Design and Modeling of Anti Wind Up PID Controllers

Ahmad Taher Azar and Fernando E. Serrano

Abstract In this chapter several anti windup control strategies for SISO and MIMO systems are proposed to diminish or eliminate the unwanted effects produced by this phenomena, when it occurs in PI or PID controllers. Windup is a phenomena found in PI and PID controllers due to the increase in the integral action when the input of the system is saturated according to the actuator limits. As it is known, the actuators have physical limits, for this reason, the input of the controller must be saturated in order to avoid damages. When a PI or PID controller saturates, the integral part of the controller increases its magnitude producing performance deterioration or even instability. In this chapter several anti windup controllers are proposed to eliminate the effects yielded by this phenomena. The first part of the chapter is devoted to explain classical anti windup architectures implemented in SISO and MIMO systems. Then in the second part of the chapter, the development of an anti windup controller for SISO systems is shown based on the approximation of the saturation model. The derivation of PID SISO (single input single output) anti windup controllers for continuous and discrete time systems is implemented adding an anti windup compensator in the feedback loop, so the unwanted effects are eliminated and the system performance is improved. Some illustrative examples are shown to test and compare the performance of the proposed techniques. In the third part of this chapter, the derivation of a suitable anti windup PID control architecture is shown for MIMO (multiple input multiple output) continuous and discrete time systems. These strategies consist in finding the controller parameters by static output feedback (SOF) solving the necessary linear matrix inequalities (LMI's) by an appropriate anti windup control scheme. In order to obtain the control gains and parameters, the saturation is modeled with describing functions for the continuous time case and a suitable model to deal with this nonlinearity in

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the discrete time case. Finally a discussion and conclusions sections are shown in this chapter to analyze the advantages and other characteristics of the proposed control algorithms explained in this work.

1 Introduction

In this chapter several control architectures of anti windup controllers are shown for the stabilization of SISO and MIMO systems in their discrete and continuous forms. Windup is a phenomena found in different kind of systems, when a PI or PID controller is implemented, produced by the integral action of the controller. This phenomenon occurs when the input of the system saturates increasing the magnitude of the integrator producing unwanted effects on the system like high overshoot and long settling time. There are several techniques and architectures found in literature to deal with this problem, for the SISO and MIMO cases, usually by suppressing the integral action of the PI or PID controller with input saturation.

For the SISO continuous case, different anti windup controller architectures are found in literature such as the tracking anti windup, conditional integration and limited integrator (Bohn and Atherton 1995), these are some of the classical anti windup control architectures implemented to eliminate the unwanted effects of windup. These classical techniques usually consist in adding an extra feedback loop to the controller from the saturated output so the effects of windup can be cancelled by implementing these control models. The back—calculation techniques is a common anti windup control architecture that ensures the system stability when the input is saturated, improving the system performance by producing smaller overshoot and acceptable settling time (Tu and Ho 2011). One issue that makes it difficult to obtain a suitable anti windup control architecture is the nonlinearity introduced by the actuator saturation, one way to design an appropriate control system when this nonlinearity is found, is the introduction of a saturation model which includes all the properties of this nonlinearity (Saeki and Wada 1996). This consideration is very important in the design of anti windup controllers for SISO and MIMO systems in the continuous and discrete time cases respectively, allowing the development of appropriate controllers including a saturation model.

In the case of SISO discrete system, there are similar anti windup control techniques as the continuous counterpart that can be implemented when a discretized model of the system is available. One of the control architectures that is very popular in the control community is the back calculation model, where the saturated signal is feedback to the controller integrator in order to suppress the windup effects yielded by the integrator action (Wittenmark 1989). Apart from this anti windup control architecture for discrete time SISO systems, the anti windup controller design by the frequency response of the model is usually implemented where a discrete time controller is obtained by the design of a continuous time SISO controller and then this controller is transformed to discrete time by one of the several well known methods (Lambeck and Sawodny 2004).

In the case of MIMO continuous and discrete time systems several anti windup controllers are synthesized usually by static output feedback (SOF) and then the controller is found by the solution of the respective linear matrix inequalities (LMI's). The SOF control law can be found by solving the LMI's to ensure the stability of the system by traditional ways or by an H_∞ controller (Wu et al. 2005; Henrion et al. 1999), allowing a flexible anti windup controller design when the input of the system is saturated.

Based on the previous explanation of different kind of anti windup controller architectures, this chapter is divided in the following sections so the first part of the chapter is devoted to SISO continuous and discrete time systems and the second part of this chapter is devoted to MIMO continuous and discrete time systems. In Sect. 2, the explanation of popular anti windup control techniques is explained to introduce the proposed strategies shown in this article, where some continuous and discrete time classical anti windup techniques found in literature are explained. It is important to notice that in this chapter, the main objective is to design and obtain stable PID controllers for the SISO and MIMO case, so in the following sections this problem is considered for analysis. Based on the previous explanation, in Sect. 3 the design of an internal model anti windup controller for continuous time systems is explained, showing that is possible to obtain a desired anti windup PID controller with an internal model controller (IMC) characteristics. In Sect. 4 an internal model anti windup controller for discrete SISO system is shown where a similar technique like the continuous counterpart is developed to eliminate the unwanted effects produced by the system saturation by implementing a scalar sign function approach (Zhang et al. 2011); an illustrative example is shown to compare the performance of the system. In Sect. 5 the derivations of an anti windup PID controller are done by SOF applying LMI's that includes the saturation of the system. The SOF control law is obtained by the stability characteristics of the system and by a H_∞ design, so the controller and system performance can be compared by the solution of these control problems. In Sect. 6 an anti windup PID controller for MIMO discrete time systems is shown and similar to its continuous counterpart, a SOF controller is implemented and then solving the LMI's based on the system stability or H_∞ the respective PID gains are found when the input of the system is saturated; in this section an illustrative example is shown to compare the systems performance. Finally, in Sects. 7 and 8 the discussion and conclusions of this chapter are shown respectively so a complete analysis of all the proposed schemes is done and then the conclusions are analyzed at the end of this chapter.

2 Previous Work

As explained in the previous section, the windup phenomena is caused by the integral action of a PI or PID controllers when the input of the system is saturated, then the performance of the system is deteriorated by the increasing of the integral action of the controller, yielding many unwanted effects such as a higher overshoot, a long settling time and even instability. This phenomena is found in SISO and MIMO systems in the continuous and discrete time representations when the input is saturated due to the physical limits of different kind of actuators such as mechanical, hydraulic and electrical systems.

In the case of SISO continuous systems there are some classical architectures implemented to avoid this unwanted effect, some of them, are based on the back calculation of the integral action and other are based on the feedback of the saturated input to the PID controller. The tracking anti windup controller is one of the well known control strategies implemented to avoid the deterioration of the system when this phenomena is found; it consists of a feedback loop generated by the saturated and non saturated inputs and then this signal is used to reduce the integrator input (Bohn and Atherton 1995). In Fig. 1 the tracking anti windup controller is shown where as can be noticed the difference of the non saturated and saturated inputs are fedback to the integrator.

Another method is conditional integration, which consist in turning on and off the integrator according on higher values of the control and error inputs (Bohn and Atherton 1995). Another anti windup control architecture is the limited integrator, this technique consist in feed the integrator output through a dead zone with high gain, reducing the effects of windup when the input saturates (Bohn and Atherton 1995).

The anti windup control architectures for discrete time SISO system are similar to their continuous counterpart, for example in Chen et al. (2003) an anti windup cascade control technique is implemented to suppress the unwanted effects yielded by windup in digital control systems, proving that is an efficient control architecture when the input is saturated. In Lambeck and Sawodny (2004) an anti windup control architecture is derived when the input of the system is constrained, the development of this strategy is based on the frequency response characteristics of

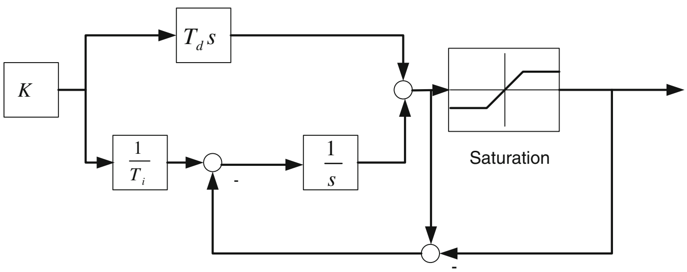


Fig. 1 Tracking anti windup controller

the closed loop system obtaining the digital controller by the conversion of an analog to digital controller. In (Wittenmark 1989) the development of different anti windup controllers are explained such as PID and cascade control for digital control system with constrained input, where the back calculation is implemented similar to the analog counterparts.

One of the common architectures for MIMO system with constrained inputs is the design of an output feedback control law that stabilizes the system while reducing the effects of windup, these control architectures can be applied in continuous and discrete MIMO systems such as explained in (Rehan et al. 2013) where an output feedback controller is implemented and the gains of the controller are found by solving the LMI's for continuous time systems. Another anti windup controller design technique is found in (Saeki and Wada 1996) where an output feedback controller is found by solving the LMI's for continuous MIMO systems with saturated inputs, the controller gains are found by solving the H_∞ optimal LMI's.

With this review about some commons anti windup architectures, in the following sections the development of this kind of novel configuration is shown, where in the first part of this chapter internal model anti windup architectures are developed for the SISO continuous and discrete cases, and the second part of the chapter, some anti windup techniques for MIMO continuous and discrete time systems are shown with illustrative examples to evince the performance of these control strategies.

