Constructive Algorithm of Optimized Arterial Networks

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Abstract— **Detailed models of complex arterial trees can be generated from optimizations principles using the computational method of Constrained Constructive Optimization (CCO). In this method, a arterial tree is modeled as dichotomously branching system of straight cylindrical tubes, with resistance and flow conditions calculated by Poiseuille's law. At bifurcations, the radii of parent and daughter segments obey Murray's law and the process of growing the tree is governed by minimizing a target function (total intravascular volume). CCO models have been shown to adequately reproduce key features of real arterial trees, such as segment radii, branching angle statistics and pressure profiles. We developed an algorithm based on CCO method to create optimized arterial networks taking into account the main feeding artery radius to be constant during the growth process.**

Keywords— **Arterial network, computational model, optimization.**

I. INTRODUCTION

Arterial trees serve the purpose of conveying blood to all sites of a tissue. They represent higly complex branching structures that have been found to follow certain optimality principles [1]. Bifurcation law [2] is fulfilled in real arterial trees and several targets functions related to functional optimality of arterial trees has been proposed [3].

In order to understand both diagnostic and probable effects of therapeutic interventions, hemodynamic simulation studies uses computational models of arterial trees as their geometrical substrate. To date these simulations can be employed one the following class of models of arterial trees: lumped parameter models [4], anatomical models [5], fractal models [6] and models generated by Constrained Constructive Optimization (CCO) [7, 8, 9, 10].

Arterial tree models generated by CCO are able to mimic important properties of real arterial trees [7, 8, 11], such as segment radii, branching angle statistics and pressure profiles. However, the CCO method is not able to take into account the value of the radius of the main artery feeding, when this morphometric data is known, for the construction of the arterial tree model.

In this context, this work contributes with an algorithm based on CCO method that generates arterial tree models maintained constant the radius of the main feeding artery. This data can usually be obtained either from anatomical atlas or from medical image reconstruction.

The remainder of this paper is organized as follows. In Section II, the algorithm proposed based on CCO method is described. In Section III, results obtained using the algorithm developed are presented. Section IV contains our conclusion and discusses the future direction of this work.

II. THE ALGORITHM BASED ON CCO METHOD

The technical approach of generating CCO trees has previously been described in detail [7, 11]. In following, we reiterate the main features of the CCO method incorpored into the algorithm proposed.

A. Model assumptions, boundary conditions and constraints

Arterial model trees are generated by CCO method based on the main assumptions listed below:

(H1) the arterial tree is modeled as a dichotomously branching (binary) system of straight cylindrical tubes (vessel segments);

(H2) the model tree starts at the root segment (main feeding artery) and it is truncated in the form of terminal segments on prearteriolar level;

(H3) the model tree should take up the space of the perfusion domain as homogeneously as possible without intersection of segments;

(H4) the blood is modeled as an incompressible, homogeneous Newtonian fluid at steady state and laminar flow conditions;

(H5) flow resistance R of each segment of the tree is assumed to follow Poiseuille's law

$$R = \left(\frac{8\eta}{\pi}\right)\frac{l}{r^4}, \tag{1}$$

where l and r are the length and radius of the segment, η is the constant blood viscosity;

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(H6) the pressure drop Δp_s along segment is given by

$$\Delta p_s = RQ, \quad (2)$$

where Q is the flow through segment.

At each stage of development, a CCO tree model satisfies a set of physiological boundary conditions and constraints:

(C1) at bifurcations the radii of parent (r_0) and daughter segments (r_1, r_2) are forced to exactly fulfill a bifurcation law derived from real coronary trees [2]

$$r_0^\gamma = r_1^\gamma + r_2^\gamma, \quad (3)$$

where γ is a constant exponent with ranging between 2.55 and 3, governing the shrinkeage of radii across bifurcations;

(C2) each terminal segment supplies an identical and equal amount of blood flow Q_{term} into the microcirculatory network, which is not modeled in detail [7];

(C3) the resistance of the resulting model tree induces a prespecified perfusion flow Q_{perf} across the overall pressure drop

$$\Delta p = p_{perf} - p_{term}, \quad (4)$$

where p_{perf} is the perfusion pressure at the inlet of the root segment, p_{term} is the pressure at the outlet of all the terminal segments and Δp is constant for the CCO method;

(C4) the radius of the root segment r_{iroot} is scaled during the growth of the tree model by CCO method as follows:

$$r_{iroot} = \left[R^*_{sub,iroot} \frac{Q_{perf}}{\Delta p} \right], \quad (5)$$

where $R^*_{sub,iroot}$ denotes the reduced hydrodynamic resistance of the whole tree (see details in [11]) and $Q_{perf} = k_{term} Q_{term}$ which k_{term} denotes the number of terminal segments to be supplied.

The algorithm proposed in this work satisfies the assumptions (H1) to (H6) and the constraints (C1) and (C2). However, it relaxes the constraints (C3) and (C4) in order to make CCO method more flexible. It supposes that the overall pressure drop Δp varies during the growth of the tree to keep constant the radius of the root segment $r_{iroot} = r_{cte}$ as follows:

$$\Delta p = \frac{R^*_{sub,iroot} k_{term}}{\Psi}, \quad (6)$$

where the constant Ψ is given by

$$\Psi = \frac{r_{cte}^4}{Q_{term}}. \quad (7)$$

where r_{cte} is provided as a parameter to the algorithm.

