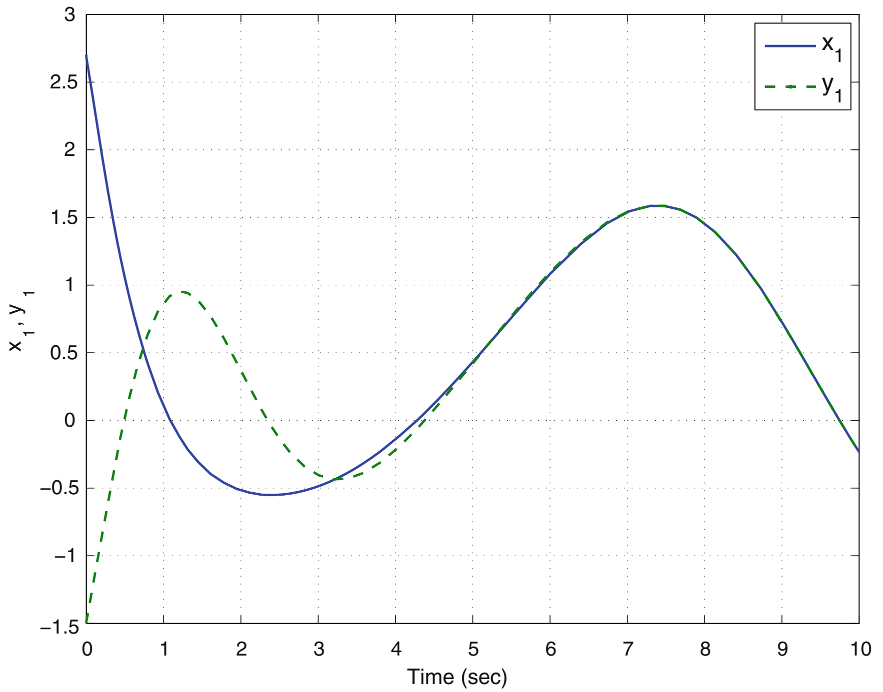

Fig. 3 Synchronization of the states $x_1(t)$ and $y_1(t)$

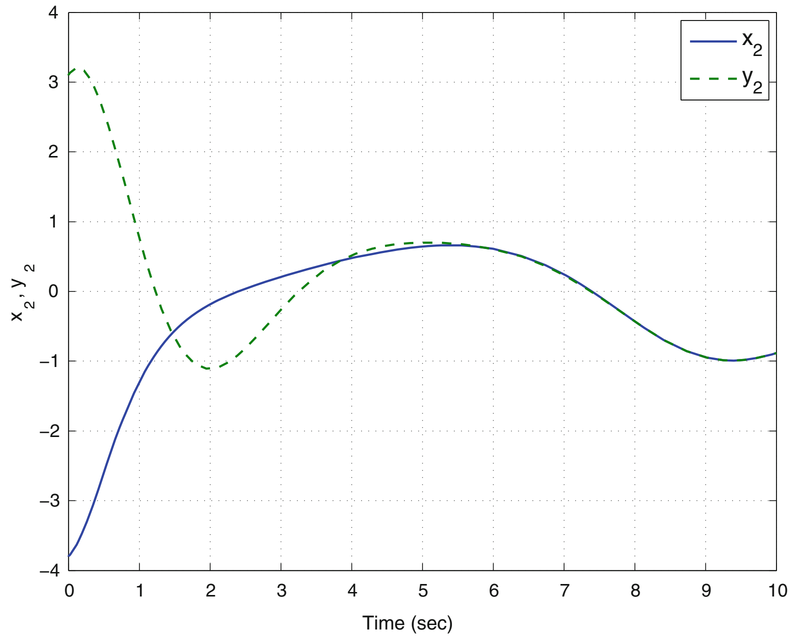

Fig. 4 Synchronization of the states $x_2(t)$ and $y_2(t)$

Figure 4 shows the complete synchronization of the states $x_2(t)$ and $y_2(t)$. From Fig. 4, it is seen that the states $x_2(t)$ and $y_2(t)$ are synchronized in 7 s. This shows the efficiency of the active backstepping controller defined by (10).