3 Internal Model Anti Windup Control of Continuous SISO Systems

In this section an anti windup control architecture is developed by implementing an internal model controller (IMC). Internal model control is a technique that consists in designing an appropriate controller according to the internal stability of the system, therefore, as it is proved in this section, this control strategy is convenient for the design of an antiwindup control architecture, reducing the unwanted effects yielded by this phenomena and improving the system performance. The anti windup control strategy shown in this section is developed by feedback the saturated input to the internal model controller so the effects of windup are minimized. The IMC PID controller synthesis is done by the minimization of the H_∞ norm of the error signal as explained in (Morari and Zafiriou 1989; Lee et al. 1998; Tu and Ho 2011) when a unit step input is implemented as a reference signal (Cockbum and Bailey 1991; Doyle III 1999). With this control technique, the resulting PID controller has anti windup properties while maintaining its robustness, so this control strategy is ideal to avoid the unwanted effects yielded by windup. In this section the derivation of an IMC PID anti windup controller is shown step by step ensuring the internal stability of the system while reducing the unwanted effects yielded by the integral action of the PID controller.

3.1 IMC PID Anti Windup Controller for Continuous Time SISO Systems

The anti windup controller architecture implemented in this section is defined in Fig. 2 and it is based on the controller architecture explained in (Saeki and Wada 1996) where a compensator is added to the feedback loop from the saturation input of the system. In Fig. 2 the description of each block is the following; $G_p(s)$ is the plant transfer function that is represented by a first order plus time delay model (FOPTD), $G_c(s)$ is the internal model PID controller and $R(s)$ is the anti windup compensator filter.

In order to obtain a simplified model of the saturation nonlinearity, it is necessary to represent this model by the following equation (Saeki and Wada 1996):

$$\begin{aligned} U &= (\alpha + \beta \Delta_\phi)\tilde{U} \\ \|\Delta_\phi\| &< 1 \\ \alpha + \beta &= 1 \\ \alpha - \beta &= a \end{aligned} \tag{1}$$

Where the saturation nonlinearity is considered to be in the interval $[a, 1]$. The filter $R(s)$ is defined by a first order system as described below:

$$R(s) = \frac{1}{a_1 s + a_0} \tag{2}$$

Then the equivalent transfer function of the nonlinearity (1) and the filter (2), depicted in Fig. 2, is given as $G_{sat}(s)$ as shown in Fig. 3

$$G_{sat}(s) = \frac{\alpha + \beta \Delta_\phi}{1 - R(s)(\alpha + \beta \Delta_\phi)} \tag{3}$$

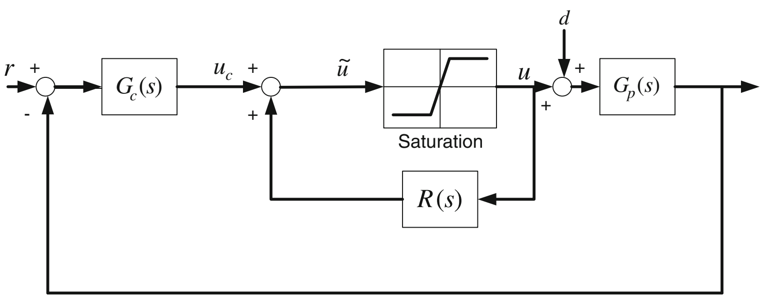


Fig. 2 Anti windup controller architecture

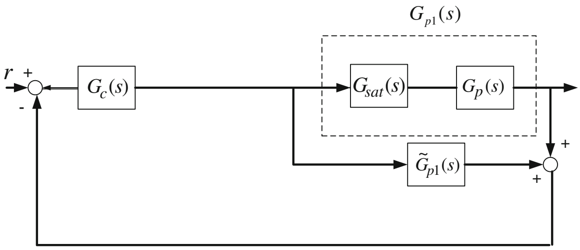


Fig. 3 Anti windup IMC PID architecture

Then the equivalent internal model anti windup control system is shown in Fig. 3. Where $G_{p1}(s)$ is the equivalent plant given by $G_{p1}(s) = G_{sat}(s)G_p(s)$

$G_p(s)$ is represented by a first order plus time delay function given by:

$$G_p(s) = \frac{ke^{-\theta s}}{\tau s + 1} \tag{4}$$

where k is the gain of the transfer function, θ is the time delay and τ is the time constant of the transfer function.

After finishing the explanation of the anti windup controller by implementing a model of the saturation nonlinearity, the IMC PID anti windup controller design can be derived using the equivalent transfer functions of the original system, considering the saturation effects on the model. To start this process it is necessary to obtain the equivalent transfer function of the anti windup controller, basically after obtaining this transfer function G_{p1}, the design of the IMC PID controller is straightforward because the equivalent transfer function is completely linear due to the implementation of an equivalent model of the saturation nonlinearity. Considering the equivalent transfer function G_{p1}

$$G_{p1}(s) = \frac{k(\alpha + \beta \Delta_\phi)(a_1 s + a_0)e^{-\theta s}}{(a_1 s + a_0 - (\alpha + \beta \Delta_\phi))(\tau s + 1)} \tag{5}$$

Then an IMC controller is obtained (Morari and Zafiriou 1989; Shamsuzzoha and Lee 2007) dividing first the transfer function G_{p1} into two parts as the process for designing a IMC controller with anti windup properties

$$G_{p1}(s) = p_{1m}p_{1A} \tag{6}$$

where p_{1a} contains all the RHP poles and zeros with time delay and the portion p_{1m} includes the rest of the transfer function. Now, define the IMC controller q_1 as shown in the following equation, considering a unit step input as the reference:

$$q_1 = p_{1m}^{-1} f \tag{7}$$

where f is a filter selected by the designer in the following form:

$$f = \frac{1}{(\lambda s + 1)^r} \tag{8}$$

for some positive constant r. The IMC PID anti windup controller $G_c(s)$ is given by the following formulae

$$G_c(s) = \frac{q_1}{1 - G_{p1} q_1} \tag{9}$$

where this controller is transformed into a PID form as shown in the rest of this section. The transfer function G_{p1} is divided in the following parts as explained in (6)

$$\begin{aligned} p_{1A}(s) &= e^{-\theta s} \\ p_{1m}(s) &= \frac{k(\alpha + \beta \Delta_\phi)(a_1 s + a_0)}{(a_1 s + a_0 - (\alpha + \beta \Delta_\phi))(\tau s + 1)} \end{aligned} \tag{10}$$

Based on these equations q_1 is given by:

$$q_1(s) = \frac{(a_1 s + a_0 - (\alpha + \beta \Delta_\phi))(\tau s + 1)}{k(\alpha + \beta \Delta_\phi)(a_1 s + a_0)(\lambda s + 1)^r} \tag{11}$$

Using these equations the controller $G_c(s)$ is given by:

$$G_c(s) = \frac{1}{p_{1m}((\lambda s + 1)^r - p_{1a})} \tag{12}$$

Substituting the functions p_{1m} and p_{1A} the following IMC anti windup controller is found:

$$G_c(s) = \frac{(a_1 s + a_0 - (\alpha + \beta \Delta_\phi))(\tau s + 1)}{k(\alpha + \beta \Delta_\phi)(a_1 s + a_0)((\lambda s + 1)^r - e^{-\theta s})} \tag{13}$$

For the PID anti windup controller synthesis it is necessary to consider a PID controller for $G_c(s)$ and then by Mclaurin series expansion the IMC anti windup controller parameters are found (Shamsuzzoha and Lee 2007). For this purposes, consider the following PID controller

$$G_c(s) = K_c(1 + \frac{1}{\tau_i s} + \tau_d s) \tag{14}$$

where K_c is the controller gain, τ_i and τ_d are the integral and derivative time constant that must be obtained in order to get the IMC anti windup controller time constants. The time constants of the IMC anti windup controller are found by the Mclaurin series expansion as shown in (Shamsuzzoha and Lee 2007). The IMC gain and constants are obtained as follow (Lee et al. 1998):

$$\begin{aligned} K_c &= \dot{f}(0) \\ \tau_i &= \frac{\dot{f}(0)}{f(0)} \\ \tau_d &= \frac{\ddot{f}(0)}{2\dot{f}(0)} \end{aligned} \tag{15}$$

The IMC anti windup controller gains are given in detail in Appendix 1, so the reader can refer to this section for detailed information. The function f and its derivatives are defined in this section according to the formulas given in (Lee et al. 1998).

With the derivation and design of an IMC anti windup controller for SISO system, the internal stability of the system while suppressing the unwanted effects of windup is ensured with the addition of a feedback loop which includes the saturated input signal through a filter that improves the system performance when the input is saturated and windup occurs in the PID controller. As it is verified later this control strategy is efficient when saturation occurs in the model, as it is noticed, this strategy is based on the implementation of a saturation model that includes all the properties of this nonlinearity. In the following section an illustrative and comparative example is done in order to test the performance of the IMC anti windup controller, the conclusions of this section are shown in order to compare the system performance with anti windup compensation and no compensation.

3.2 Example 1

In this subsection an illustrative example of the internal model anti windup controller for SISO continuous time system is shown. Consider the following FOPTD system:

$$G_p(s) = \frac{e^{-0.0000001s}}{0.001s + 1} \tag{16}$$

and the following parameters for the anti windup filter and saturation model as shown in Table 1.

Now implementing the formulae found in Appendix 1, the following IMC parameters are found for the IMC PID controller with anti windup compensation and when there is no anti windup compensation. These parameters are shown in Table 2.

The system response of the IMC anti windup controller is depicted in Fig. 4.

