A Truthful-in-Expectation Mechanism for the Generalized Assignment Problem

Salman Fadaei and Martin Bichler

Department of Informatics, TU München, Munich, Germany
`{salman.fadaei,bichler}@in.tum.de`

Abstract. We propose a truthful-in-expectation, $(1-\frac{1}{e})$-approximation mechanism for the generalized assignment auction. In such an auction, each bidder has a knapsack valuation function and bidders' values for items are private. We present a novel convex optimization program for the auction which results in a maximal-in-distributional-range (MIDR) allocation rule. The presented program contains at least a $(1-\frac{1}{e})$ ratio of the optimal social welfare. We show how to implement the convex program in polynomial time using a fractional local search algorithm which approximates the optimal solution within an arbitrarily small error. This leads to an approximately MIDR allocation rule which in turn can be transformed to an approximately truthful-in-expectation mechanism. Our contribution has algorithmic importance, as well; it simplifies the existing optimization algorithms for the GAP while the approximation ratio is comparable to the best given approximation.[1]

Keywords: Generalized assignment problem, Truthful-in-expectation, Mechanism design, Convex optimization.

In *algorithmic mechanism design*, a mechanism designer wishes to solve an optimization problem, but the inputs to this problem are the private information of self-interested players. The mechanism designer must thus design a mechanism that solves the optimization problem while encouraging the agents to reveal their information truthfully. The game-theoretic solution concept of truthfulness guarantees that an agent is better off truthfully interacting with the mechanism regardless of what the other agents do.

We consider the generalized assignment problem as a combinatorial auction. In the generalized assignment problem (GAP), a set of items should be assigned to a set of bidders in order to maximize total valuation. Each bidder associates a different value and weight to each item and has a limited capacity. We can assign each bidder any subset of items that does not exceed the bidder's capacity. For every such subset, the bidder's valuation is additive in the values of items in the subset. We assume bidders' valuations for items to be private while weights and capacities are publicly known. Our goal is to find an allocation and payment rule which constitute a truthful-in-expectation mechanism for the GAP.

---

[1] A full version at `http://dss.in.tum.de/files/bichler-research/wine-gaptie.pdf`.

T.-Y. Liu et al. (Eds.): WINE 2014, LNCS 8877, pp. 247–248, 2014.

The well-known *Vickrey-Clarke-Groves* (VCG) technique provides truthfulness as well as social welfare maximization in every combinatorial auction. The VCG technique, however, is applicable only when the optimal social welfare can be computed efficiently. Yet, in many cases, including our problem, optimizing social welfare is computationally intractable which makes the VCG technique inapplicable.

From an algorithmic point of view, the generalized assignment problem has been studied extensively in the literature. An approximation factor of $(1 - \frac{1}{e}) + \rho$ with $\rho > 0$, is the best given approximation ratio for the GAP. The presented algorithms, however, are not directly applicable for mechanism design as they rely on non-monotone rounding procedures.

MIDR or maximal-in-distributional-range is the only known general approach for designing randomized truthful mechanisms. An MIDR algorithm fixes a set of distributions over feasible solutions (the distributional range) independently of the valuations reported by the self-interested players, and outputs a random sample from the distribution that maximizes expected (reported) welfare.

In order to achieve a MIDR, we directly optimize over the outcome of the rounding procedure, rather than over the outcome of the relaxation algorithm. To this end, we formulate the GAP as a convex optimization problem where the objective function equals the expected value of the rounding procedure. This is similar to the technique used in [1] for finding a truthful-in-expectation mechanism for players whose valuations are of a special type of submodular functions. We notice that our technique allows to guarantee *non-negativity of payments* and *individual rationality*, *ex post*, while in [1], these important properties are provided only *ex ante*.

We are able to approximate the proposed convex optimization problem within an arbitrarily small error, in the sense of an FPTAS. This in fact leads to an approximate MIDR. Taking into account the black box transformation of an approximately MIDR allocation rule to an approximately truthful-in-expectation mechanism known in the literature, we immediately achieve a $(1 - \epsilon)$-truthful-in-expectation mechanism for the GAP.

We also emphasize the algorithmic importance of our result. Our algorithm has advantages over the existing optimization algorithms in terms of runtime and simplicity. It does not employ the ellipsoid method or any other LP solving algorithms. Additionally, the exact specification of the proposed objective function enables to calculate the gradient of the objective function explicitly which helps in designing a simpler algorithm for the problem.

Acknowledgement. The authors gratefully acknowledge support of the Deutsche Forschungsgemeinschaft (DFG) (BI 1057/7-1).

References

1. Dughmi, S., Roughgarden, T., Yan, Q.: From convex optimization to randomized mechanisms: toward optimal combinatorial auctions. In: Proceedings of the 43rd Annual ACM Symposium on Theory of Computing, pp. 149–158. ACM (2011)