Figure 5 shows the complete synchronization of the states $x_3(t)$ and $y_3(t)$. From Fig. 5, it is seen that the states $x_3(t)$ and $y_3(t)$ are synchronized in 7 s. This shows the efficiency of the active backstepping controller defined by (10).

Figure 6 shows the time-history of the synchronization errors $e_1(t)$, $e_2(t)$, $e_3(t)$. From Fig. 6, it is seen that the errors $e_1(t)$, $e_2(t)$, $e_3(t)$ exponentially converge to zero in 7 s. This shows the efficiency of the active backstepping controller defined by (10).

4 Adaptive Backstepping Design for the Synchronization of Identical Sprott Jerk Chaotic Systems

This section derives new results for the adaptive backstepping design for the global chaos synchronization of the identical Sprott jerk systems with unknown parameters. The main result of this section is proved using Lyapunov stability theory.

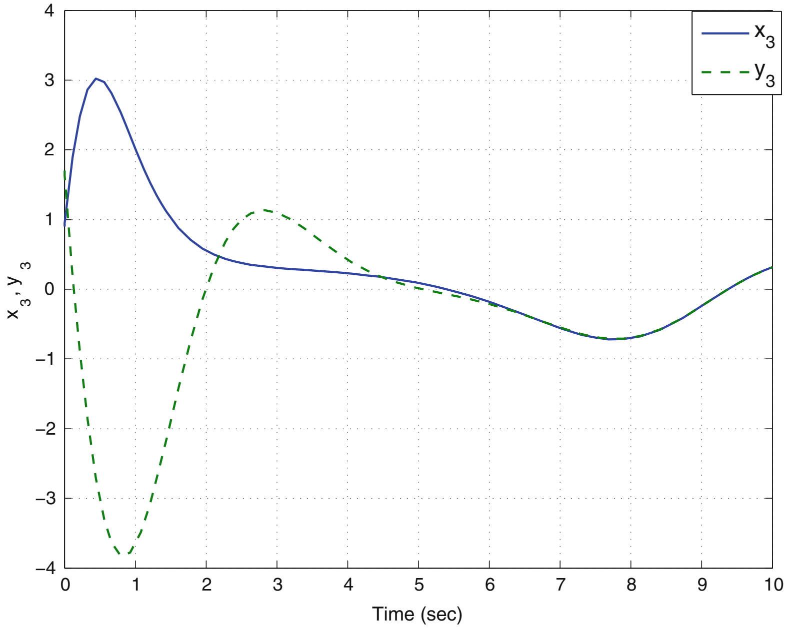

Fig. 5 Synchronization of the states $x_3(t)$ and $y_3(t)$

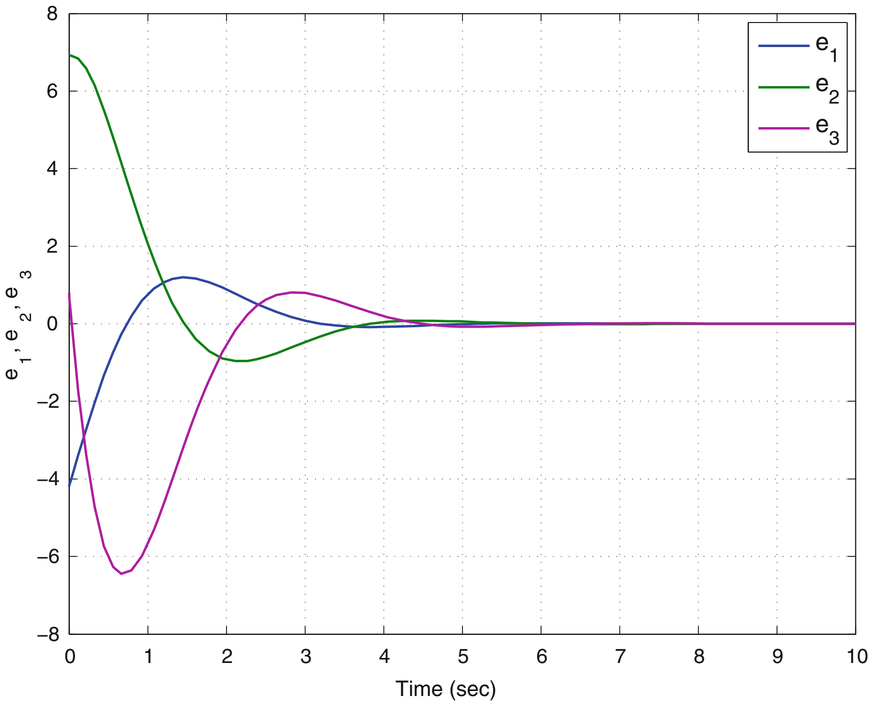

Fig. 6 Time history of the synchronization errors $e_1(t)$, $e_2(t)$, $e_3(t)$

The master system is described by the Sprott chaotic system

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= -ax_1 + x_2^2 - bx_3 \end{aligned} \tag{26}$$

where x_1, x_2, x_3 are state variables and a, b are unknown parameters.

The slave system is described by the controlled Sprott chaotic system