It can be noticed that when the AWC is implemented the system response has almost no overshoot and a small settling time in comparison when no AWC is

Table 1 Filter and saturation parameters

Parameter	Value
a_0	5
a_1	0.1
α	1
β	0.06
Δ	100
λ	0.9
r	1

Table 2 Parameters with AW and no AW compensation

Parameter	Value with AW compensation	Value with no AW compensation
K_c	0.0635	0.0036
τ_i	0.9503	33,327.8
τ_d	1	0.0005

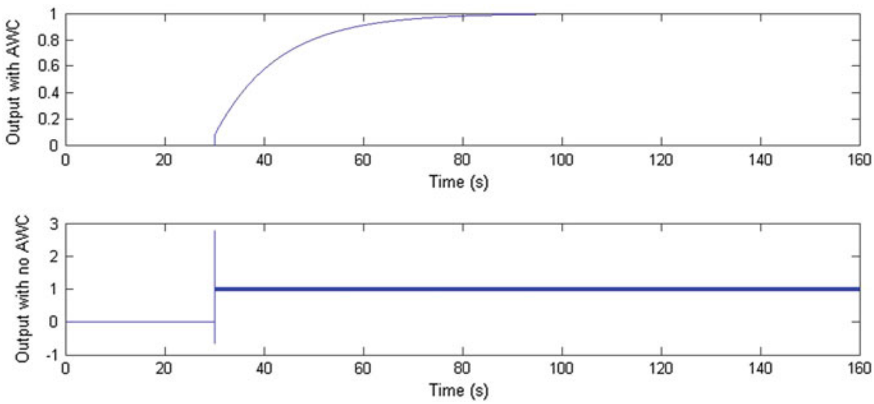


Fig. 4 System response with the IMC AWC (*upper*) and with no AWC (*lower*)

implemented where a high overshoot, a large settling time and higher oscillations are shown proving that the system has a better performance when the anti windup controller is implemented. These results are yielded due to the feedback compensation applied to the PID controller reducing the unwanted effects produced by windup, in comparison when there is not compensation where the system performance is deteriorated due to the increasing in the integrator output when the input of the system is saturated.

In Fig. 5 the input $\tilde{U}$ for the system with AWC is shown where the input is generated according to the reference signal. This signal is the non saturated signal generated by the IMC PID AWC, so the signal follows a designated trajectory according to the required control input necessary to control the system.

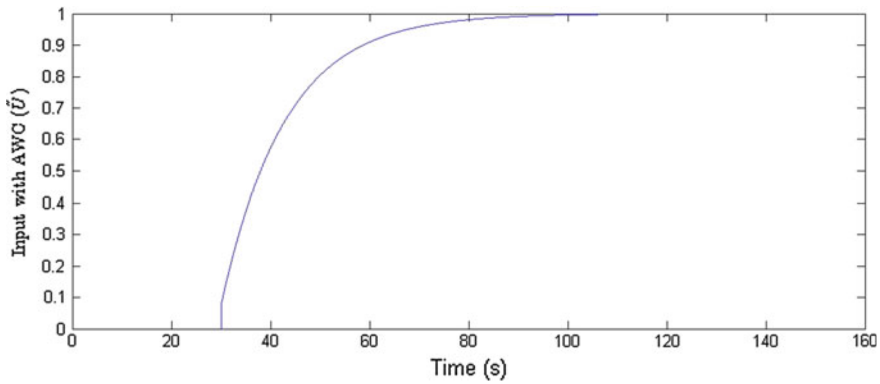


Fig. 5 Control input $\tilde{u}$ of the anti windup controller

In Fig. 6 the control input $\tilde{U}$ with no AWC is shown, where the nonsaturated signal applied to the system is depicted proving that this signal is more irregular than in the AWC version due to the increasing of the integral action producing an abrupt change in the input signal deteriorating the system response.

As it is corroborated in Figs. 7 and 8 these results are affected by the non saturated signals, especially when there is not AWC compensation due to the compensators improves the system performance considerably in comparison when there is no AWC compensation.

In Figs. 7 and 8 the respective control inputs with AWC and AWC compensation are shown, where as it is expected, the control input of the saturated system with no anti windup compensation is deteriorated due to the increasing of the integral action when the input of the system is saturated. This effect is improved by the IMC PID AWC compensation, because the extra feedback added to the model reduces the integral action when the system is saturated.

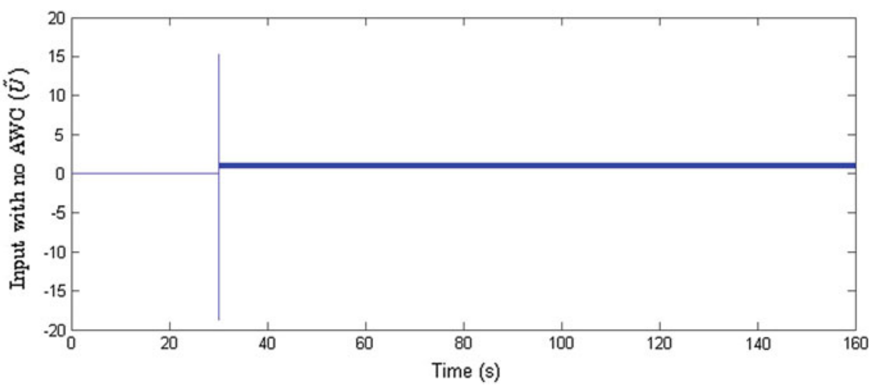


Fig. 6 Control input $\tilde{u}$ when there is no anti windup controller compensation

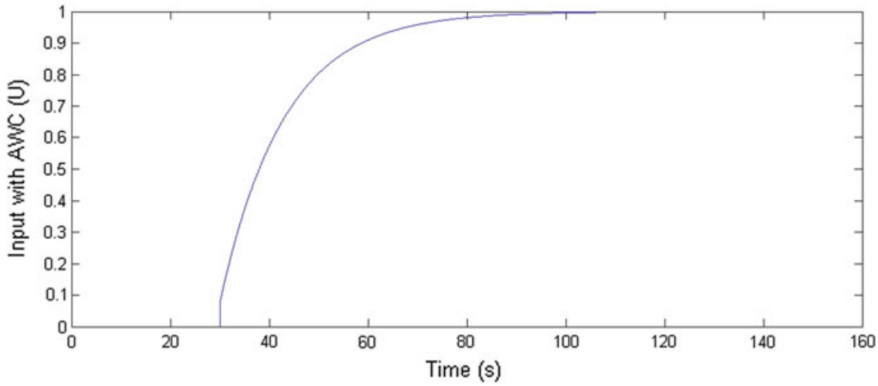


Fig. 7 Saturated input with AWC controller compensation

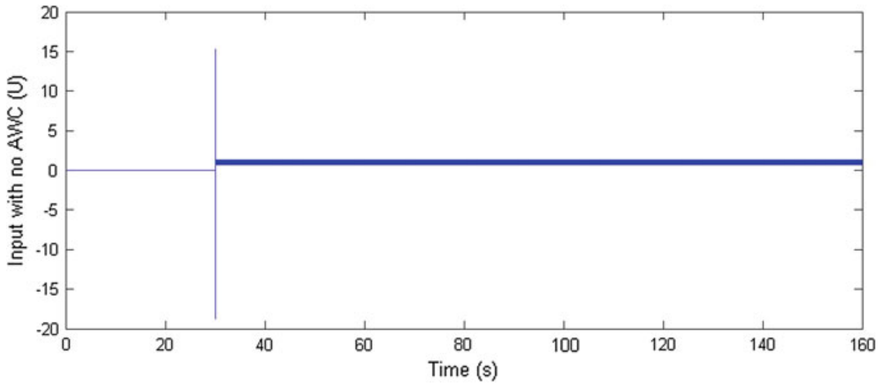


Fig. 8 Saturated input with no AWC controller compensation

These unwanted effects lead to the system performance deterioration, as explained before, Therefore a correction signal send to the internal model controller corrects and improves the system performance deterioration, yielding better system characteristics in comparison when there is no anti windup compensation.

Finally, as a conclusion of this section, it was proved that is possible to stabilize a saturated system by anti windup control compensation, when the system is a single input single output continuous time model, independently of the saturation and the unwanted effects yielded by the windup, generated by the increasing of the integral action. In the next section the discrete time counterpart of the IMC PID anti windup controller is derived, following the internal model control guidelines for the design of an appropriate anti windup controller for this kind of models.

4 Internal Model Anti Windup Control of Discrete Time SISO Systems

In this section the design of a discrete time anti windup controller for discrete time SISO system is explained. In this case, an internal model PID controller compensator is proposed to suppress the unwanted effects yielded by windup when the integrator output is increased due to the actuator saturation. The nonlinearities found in many control systems, specially saturation, deteriorates the system performance similar as it occurs in the SISO continuous time counterpart. As explained before, there are several anti windup control architectures for discrete time systems, some of them are derived from the system frequency response as shown in (Chen et al. 2003; Lambeck and Sawodny 2004) where the design of a frequency response method in a cascade configuration, eliminates the effects yielded by windup. As shown in (Wittenmark 1989) the incorporation of a back calculation compensator improves the system performance and reduces the unwanted effects yielded by saturation. This anti windup controller compensation is shown in (Baheti 1989) where a digital PID controller implementation is used to eliminate the unwanted effects of windup when the input of the system is saturated.

The anti windup controller strategy shown in this section is based on the theoretical background shown in (Morales et al. 2009) where a standard IMC anti windup compensator is implemented where the robustness of the control system is analyzed and the stabilization of the system is done by an internal model controller. The proposed strategy shown in this section is based on an IMC PID anti windup compensator, where due to integral characteristic of the PID controller it is necessary to cancel the windup effects yielded by saturation. The saturation nonlinearity model is obtained by a scalar function approach as explained in (Zhang et al. 2011), so the IMC PID controller can be derived in order to avoid the unwanted effects yielded by windup.

4.1 IMC PID Anti Windup Controller for Discrete Time SISO Systems

The anti windup internal model PID controller architecture is shown in Fig. 9.