B. Algorithm of tree generation

The optimized tree model generation starts by planting the root segment with its proximal end $\mathbf{x}_{prox}$ fixed at the perfusion domain Ω_{perf} and the distal $\mathbf{x}_{inew}$ selected randomly within this domain. If this position is not too close to $\mathbf{x}_{prox}$, it is connected $\mathbf{x}_{inew}$ to $\mathbf{x}_{prox}$ resulting root segment length (l_{root}). The radius r_{cte} (constant) of the root segment is such that the hydrodynamic resistance $R^*_{sub,iroot}$ yields the flow Q_{term} through one terminal segment ($k_{term} = 1$) into the microcirculatory network. Equations (6) and (7) are calculated.

Given a tree with k_{term} terminal segments, the stepwise growing of the tree is as follows. First, the location $\mathbf{x}_{term}$ for a new terminal is selected from a pseudorandom number sequence, uniformly distributed inside the perfusion domain. In this work, we used the dSFMT (double precision SIMD-oriented Fast Mersenne Twister) free generator developed by Saito and Matsumoto [12] to produce the pseudorandom number sequence for tossing.

The prospective location $\mathbf{x}_{term}$ is accepted as a candidate for a new terminal site only if $\mathbf{x}_{term}$ satisfy a distance criterion (see [7, 13]).

Since $\mathbf{x}_{term}$ has been accepted as a distal end of a new terminal segment, it is temporarily connected to each of the neighboring segments, one after the other. Connecting the new terminal segment to a preexisting segment, consequently cause violation in the boundary condition regarding the terminal flows. In order to return the proper terminal flows, the flow resistance of the tree must be adjusted for each temporary connection and overall pressure drop is updated by equation (6) to ensure constant the radius of the root segment $r_{iroot} = r_{cte}$. This can only be performed by rescaling of the segments' radii [11].

The bifurcation site resulting in each temporary connection is optimized in order to minimize the total intravascular volume and dissolved again. After having investigated all possible connections in the neighborhood of $\mathbf{x}_{term}$, the connection that provided the lowest optimization target is adopted as permanent for the new terminal site $\mathbf{x}_{term}$. Thus, the tree is grown to $k_{term} + 1$ terminal segments. The process of growing the tree summarized above is repeated until $k_{term} = N_{term}$, i.e., the preset number of terminals N_{term} is achieved. The steps described previously are systematized in the **Algorithm 1**.

Algorithm 1: Tree generation inspired in the CCO.

Data: Ω_{perf}, $\mathbf{x}_{prox}$, Q_{perf}, N_{term}, γ, η, r_{cte}.

Fix the proximal position $\mathbf{x}_{prox}$ of the root segment into domain Ω_{perf};
repeat
  Generate the distal position $\mathbf{x}_{inew}$ for the root segment into domain Ω_{perf};
  Distance criterion: check if this position is not too close to $\mathbf{x}_{prox}$ (adaptive threshold);
until *(distance criterion is met)*;
Connect $\mathbf{x}_{inew}$ to $\mathbf{x}_{prox}$ (*planting the root segment*);
$k_{term} \leftarrow 1$;
Calculate the value of Δp and Ψ (equations (6) and (7), respectively);
while *($k_{term} < N_{term}$)* **do**
  repeat
    Generate the distal position $\mathbf{x}_{inew}$ for a new terminal segment from dSFMT;
    Distance criterion: check if this position is not too close to any of the existing segments (adaptive threshold);
  until *(distance criterion is met)*;
  Obtain the neighboring segments N_{con} of $\mathbf{x}_{inew}$ for temporary connection;
  for $j \leftarrow 1$ ***to*** N_{con} **do**
    Connect $\mathbf{x}_{inew}$ to the midpoint $\mathbf{x}_{ibif}$ of segment j (*creating a new bifurcation*);
    Calculate the overall pressure drop Δp of the tree model (equation (6));
    Optimize the bifurcation position $\mathbf{x}_{ibif}$ (*geometric optimization*);
    Perform restriction checks (e.g. segment intersection, segments traversing forbidden domains);
    Save value of target function, position $\mathbf{x}_{ibif}$ and results of check in line j of the *Connection Evaluation Table* (*CET*);
    Remove the bifurcation created;
  Restrict *CET* to allowed connections: CET^v;
  if *(CET^v is not an empty set)* **then**
    Find optimal connection j_{opt} from CET^v (*structural optimization*);
    Make connection from $\mathbf{x}_{inew}$ to j_{opt} permanent (*create optimum bifurcation*);
    Calculate the overall pressure drop Δp of the tree model (equation (6));
    $k_{term} \leftarrow k_{term} + 1$;
  else
    Refuse the position $\mathbf{x}_{inew}$;
Obtain computed quantities (length, radius, resistance, pressure drop);

III. RESULTS

The **Algorithm 1** inspired in the CCO method was implemented using the C programming language. Results obtained in the simulations performed are here presented.