$$\begin{aligned} \dot{y}_1 &= y_2 \\ \dot{y}_2 &= y_3 \\ \dot{y}_3 &= -ay_1 + y_2^2 - by_3 + u \end{aligned} \tag{27}$$

where y_1, y_2, y_3 are state variables and $u(t)$ is the adaptive backstepping controller to be designed.

The synchronization error between the master system (26) and the slave system (27) is defined as

$$\begin{aligned} e_1(t) &= y_1(t) - x_1(t) \\ e_2(t) &= y_2(t) - x_2(t) \\ e_3(t) &= y_3(t) - x_3(t) \end{aligned} \tag{28}$$

The error dynamics is obtained as

$$\begin{aligned} \dot{e}_1 &= e_2 \\ \dot{e}_2 &= e_3 \\ \dot{e}_3 &= -ae_1 + y_2^2 - x_2^2 - be_3 + u \end{aligned} \tag{29}$$

The design problem is to find $u(t)$ so that the error converges to zero asymptotically, i.e. $e_i(t) \to 0$ as $t \to \infty$ for $i = 1, 2, 3$.

Inspired by the control law defined by (10) in the active control case, we may consider the adaptive control law

$$u(t) = (\hat{a}(t) - 3)e_1 - 5e_2 + (\hat{b}(t) - 3)e_3 - y_2^2 + x_2^2 \tag{30}$$

where $\hat{a}(t)$ and $\hat{b}(t)$ are estimates of the unknown parameters a and b, respectively.

We define the parameter estimation errors as

$$\begin{aligned} e_a(t) &= a - \hat{a}(t) \\ e_b(t) &= b - \hat{b}(t) \end{aligned} \tag{31}$$

We note that

$$\begin{aligned}\dot{e}_a(t) &= -\dot{\hat{a}}(t)\\ \dot{e}_b(t) &= -\dot{\hat{b}}(t)\end{aligned} \tag{32}$$

Next, we shall prove the main result of this section.