Where $G_c(z)$ is the digital internal model controller, $R(z)$ is the anti windup compensator filter, $G_p(s)$ is the continuous time transfer function discretized by a sampler and $p_\gamma^*(z)$ is the equivalent discrete time transfer function implemented in the internal model PID anti windup controller design. In Fig. 10 the equivalent discrete time transfer function is shown, where this transfer function is obtained by the implementation of the scalar sign function approach.

The resulting transfer function $p_\gamma^*(z)$ is implemented to design the anti windup internal model controller with the robustness and internal stability requirements

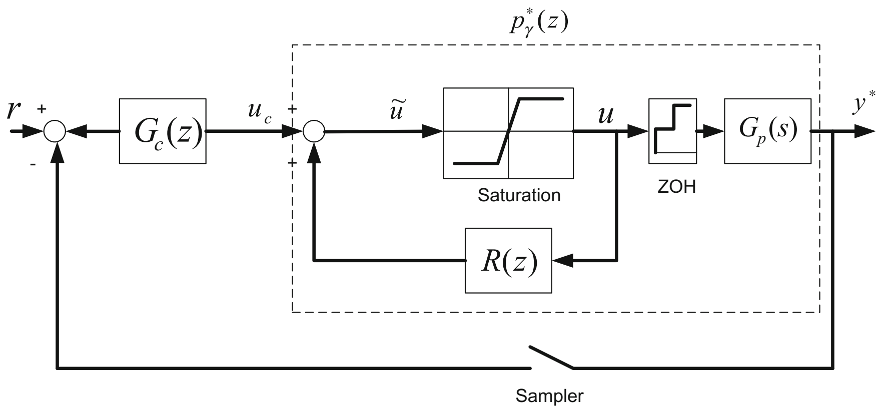


Fig. 9 Anti windup controller architecture

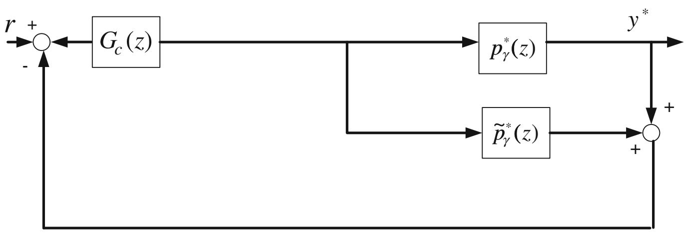


Fig. 10 Equivalent IMC controller architecture

including the anti windup compensator to eliminate the unwanted effects yielded by windup when the input signal is saturated.

Similar as the continuous time counterpart can be divided into two parts, $p_{\gamma A}^*(z)$ and $p_{\gamma M}^*(z)$ as shown in the following equation:

$$p_\gamma^*(z) = p_{\gamma A}^*(z) p_{\gamma M}^*(z) \tag{17}$$

where

$$p_{\gamma A}^*(z) = z^{-N} \prod_{j=1}^{h} \frac{(1 - (\zeta_j^H)^{-1})(z - \zeta_j)}{(1 - \zeta_j)(z - (\zeta_j^H)^{-1})} \tag{18}$$

ζ_j are the zeros of $p_{\gamma A}^*(z)$ outside the unit circle for $j = 1 \ldots h$. N is selected to make $p_{\gamma M}^*(z)$ semiproper and H denotes the complex conjugate (Morari and Zafiriou 1989). In order to design the internal model anti windup controller the following controller must be implemented:

$$G_c(z) = \frac{q(z)}{1 - p_\gamma^*(z)q(z)} \tag{19}$$

where

$$q(z) = p_{\gamma M}^{*-1}(z)f(z) \tag{20}$$

and the filter $f(z)$ is given by:

$$f(z) = \frac{(1-\alpha)z}{z-\alpha} \tag{21}$$

for a given value of α. Meanwhile, the anti windup compensator filter $R(z)$ is given by:

$$R(z) = \frac{1}{a_1 z + a_0} \tag{22}$$

where a_1, $a_0 > 0$. The saturation function is obtained by the scalar sign function (Zhang et al. 2011) taking into account the following *sign* function representation:

$$sign(z) = \begin{cases} 1 & \text{if Re(z)} > 0 \\ -1 & \text{if Re(z)} < 0 \end{cases} \tag{23}$$

so for $j = 1$ the following representation of the saturation model is implemented:

$$saturation(z) = U_{max} sign_1(z) \tag{24}$$

where U_{max} is the saturation limit and

$$sign_1(z) = z \tag{25}$$

In this section in order to design the anti windup control system for discrete time models the following first order plus time delay discrete time model is implemented:

$$G_p(z) = \frac{k}{\tau z + 1} z^{-N} \tag{26}$$

Where k is the system gain, τ is the time constant and $N > 0$ is an integer which indicates the number of time delays. In order to obtain the internal model controller it is necessary to get the equivalent transfer function $p_\gamma^*(z)$ taking in count the compensator and saturation in order to obtain this transfer function:

$$p_\gamma^*(z) = \frac{k(U_{max}a_1 z^{2-N} + U_{max}a_0 z^{1-N})}{((a_1 - U_{max})z + a_0)(\tau z + 1)} \tag{27}$$

With a sample time T. Where this transfer function is divided as explained in (17) and (18) as:

$$p_{\gamma M}^*(z) = \frac{k(U_{max}a_1 z^2 + U_{max}a_0 z)}{((a_1 - U_{max})z + a_0)(\tau z + 1)} \tag{28}$$

$$p_{\gamma A}^*(z) = z^{-N} \tag{29}$$

Then using (19) the following internal model controller is obtained:

$$G_c(z) = \frac{(1-\alpha)z((a_1 - U_{max})z + a_0)(\tau z + 1)}{(U_{max}a_1 z^2 + U_{max}a_0 z)(z-\alpha) - (U_{max}a_1 z^{3-N} + U_{max}a_0 z^{2-N})} \tag{30}$$

In order to obtain the internal model anti windup controller, it is necessary to define the following standard PID controller:

$$G_c(z) = K_c(1 + \frac{1}{\tau_i(z-1)} + \tau_d(z-1)) \tag{31}$$

Due to the integral term of $G_c(z)$ the controller gain and parameters using a similar procedure like the continuous time counterpart. Implementing the Taylor series expansion, similar as the previous section the following constant and time constants of the PID controller are found:

$$\begin{aligned} K_c &= f'(1) \\ \tau_i &= \frac{f'(1)}{f(1)} \\ \tau_d &= \frac{f''(1)}{2f'(1)} \end{aligned} \tag{32}$$

where f and its derivatives are defined in Appendix 2. This equations are valid for any sampling period T and the resulting equations are shown in Appendix 2. The proposed control strategy explained in this section meets the robustness and internal stability properties that make them suitable for the anti windup control of discrete time SISO systems. In the next subsection, an illustrative example is shown, to test the system performance by a numerical example.

4.2 Example 2

In this subsection the stabilization of a discrete time SISO system is done when saturation is found in the model. The system to be stabilized is the following:

$$G_p(z) = \frac{z^{-2}}{10z + 1} \tag{33}$$

with the following saturation, filter and anti windup compensator parameters (Table 3).

Using the formulae shown in Appendix 2and a sampling period of $T = 1$ s, the controller gain and parameters are found as shown in Table 4.

With this control systems parameters, the system output with AWC compensation and with no AWC compensation are shown in the figure below.

The system response shown in Fig. 11 corroborates that a small overshoot and small settling time is obtained when an internal model anti windup controller is implemented, in contrast when there is not anti windup compensation. These results are expected due to the anti windup compensator reduces the integral action when the system is saturated, so a smaller overshoot and smaller settling time is obtained when the internal model controller is implemented.

In Fig. 12 the non saturated input of the system with anti windup compensation is shown where this signal reaches the necessary output value to obtain the required value.

In Fig. 13 the non saturated input, when there is no anti windup compensation, is shown. As can be noticed, the required input signal is applied to the system until the required output value is obtained.

In Fig. 14 the saturated input value is depicted, where the limiter imposed by the saturation makes the system to reach the desired value and as it is compared with Fig. 15 the saturated signal is better when an anti windup controller is implemented.

Table 3 Filter, compensator and saturation parameters

Parameter	Value
α	0.2
a_0	4×10^{-7}
a_1	5×10^{-7}
U_{max}	10.9

Table 4 Parameters with AW and no AW compensation

Parameter	Value with AW compensation	Value with no AW compensation
K_c	3.4362×10^{7}	3.1062×10^{8}
τ_i	1.6869	2
τ_d	0.4265	1×10^{-6}