A. Comparison with real coronary arterial trees

For morphometric comparison with real coronary arterial trees, the algorithm based on CCO was applied to generate arterial trees with 250 terminal segments (499 in total) in order to represent the tree of the left anterior descending (LAD) coronary artery. In Figure 1 observe that the LAD orifice has level 1 in a tree starting with the main left coronary artery (level 0). In this figure, the bifurcation level indicates the number of proximal bifurcations of a segment.

The arterial tree models were generated under the following conditions: total perfusion flow $Q_{perf} = 500$ mL/min, terminal flows $Q_{term} = 2$ mL/min, bifurcation exponent $\gamma = 3$, viscosity blood $\eta = 3.6$ cP, circular area $\Omega_{perf} = 78.54$ cm^2 representing tissue to be perfused (LAD region). In addition to these parameters, two values for the radius (constant) of the root segment were adopted: $r_{cte} = 1.2536$ mm (measured from the heart A, see Figure 1) and $r_{cte} = 1.3478$ mm (measured from the heart B).

For each fixed radius r_{cte}, ten replicates of the tree with 250 terminal segments were generated on the same predefined parameters and the same optimization target function (total intrascular volume). Each tree was generated using a different sequence of pseudorandom numbers obtained using dSFMT for casting the distal ends of its terminal segments.

The morphometric result of the tree models was compared with experimental data acquired from corrosion casts from two human hearts by Zamir and Chee [14]. They classified each vessel segment according to the bifurcation level, measured the diameter, and depicted average diameter versus bifurcation level.

Figure 1 displays the mean diameter and standard deviation of this mean diameter (SDM) of all vessel segments at a certain bifurcation level. Close agreement with the experimental data is found for the levels 1-2, 10-16 and 25-33. The overall pressure drop obtained for each tree model generated by algorithm was approximately 37 mmHg. It is important to inform that this pressure drop is set for the classical CCO simulations in [7].

B. Application: 3D perfusion domain

The algorithm proposed was here employed to generate the arterial tree within a cubic domain under the conditions: $Q_{perf} = 1000$ mL/min, $\gamma = 3$ and $\eta = 3.6$ cP. The radius of

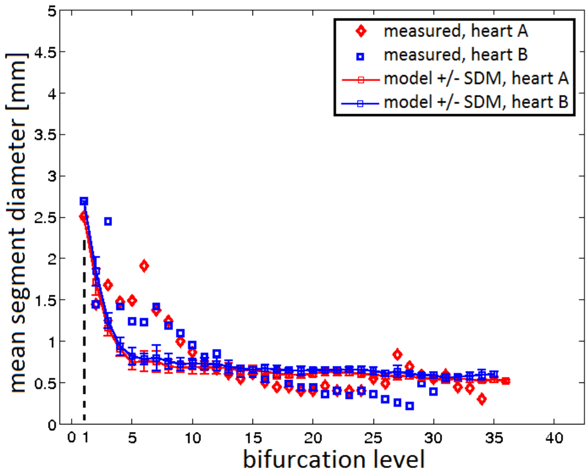

Fig. 1: Morphometric comparison between models and the left coronary arterial trees of two humans.

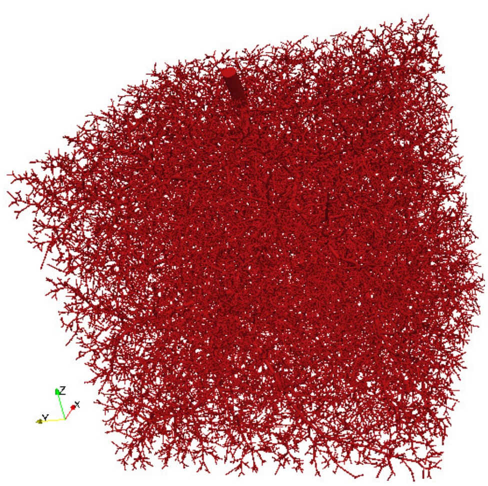

Fig. 2: Detailed arterial tree model generated with 50000 terminal segments.

the root segment r_{cte} is set at 2.4514 mm and the perfusion volume 409.6 cm^3.

Figure 2 illustrates the tree model generated with N_{term} = 50000 terminal segments (99999 in total) optimized according to minimum intravascular volume at equal terminal flows Q_{term} = 0.02 mL/min. Note in this figure that the model tree obtained is highly detailed and it has pressure drop Δp = 9.138 mmHg and the Strahler order is 9. This model reasonably mimics the structure of real arterial trees.

IV. Conclusion and future work

The comparison between the resulting models generated by algorithm proposed and real vascular trees shows satisfactory agreement regarding morphometric parameter. The main result of the present work shows that the algorithm developed based on CCO method is capable of generating detailed arterial trees from the root segment whose radius is maintained constant during the growth model. It should be emphasized that the described algorithm can be improved. In future work, we plan to incorporate elastic vessel representation. Furthermore, hemodynamic simulations will be performed taking account as geometric substrate the tree models here constructed.

Conflict of Interest

The authors declare that they have no conflict of interest.

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