Theorem 2 *The identical Sprott jerk chaotic systems* (26) *and (*27*) with unknown system parameters are globally and exponentially synchronized for all initial conditions* $x(0), y(0) \in \mathbb{R}^3$ *and* $\hat{a}(0), \hat{b}(0) \in \mathbb{R}$ *by the adaptive controller law*

$$u(t) = (\hat{a}(t) - 3)e_1 - 5e_2 + (\hat{b}(t) - 3)e_3 - y_2^2 + x_2^2 \tag{33}$$

where the parameter update law is given by

$$\begin{aligned}\dot{\hat{a}}(t) &= -(2e_1 + 2e_2 + e_3)e_1\\ \dot{\hat{b}}(t) &= -(2e_1 + 2e_2 + e_3)e_2\end{aligned} \tag{34}$$

Proof We prove this main result by using Lyapunov stability theory Khalil. First, we define a Lyapunov function candidate

$$V_1(z_1) = \frac{1}{2} z_1^2 \tag{35}$$

where

$$z_1 = e_1 \tag{36}$$

Differentiating V_1 along the dynamics (29), we get

$$\dot{V}_1 = z_1 \dot{z}_1 = e_1 e_2 = -z_1^2 + z_1(e_1 + e_2) \tag{37}$$

Next, we define

$$z_2 = e_1 + e_2 \tag{38}$$

Then the Eq. (37) can be simplified as

$$\dot{V}_1 = -z_1^2 + z_1 z_2 \tag{39}$$

Secondly, we choose the Lyapunov function candidate as

$$V_2(z_1, z_2) = V_1(z_1) + \frac{1}{2} z_2^2 = \frac{1}{2}\left(z_1^2 + z_2^2\right) \tag{40}$$

Differentiating V_2 along the dynamics (29), we get

$$\dot{V}_2 = \dot{V}_1 + z_2\dot{z}_2 = -z_1^2 - z_2^2 + z_2(2e_1 + 2e_2 + e_3) \tag{41}$$

Next, we define

$$z_3 = 2e_1 + 2e_2 + e_3 \tag{42}$$

Then the Eq. (41) can be simplified as

$$\dot{V}_2 = -z_1^2 - z_2^2 + z_2z_3 \tag{43}$$

Finally, we choose the Lyapunov function candidate as

$$V(z_1, z_2, z_3, e_a, e_b) = V_2(z_1, z_2) + \frac{1}{2}z_3^2 + \frac{1}{2}e_a^2 + \frac{1}{2}e_b^2 \tag{44}$$

That is, V is defined as

$$V(z_1, z_2, z_3, e_a, e_b) = \frac{1}{2}\left(z_1^2 + z_2^2 + z_3^2 + e_a^2 + e_b^2\right) \tag{45}$$

By definition, V is a quadratic, positive-definite function on $\mathbb{R}^5$. Differentiating V along the dynamics (29) and (32), we get

$$\begin{aligned} \dot{V} = &-z_1^2 - z_2^2 - z_3^2 + z_3\left[(3-a)e_1 + 5e_2 + (3-b)e_2 + y_2^2 - x_2^2 + u\right] \\ &- e_a\hat{a}(t) - e_b\hat{b}(t) \end{aligned} \tag{46}$$

Substituting the adaptive controller (33) into (46), we obtain

$$\dot{V} = -z_1^2 - z_2^2 - z_3^2 + e_a(-z_3e_1 - \dot{\hat{a}}) + e_b(-z_3e_2 - \dot{\hat{b}}) \tag{47}$$

Substituting the parameter update law (32) into (47), we obtain

$$\dot{V} = -z_1^2 - z_2^2 - z_3^2 \tag{48}$$

which is a negative semi-definite function on $\mathbb{R}^5$.

If we define

$$z(t) = \begin{bmatrix} z_1(t) \\ z_2(t) \\ z_3(t) \end{bmatrix}, \tag{49}$$

then it can be concluded from (48) that the vector $z(t)$ and the parameter estimation error are globally bounded, i.e.

$$[\, z_1(t) \quad z_2(t) \quad z_3(t) \quad e_a(t) \quad e_b(t) \,]^T \in L_\infty. \tag{50}$$

From (48), it follows that

$$\dot{V} \leq -\|z\|^2 \quad \text{or} \quad \|z\|^2 \leq -\dot{V} \tag{51}$$

Integrating the inequality (51) from 0 to t, we get

$$\int_0^t \|z(\tau)\|^2 d\tau \leq -\int_0^t \dot{V}(\tau) d\tau = V(0) - V(t) \tag{52}$$

From (52), it follows that $z(t) \in L_2$.