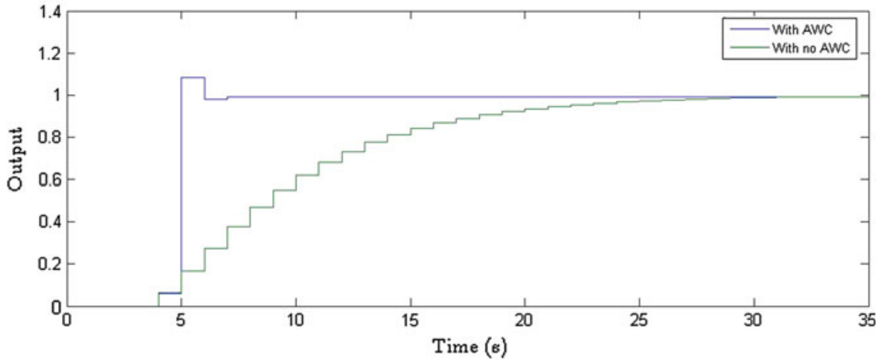


Fig. 11 System output with AWC and no AWC

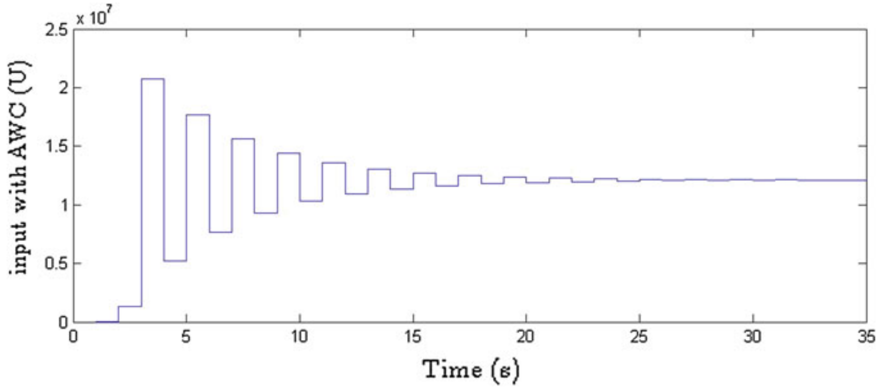


Fig. 12 Non saturated input with AWC

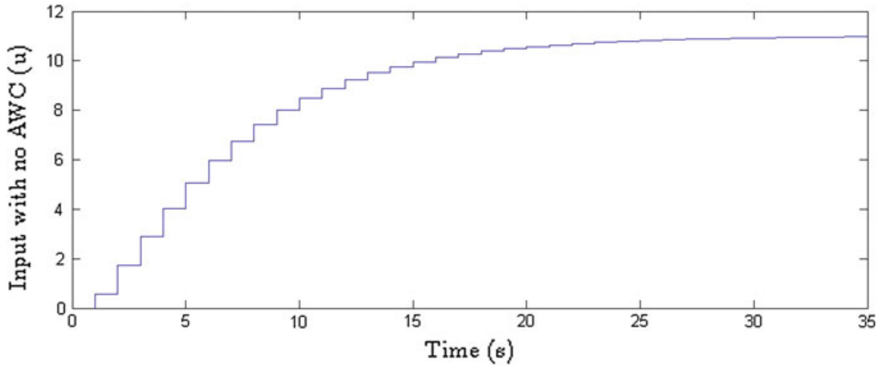


Fig. 13 Non saturated input with no AWC

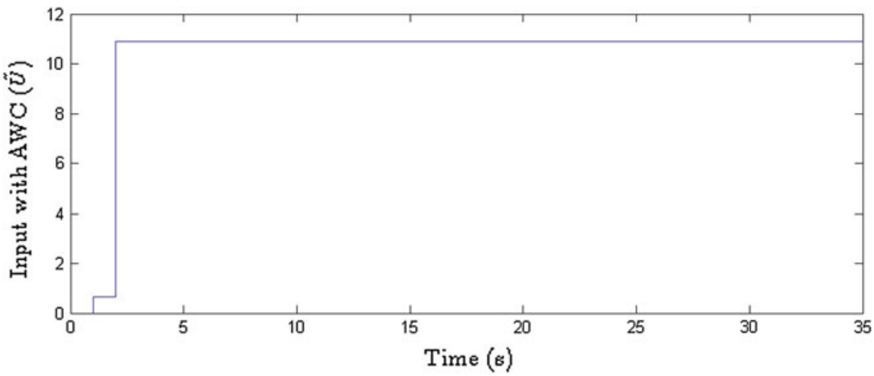


Fig. 14 Saturated input with AWC

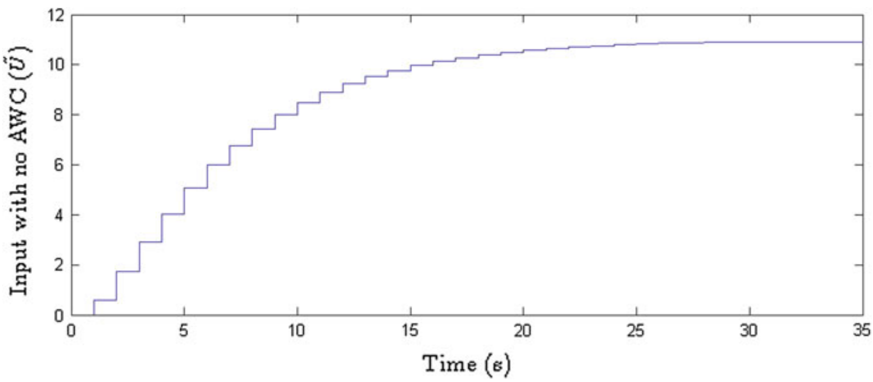


Fig. 15 Saturated input with AWC

In this section an internal model PID anti windup controller is designed in order to improve the system performance by reducing the windup effect. As it is noticed, the controller design is very similar to the continuous time counterpart taking in account the saturated signal and then this signal is sent through a feedback loop by a compensator. The main idea behind this controller is to apply the robust controller characteristics of internal model control in order to obtain the desired gain and time constants of the PID controller to make the system to follow a step reference signal. With the control strategies derived in Sects. 3 and 4 a complete design and analysis of anti windup controllers for continuous and discrete time SISO systems is deployed. Where it was proved that efficient anti windup control strategies can be derived implementing the internal model control strategy for any kind of SISO systems while ensuring internal stability and the improvement of the system output performance.

In the following sections, the design and analysis of anti windup techniques for discrete and continuous time MIMO systems is shown, where different approaches are implemented in order to improve the control system performance when saturation or constrained inputs are present in the system. Generally, the design of anti windup control strategies for MIMO systems are more difficult than the anti windup control of SISO system, for this reason, the solution of this problem is done by static output feedback control law design, where MIMO PID controllers are designed in the continuous and discrete time cases.

It will be proved that as similar to the SISO system cases, the modeling and design of effective anti windup control techniques is possible improving the system performance when some kind of compensation is added to the controller.

5 Anti Windup Control of Continuous MIMO Systems by Static Output Feedback (SOF)

In this subsection the design of an anti windup PID controller for continuous time MIMO system is derived based on static output feedback (SOF) controller. This work is based on the solution of the specified linear matrix inequalities (Cao et al. 2002; Wu et al. 2005; Rehan et al. 2013) where a static output feedback controller is defined in order to improve the anti windup characteristics of this MIMO controller (Neto and Kucera 1991; Henrion et al. 1999; Fujimori 2004; He and Wang 2006). A PID control law is obtained by solving the required LMI's in order to find the PID controller gains. The controller gains are found by two static output feedback solutions, by solving an standard LMI and a H_∞ problem. With these two control strategies it is possible to find appropriate controller gains for the PID anti windup controller taking in count the saturation nonlinearity.

In order to design the anti windup PID controller it is necessary to model the saturation nonlinearity by a describing function approach (Taylor and O'Donnell 1990) in order to deal with the nonlinearities added to the system by the actuators saturation.

The intention of this control approach is to design an efficient anti windup controller system for MIMO continuous time systems when the inputs are constrained or saturated. It is proved that solving the system constraints by LMI's in order to obtain a stable PID control law, the addition of anti windup compensation similar as the SISO time systems, improves the system performance and avoids the deterioration of the output signal. In the following subsections the design of an anti windup controller is explained in detail, and in order to test the system performance an illustrative example of the stabilization and control of a DC motor is evinced.

5.1 PID Anti Windup Controller Design for MIMO Continuous Time Systems

The PID anti windup controller design for MIMO continuous time systems, consist of a back calculation PID controller by a loop which includes the saturated and non saturated input signal of a linear time invariant MIMO system. In Fig. 16 the anti windup controller is shown where v is the back calculation signal that is implemented to avoid the windup effects which deteriorates the system performance. The MIMO system is represented by $G(s)$ and the anti windup PID controller and compensator is represented by $G_c(s)$.

The anti windup controller takes the non saturated and saturated difference signal v to suppress the increment of the integral action when the system saturates. As occurs in the SISO case, the windup phenomena yields unwanted effects that deteriorates the system performance; the settling time and overshoot generally are damage when the input signal is saturated, so the compensator corrects the effects of windup by the back calculation of the input signal added by an extra loop.

The approach explained in this section consist in obtaining a saturation representation by a describing function approach (Taylor and O'Donnell 1990), where this method simplifies the PID controller synthesis and provides an accurate representation of the equivalent control systems.

In Fig. 17 the saturation model is depicted in order to be represented by a describing function that helps to obtain an equivalent anti windup controller.