From (29), it can be deduced that $\dot{z}(t) \in L_\infty$.

Hence, using Barbalat's lemma (19), it can be concluded that $z(t) \to 0$ exponentially as $t \to \infty$ for all initial conditions $z(0) \in \mathbb{R}^3$.

Hence, the identical Sprott jerk chaotic systems (26) and (27) with unknown system parameters are globally and exponentially synchronized for all initial conditions $x(0), y(0) \in \mathbb{R}^3$ and $\hat{a}(0), \hat{b}(0) \in \mathbb{R}$.

This completes the proof. □

For numerical simulations, the fourth-order classical Runge-Kutta method with initial step $h = 10^{-8}$ has been used to solve the two systems of differential Eqs. (26) and (27) with the adaptive backstepping controller defined by (33) and the parameter update law (32).

The parameter values are taken as in the chaotic case, i.e.

$$a = 1, \quad b = 2.02 \tag{53}$$

The parameter estimates are taken as

$$\hat{a}(0) = 4.7, \quad \hat{b} = 2.3 \tag{54}$$

The initial values of the master system are taken as

$$x_1(0) = 1.0, \quad x_2(0) = 0.7, \quad x_3(0) = 0.4 \tag{55}$$

The initial values of the slave system are taken as

$$y_1(0) = 0.3, \quad y_2(0) = 0.1, \quad y_3(0) = 0.2 \tag{56}$$

Figure 7 shows the complete synchronization of the states $x_1(t)$ and $y_1(t)$. From Fig. 7, it is seen that the states $x_1(t)$ and $y_1(t)$ are synchronized in 10 s. This shows the efficiency of the active backstepping controller defined by (33).

Figure 8 shows the complete synchronization of the states $x_2(t)$ and $y_2(t)$. From Fig. 8, it is seen that the states $x_2(t)$ and $y_2(t)$ are synchronized in 10 s. This shows the efficiency of the active backstepping controller defined by (33).

Figure 9 shows the complete synchronization of the states $x_3(t)$ and $y_3(t)$. From Fig. 9, it is seen that the states $x_3(t)$ and $y_3(t)$ are synchronized in 10 s. This shows the efficiency of the active backstepping controller defined by (33).

Figure 10 shows the time-history of the synchronization errors $e_1(t)$, $e_2(t)$, $e_3(t)$. From Fig. 10, it is seen that the errors $e_1(t)$, $e_2(t)$, $e_3(t)$ exponentially converge to zero in 10 s. This shows the efficiency of the active backstepping controller defined by (33).