The saturation model shown in Fig. 17 depicts the parts in which this model is divided in order to obtain the Fourier series coefficients of the describing function. $n(e)$ is the saturation output, e is the saturation input, α is the input limit and M is the saturation output limit. The describing function is given by the following transfer function:

$$\phi(s) = \frac{a_1 + b_1 s}{E} \tag{34}$$

where a_1 and b_1 are the Fourier series coefficients when a sinusoidal input signal with amplitude E is implemented. The coefficients of the Fourier series implemented in this analysis are:

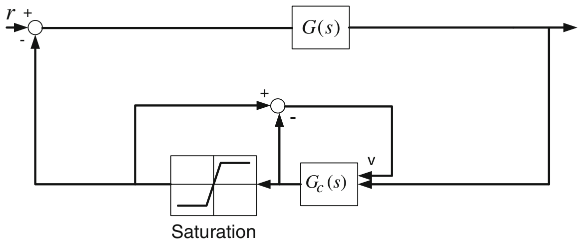
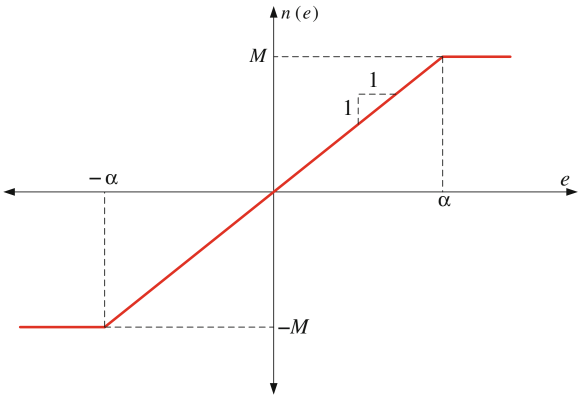


Fig. 16 Multivariable control system

Fig. 17 Saturation model



$$
\begin{aligned}
a_1 &= \frac{1}{T}\int_{-T}^{T} \sin(\omega t)n(t)dt \\
b_1 &= \frac{1}{T}\int_{-T}^{T} \cos(\omega t)n(t)dt
\end{aligned}
\tag{35}
$$

where ω is the angular frequency of the input signal e and in order to obtain the Fourier series coefficients a sinusoidal input signal of amplitude E and period T must be assumed as the input of the saturation. Considering that e is 2π periodic or $T = 2\pi$ the following Fourier coefficients are obtained:

$$
\begin{aligned}
a_1 &= (\frac{-\alpha}{2E}\sqrt{1+(\frac{\alpha}{E})^2} + sin^{-1}(\frac{\alpha}{E}) + \frac{\pi}{4}) + \frac{M}{2\pi}(\frac{2\alpha}{E}) \\
b_1 &= \frac{-E}{4\pi}\left(2\left(1-\left(\frac{\alpha}{E}\right)^2\right) - \left(\frac{\alpha}{E}\right)^2 - 1\right) - \frac{M}{\pi}\left(\frac{\alpha}{E}\right)
\end{aligned}
\tag{36}
$$

Considering the following PID controller:

$$
y_c(s) = F_1 u_c(s) + F_2\frac{u_c(s)}{S} + F_3 u_c(s)\left(\frac{s-1}{s}\right) \tag{37}
$$

where F_1, F_2 and F_3 are diagonal matrices of appropriate dimensions (He and Wang 2006) for the proportional, integral and derivative parts of the controller.

In order to obtain the anti wind up controller, consider the following linear time invariant system $G(c)$ given by:

$$\begin{aligned} \dot{x} &= Ax - B\phi(u) \\ y &= Cx \end{aligned} \tag{38}$$

where $x \in \Re^n, u \in \Re^m$ and A, B and C are matrices of appropriate dimensions. From (37) The controller and anti windup compensator can be represented in state space by (Cao et al. 2002):

$$\begin{aligned} \dot{x}_c &= k_1 y + \Delta(\phi(u) - u) \\ u &= x_c + k_2 y = x_c + k_2 Cx \end{aligned} \tag{39}$$

where $k_1 = F_2 - F_3$, $k_2 = F_1 + F_3$, and Δ is a positive definite diagonal matrix that is part of the anti windup controller and compensator. Usually a correction term $\Delta(\phi(u) - u)$ is needed in the controller to compensate the saturation effects.

With (38) and (39) a closed loop augmented control system is obtained as:

$$\begin{aligned} \dot{\overline{x}} &= \overline{A}\overline{x} + \overline{B}w \\ u &= F\overline{x} \end{aligned} \tag{40}$$

where:

$$\begin{aligned} \overline{x} &= \begin{bmatrix} x \\ x_c \end{bmatrix} \\ \overline{A} &= \begin{bmatrix} A & 0 \\ k_1 C & 0 \end{bmatrix} \\ \overline{B} &= \begin{bmatrix} -B & 0 \\ \Delta & -\Delta \end{bmatrix} \\ F &= \begin{bmatrix} k_2 C & I \end{bmatrix} \\ w &= \begin{bmatrix} \phi(u) & u \end{bmatrix}^T \end{aligned} \tag{41}$$

Using the saturation model ϕ, the obtained input vector w is:

$$w = \begin{bmatrix} F(\frac{a_1}{E}\overline{x} + \frac{b_1}{E}\dot{\overline{x}}) \\ F\overline{x} \end{bmatrix}$$

Making another change of variable with $z = \begin{bmatrix} \overline{x} & \dot{\overline{x}} \end{bmatrix}^T$ the following system is obtained:

$$\dot{z} = A'z + B'Mz \tag{42}$$

that yields the following system's equation:

$$\dot{z} = (A' + B'M)z \tag{43}$$

where:

$$A' = \begin{bmatrix} \overline{A} & 0 \\ 0 & 0 \end{bmatrix}$$
$$B' = \begin{bmatrix} \overline{B} \\ 0 \end{bmatrix} \tag{44}$$
$$M = \begin{bmatrix} Fa_1/E & Fb_1/E \\ F & 0 \end{bmatrix}$$

In order to obtain the linear matrix inequality to solve the gains of the PID anti windup compensator the following Lyapunov function must be considered:

$$V(z) = z^T P z \tag{45}$$

where P is a positive definite matrix, used in order to ensure the stability of the system. Deriving the Lyapunov function the following result is obtained:

$$\dot{V}(z) = z^T (A' + B'M)^T Pz + z^T P(A' + B'M)z \tag{46}$$

where in linear matrix inequality form (45) is represented as:

$$(A' + B'M)^T P + P(A' + B'M) < 0 \tag{47}$$

So by solving the following LMI the controller parameters of the equivalent system are found (Fujimori 2004):

$$\begin{bmatrix} (A' + B'M)^T P + P(A' + B'M) & 0 \\ 0 & 0 \end{bmatrix} < 0 \tag{48}$$

For the H_∞ synthesis, a similar approach is implemented to find the controller gains, considering the following criteria:

$$\|T_{2\omega}(s)\|_\infty < \gamma \tag{49}$$

where $T_{2\omega}(s)$ is the closed loop transfer function of the model (Fujimori 2004; He and Wang 2006; Rehan et al. 2013) and $\gamma > 0$ is a positive constant that indicates the desired performance. Then the respective LMI is needed to find the gain F and the solutions of the anti windup PID controller.

$$\begin{bmatrix} PA_{cl} + P^T A_{cl} & 0 & C_{cl} \\ 0 & -\gamma I & 0 \\ C_{cl} & 0 & -\gamma I \end{bmatrix} < 0 \tag{50}$$

where:

$$A_{cl} = (A' + B'M)$$
$$C_{cl} = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \tag{51}$$

where I is an identity matrix of an appropriate dimension. Solving the LMI shown in (48) and (50) for P and M the controller parameters can be extracted from M obtaining all the PID controller gain matrices found by these optimization techniques.

In this subsection is proved that an anti windup PID controller for MIMO continuous time system can be implemented by solving a LMI based optimization problem. In the following subsection, the control of a DC motor is done in order to show by an illustrative example the application of these control strategies, it is proved that finding the respective matrix M the rest of the controller variables can be obtained. The solution of these LMI can be obtained by several numerical methods found in literature, such as shown in (He and Wang 2006) for example.

5.2 Example 3

In this section the stabilization and control of a DC motor by an anti windup PID controller for MIMO systems is shown to illustrate the advantages of the proposed technique.

Consider the following DC motor transfer function (Cockbum and Bailey 1991):

$$\frac{\omega_L(s)}{v_a(s)} = \frac{k_m}{(J_m L + J_L L)s^2 + (J_m R + J_L B_L)s + k_m^2} \tag{52}$$

where ω_L is the angular velocity of the model, v_a is the applied armature voltage, J_L is the inertial load, J_m is the motor inertia, L is the inductance, R is the resistance, B_L is the viscous friction constant and k_m is the motor constant. Converting (52) to state space the following equation is obtained:

$$\begin{aligned} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} &= \begin{bmatrix} \frac{J_m R + J_L B_L}{J_m + J_L L} & 1 \\ -\frac{K_m^2}{J_m + J_L L} & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \\ &+ \begin{bmatrix} 0 & 0 \\ 0 & \frac{K_m}{J_m + J_L L} \end{bmatrix} \begin{bmatrix} 0 \\ v_a \end{bmatrix} \end{aligned} \tag{53}$$

Table 5 Parameters of the DC motor

Parameter	Value
J_m	0.02215 kg m^2
J_L	0.01 kg m^2
B_L	0.002953 Nm s
R	11.2 Ohms
L	0.1215 H
K_m	0.5161 Nm/A
Nominal speed	1,750 RPM

and

$$Y = Cx \tag{54}$$

where x_1 is the angular velocity of the motor, x_2 is the armature current and C is a 2×2 identity matrix. The motor parameters are shown in Table 5.