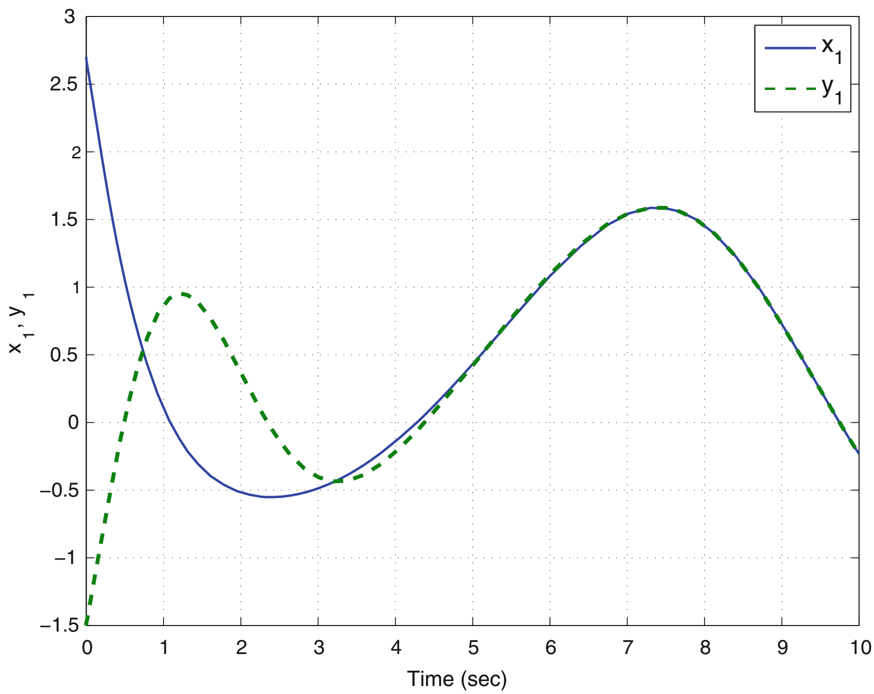

Fig. 7 Synchronization of the states $x_1(t)$ and $y_1(t)$

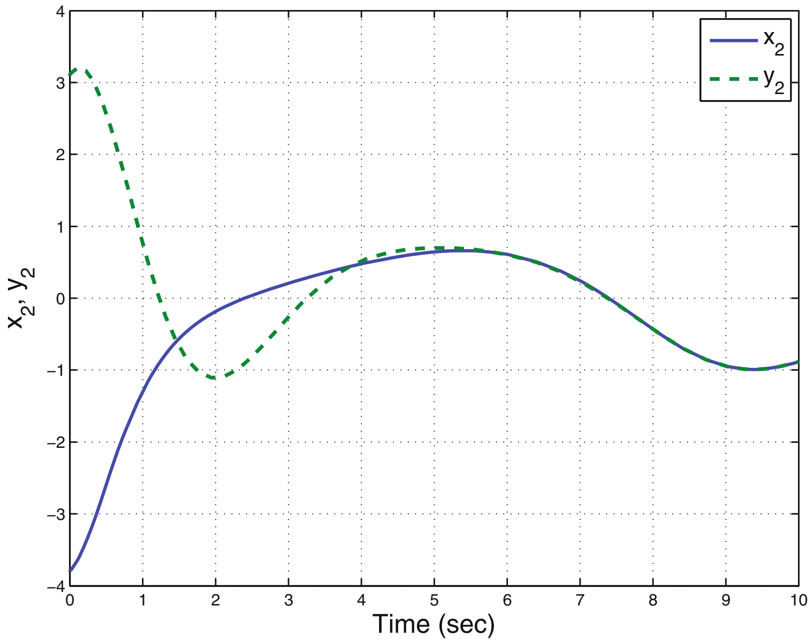

Fig. 8 Synchronization of the states $x_2(t)$ and $y_2(t)$

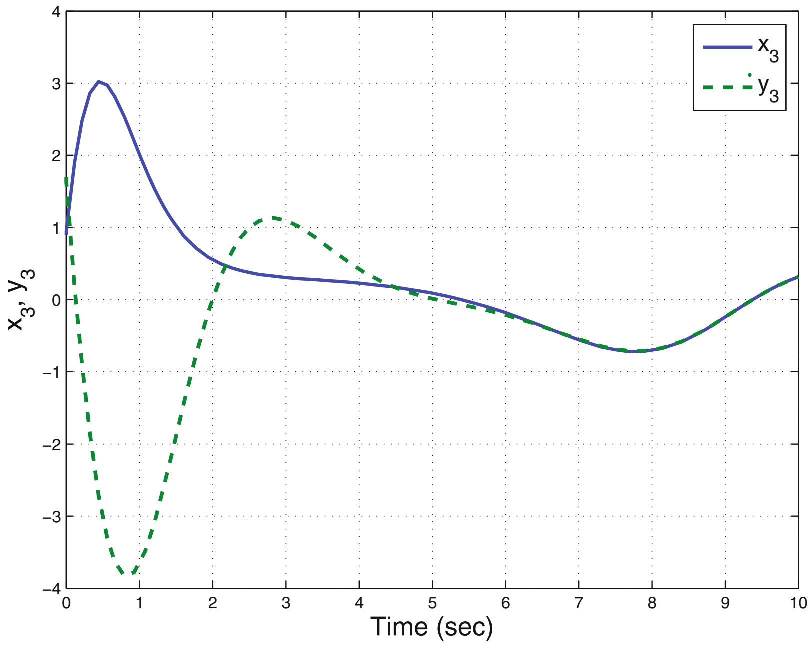

Fig. 9 Synchronization of the states $x_3(t)$ and $y_3(t)$

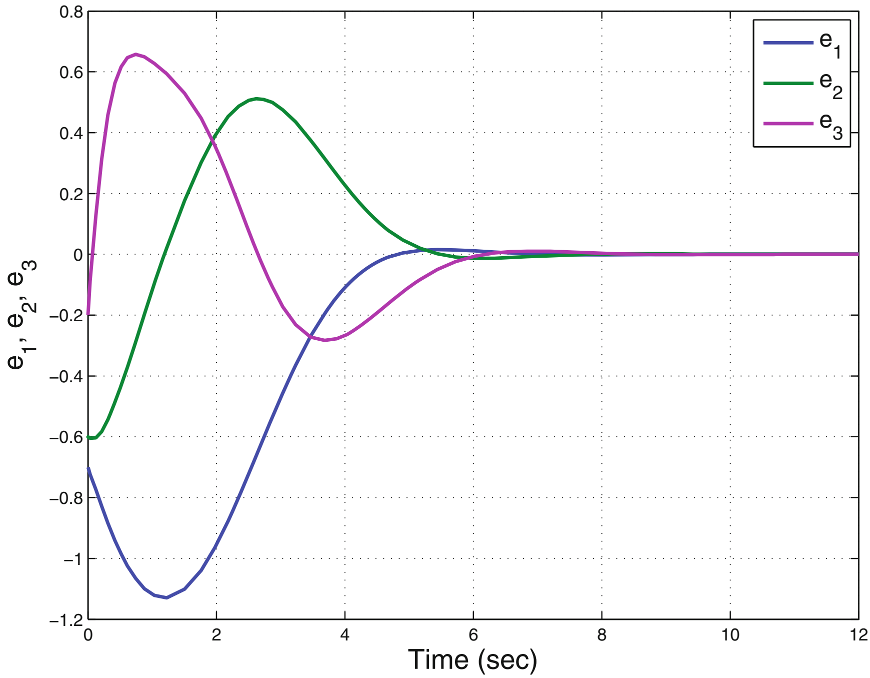

Fig. 10 Time history of the synchronization errors $e_1(t)$, $e_2(t)$, $e_3(t)$

5 Conclusions

In this research work, active and adaptive backstepping controllers were derived the global chaos synchronization of Sprott's jerk chaotic system and the main results were proved using Lyapunov stability theory. This paper investigated synchronization properties for the Sprott's jerk system (1997), which is algebraically the simplest dissipative chaotic system consisting of five terms and a quadratic nonlinearity. Sprott's chaotic system involves only five terms and one quadratic nonlinearity, while Rössler's chaotic system (1976) involves seven terms and one quadratic nonlinearity. This work first detailed the properties of the Sprott's jerk chaotic system. The phase portraits of the Sprott's jerk system were described. The Lyapunov exponents of the Sprott's jerk system were obtained as $L_1 = 0.0525$, $L_2 = 0$ and $L_3 = -2.0727$. The Lyapunov dimension of the Sprott's jerk system was obtained as $D_L = 2.0253$. An active backstepping controller was derived for the global chaos synchronization of identical Sprott's jerk systems with known parameters. The backstepping control method is a recursive procedure that links the choice of a Lyapunov function with the design of a controller and guarantees global asymptotic stability of strict-feedback chaotic systems. Finally, an adaptive backstepping controller was derived for the global chaos synchronization of identical Sprott's jerk systems with unknown parameters. MATLAB simulations are provided in detail to illustrate all the main results presented in this work.

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