Solving the LMI (48) for P by an optimization algorithm, a matrix F can be found from M in order to obtain the gain matrices for the PID controller. The gain matrices of the PID anti windup controller can be obtained by (55).

$$F = \begin{bmatrix} 800 & 0 & 1 & 0 \\ 0 & 800 & 0 & 1 \end{bmatrix} \tag{55}$$

From F the following PID anti windup controller parameters are found

$$\begin{aligned} F_1 &= \begin{bmatrix} -200 & 0 \\ 0 & -200 \end{bmatrix} \\ F_2 &= \begin{bmatrix} 2000 & 0 \\ 0 & 2000 \end{bmatrix} \\ F_3 &= \begin{bmatrix} 1000 & 0 \\ 0 & 1000 \end{bmatrix} \end{aligned} \tag{56}$$

The gain matrices when there is no anti windup compensation are the following:

$$\begin{aligned} F_1 &= \begin{bmatrix} -100000 & 0 \\ 0 & -100000 \end{bmatrix} \\ F_2 &= \begin{bmatrix} 2000 & 0 \\ 0 & 2000 \end{bmatrix} \\ F_3 &= \begin{bmatrix} 1000 & 0 \\ 0 & 1000 \end{bmatrix} \end{aligned} \tag{57}$$

The matrix F for the H_∞ controller are given by:

$$F = \begin{bmatrix} -4.0825 & 0 & 0 & 0 \\ 0 & -4.0825 & 0 & 0 \end{bmatrix} \times 10^8 \tag{58}$$

and the following gain matrices for the H_∞ anti windup controllers are given by:

$$\begin{aligned} F_1 &= \begin{bmatrix} -4.0825 & 0 \\ 0 & -4.0825 \end{bmatrix} \times 10^8 \\ F_2 &= \begin{bmatrix} 2000 & 0 \\ 0 & 2000 \end{bmatrix} \\ F_3 &= \begin{bmatrix} 1000 & 0 \\ 0 & 1000 \end{bmatrix} \end{aligned} \tag{59}$$

and the compensator gain Δ is given by:

$$\Delta = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \tag{60}$$

The main idea of this example is to keep the nominal angular velocity(1,750 RPM) or (183.26 rad/s) while applying a disturbance torque of (100 Nm) at 0 s, so the anti windup PID controller must be able to keep this velocity even when an external disturbance is applied on the model.

In Fig. 18 the angular velocity of the DC motor in three cases; with anti windup, no anti windup and H_∞ anti windup PID controllers are shown; where in the H_∞ and standard static feedback anti windup controller better results were obtained with smaller settling time, smaller steady state error and smaller overshoot in comparison when there is not anti windup compensation. For these reasons, the anti windup controllers and compensator are better than the uncompensated controller version.

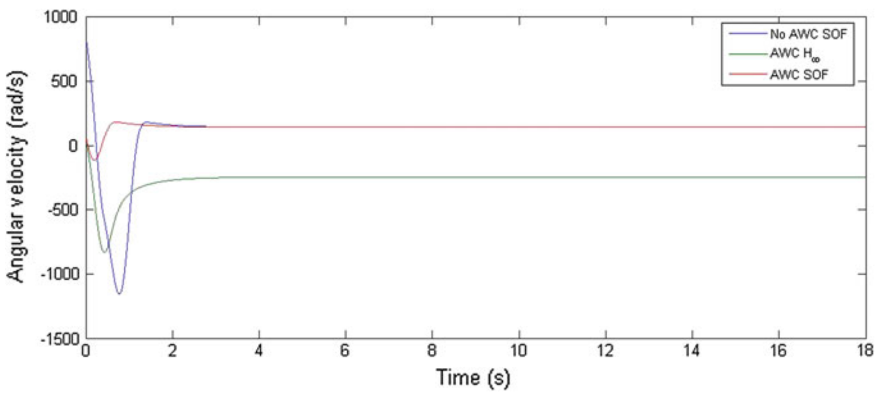


Fig. 18 Angular velocities with AWC, H_∞ and no AWC

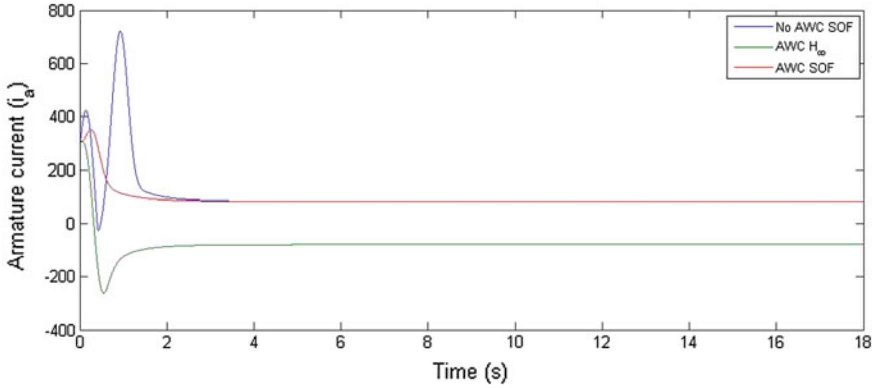


Fig. 19 Armature current i_a

It is clear when an anti windup compensator is implemented the performances is improved significantly, this means, that the settling time, steady state error and overshoot are smaller than the performance indexes of the uncompensated system.

In Fig. 19 the armature current i_a is depicted for the three cases of static output feedback (SOF) controller. It can be noticed that even than in the standard and H_∞ SOF the armature current is greater in comparison when no compensation is implemented in the SOF controller. These results are obtained due to the better performance of the standard and H_∞ SOF in comparison with the uncompensated controller, so more control effort is necessary in order to obtain an acceptable performance.

In Fig. 20 the input voltage of the DC motor (field voltage) is depicted where the input voltage of the non compensated systems increases to higher values than the compensated control systems. The input voltage for the uncompensated controller raises to higher values due to the windup effects that increases the integral action, similar to the SISO case, deteriorating the system performance.

The anti windup PID compensator by SOF improves the system response and performance significantly due to the compensator added to the MIMO PID controller, The windup effects are suppressed by the compensator action, reducing the integral action when the input of the system is saturated.

Finally, in Fig. 21 the mechanical torque of the DC motor is depicted where this variable reaches the final value of 100 Nm, which is the value of the disturbance input applied to the motor at 0 s while keeping the desired nominal velocity.

In this section the design of an anti windup PID controller for MIMO system by standard and H_∞ SOF is explained in order to obtain a suitable controller that eliminates the unwanted effects yielded by windup. As it occurs in the SISO case, the windup phenomena occurs when the input of the system is saturated increasing the integrator action, this effects damage the system performance, specially, it yields higher overshoot and longer settling times.

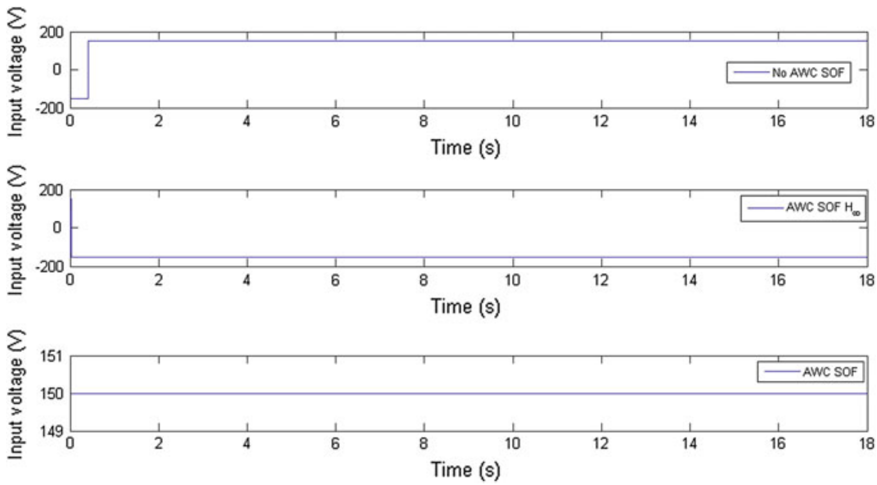


Fig. 20 Input voltage

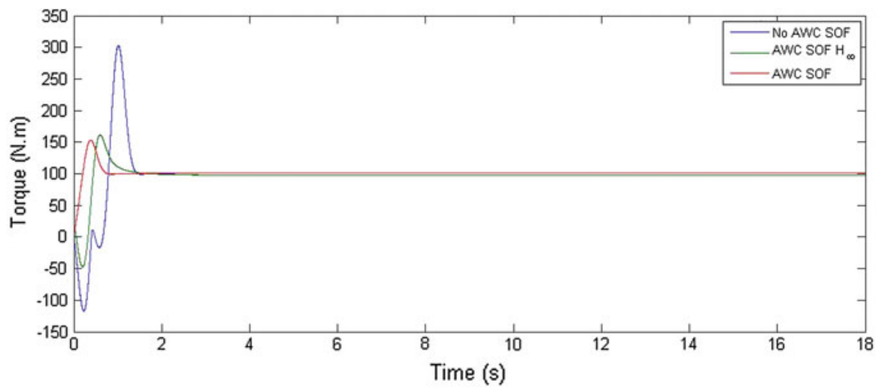


Fig. 21 Mechanical torque

In this section the design of two control strategy to deal with windup reducing the integral action and improving the system performance significantly. The saturation nonlinearity is implemented by the describing function method simplifying the design of the proposed control strategies. It is confirmed by an example, that the standard anti windup SOF controller yields better results than the uncompensated systems in which the system output is deteriorated by the windup effect. The standard and H_∞ SOF PID controllers are a perfect option for the control and compensation of saturated or constrained input MIMO systems.

In the next section, the MIMO counterpart of the control strategy presented in this section is shown. A discrete time MIMO system is obtained by a static output feedback, a PID compensator is selected similar as the continuous counterpart. In this section it is shown that an efficient control strategy is developed for the suppression of the unwanted effects yielded by windup, and the controller synthesis is done by a different saturation model.

6 Anti Windup Control of Discrete MIMO Systems by Static Output Feedback (SOF)

In this section the derivation of an anti windup PID controller for discrete MIMO system is proposed. The main idea behind this controller is to design an anti windup controller/compensator that minimizes the windup effects when the input of the system saturates producing an increasing of the integral action that deteriorates the system performance. The controller design for this kind of systems consist in deriving a static output feedback (SOF) control law, similar as the continuous time counterpart (Bateman and Zongli 2002; Kwan Ho et al. 2006; Matsuda and Ohse 2006) and then the SOF gain is obtained by solving the LMI's as an optimization problem.

In order to achieve suitable control gains for the PID controller, it is necessary to implement a saturation model (Li-Sheng et al. 2004; Zongli and Liang 2006; Shuping and Boukas 2009) where sufficient conditions are established in order to solve the LMI's by a convex optimization problem (Shuping and Boukas 2009). For the AWC design it is necessary to add a back calculation loop which consists in the difference between the non saturated and saturated input signal, similar as the continuous time counterpart, to reduce the effects of windup when the input system saturates. Then using the saturation model (Li-Sheng et al. 2004) this nonlinearity form is implemented to obtain the respective LMI's solved by a convex optimization problem. Beside from the standard solution of static output feedback controllers (SOF) a h_∞ SOF controller synthesis is obtained by solving the required LMI's (Lim and Lee 2008). In this section it is proved that a discrete time PID controller can be obtained by a static output feedback control law, simplifying the anti windup controller design and then the PID controller gains can be found by solving the linear matrix inequalities for SOF and H_∞ SOF.

As occurs in the continuous time case, there are several numerical methods to solve discrete time SOF problems by LMI's so with this method an optimal solution of the LMI's can be found. By implementing the appropriate LMI's and the saturation nonlinearity model an optimal solution can be found by any of the algorithm found in literature such as (Matsuda and Ohse 2006) for continuous time and (Kwan Ho et al. 2006) for discrete time systems. The proposed anti windup PID controller is designed taking into account the stability properties and characteristic of the closed loop system and for the H_∞ SOF problem the robustness of the closed loop system improves the system performance and reduces the deterioration of the system operation when a reference signal needs to be tracked.

This section is divided in two subsections, where in the first part the design of a PID anti windup controller is derived by adding a back calculation signal to the controller and converting the anti windup PID controller in a static output feedback problem and then this problem is solved by LMI's. Another anti windup PID controller is designed by a H_∞ synthesis where the stability and robustness of the system is considered, then the closed loop system is robust when unmodeled dynamics and disturbances are found in the system. Finally, an illustrative example is explained in the last subsection where the PID anti windup controller for a DC motor is shown, where the main objective is to maintain a constant nominal angular velocity by following a desired profile torque. With the theoretical background and the illustrative example shown in this section a complete demonstration of a PID anti windup control strategy for discrete time system is shown where the stability and robustness condition are met by selecting an appropriate static output feedback controller.

6.1 PID Anti Windup Controller Design for MIMO Discrete Time Systems

Consider the PID antiwindup controller shown in Fig. 22.

Where $G(z)$ is the discrete time transfer matrix, and v is the back calculation input signal of the anti windup PID controller. Consider the transfer matrix $G(z)$ in state space form

$$\begin{aligned} x(k+1) &= Ax(k) - B\sigma(u(k)) \\ y(k) &= Cx(k) \end{aligned} \tag{61}$$

where $x \in \Re^m$, A is a $\Re^{m\times m}$ matrix x is a $\Re^m$ vector, B is a $\Re^{n\times m}$ and C is a $\Re^{l\times m}$ matrix. $m > 0$ denotes the number of states, $n > 0$ is the number of inputs, l is the number of outputs and $\sigma(\cdot)$ is the saturation input. Consider the following anti windup PID controller given by:

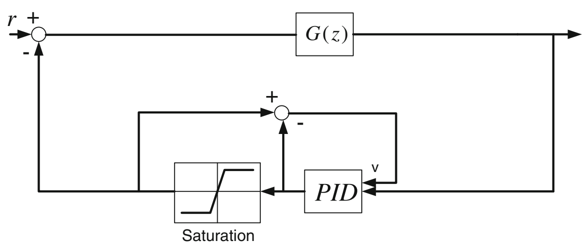


Fig. 22 Anti windup PID controller

$$\begin{aligned} x(k+1) &= Ax(k) - B\sigma(u(k)) \\ y(k) &= Cx(k) \end{aligned} \tag{62}$$

Consider the following PID controller with anti windup compensation (Lim and Lee 2008):

$$\begin{aligned} \sum_{i=0}^{k-1} y(k+1) &= y(k) + \sum_{i=0}^{k-1} y(k) - \Gamma(\sigma(u(k)) - u(k)) \\ u(k) &= -k_p y(k) - k_I \sum_{i=0}^{k-1} y(i) - k_D \Delta y(k) \end{aligned} \tag{63}$$

Due to $\sum_{i=0}^{k-1} y(k+1) - \sum_{i=0}^{k-1} y(k) = y(k)$ with $y(0) = 0$ subtracting the correction signal $\Gamma(\sigma(u(k)) - u(k))$ as done in the continuous time case explained in the previous section (Cao et al. 2002). Where k_p, k_I, k_D are diagonal matrices for the proportional, integral and derivative parts of the PID controller, Γ is a positive definite matrix and $\Delta y(k) = y(k) - y(k-1)$. In order to design the PID controller the following augmented variables are introduced to represent (62) and (63)

$$\begin{aligned} x_a(k) &= \begin{bmatrix} x(k) \\ \sum_{i=0}^{k-1} y(k) \\ y(k-1) \end{bmatrix} \\ y_a(k) &= \begin{bmatrix} y(k) \\ \sum_{i=0}^{k-1} y(k) \\ \Delta y(k) \end{bmatrix} \end{aligned} \tag{64}$$

Then the augmented system is (Lim and Lee 2008):

$$\begin{bmatrix} x(k+1) \\ \sum_{i=0}^{k-1} y(k+1) \\ y(k) \end{bmatrix} = \begin{bmatrix} A & 0 & 0 \\ C & I & 0 \\ C & 0 & 0 \end{bmatrix} \begin{bmatrix} x(k) \\ \sum_{i=0}^{k-1} y(k) \\ y(k-1) \end{bmatrix} + \begin{bmatrix} -B \\ -\Gamma \\ 0 \end{bmatrix} \sigma(u(k)) + \begin{bmatrix} 0 \\ \Gamma \\ 0 \end{bmatrix} u(k) \tag{65}$$

$$y_a(k) = \begin{bmatrix} C & 0 & 0 \\ 0 & I & 0 \\ C & 0 & -I \end{bmatrix} x_a(k) \tag{66}$$

where

$$A_{cl} = \begin{bmatrix} A & 0 & 0 \\ C & I & 0 \\ C & 0 & 0 \end{bmatrix}$$
$$B_{cl} = \begin{bmatrix} -B \\ -\Gamma \\ 0 \end{bmatrix}$$
$$C_{cl} = \begin{bmatrix} C & 0 & 0 \\ 0 & I & 0 \\ C & 0 & -I \end{bmatrix} \tag{67}$$
$$D_{cl} = \begin{bmatrix} 0 \\ \Gamma \\ 0 \end{bmatrix}$$
$$F = \begin{bmatrix} -k_p & -k_I & -k_D \end{bmatrix}$$

The saturation nonlinearity can be modeled by the following definition (Li-Sheng et al. 2004; Zongli and Liang 2006; Shuping and Boukas 2009).

Definition 1 Let $F, H \in \Re^{m \times n}$ be given. For $x \in \Re^n$, if $\|Hy_a\|_\infty \le 1$ then $\sigma(Fy_a) \in co\left\{E_j F y_a + E_j^- H y_a : j \in [1, 2]\right\}$ where $co\{\cdot\}$ denotes the convex hull and $E_j^- = I - E_j$ where E_j is the set of $m \times m$ diagonal matrices where all their elements are 1 or 0. With Definition 1 (65) can be transformed into:

$$x_a(k+1) = \Phi x_a(k) \tag{68}$$

where:

$$\Phi = A_{cl} + B_{cl} E_j F C_{cl} + B_{cl} E_j^- H C_{cl} + D_{cl} F C_{cl} \tag{69}$$

Considering definition 1 and the following Lyapunov function:

$$V(k) = x_a^T(k) P x_a(k) \tag{70}$$

where P is a positive definite function. The derivative of (70) is given by:

$$\begin{aligned} \Delta V(k) &= V(k+1) - V(k) \\ \Delta V(k) &= x_a^T(k) \Phi^T P \Phi x_a(k) - x_a^T(k) P x_a(k) \end{aligned} \tag{71}$$

where in LMI equivalent is given by

$$x_a^T(k)\Phi^T P\Phi x_a(k) - x_a^T(k)Px_a(k) < 0 \tag{72}$$

For the static output feedback problem the following LMI must be solved for P and F (Mayer et al. 2013)

$$\begin{bmatrix} P^{-1} & \Phi \\ \Phi^T & P \end{bmatrix} > 0 \tag{73}$$

with $P^{-1} > 0$ and for the H_∞ controller synthesis the following LMI must be solved considering

$$\|T_{2\omega}(z)\|_\infty < \gamma \tag{74}$$

where γ is a robustness parameter that indicates the disturbance rejection of the system and $T_{2\omega}(z)$ is the discrete time transfer function of the closed loop system (Kwan Ho et al. 2006). Then by solving for F and P in the following LMI the H_∞ static output feedback PID controller can be obtained.

$$\begin{bmatrix} P & 0 & \Phi^T P & C_{cl}^T \\ 0 & \gamma I & 0 & 0 \\ P\Phi & 0 & P & 0 \\ C_{cl} & 0 & 0 & \gamma I \end{bmatrix} > 0 \tag{75}$$

With these explanations the PID controller gains can be obtained by any of the SOF controllers. In the next subsection an illustrative example is shown to evince the numerical simulation of an anti windup controller for a DC motor in discrete time.

6.2 Example 4

In this subsection a DC motor is stabilized with a PID anti windup controller in MIMO form. The same DC motor model of example 3 is considered in this section, so the following discretized state space model of the DC motor is obtained with a sampling period $T = 0.1$ s

$$x(k+1) = \begin{bmatrix} 2.774 & 0.1749 \\ -1.993 & 0.9169 \end{bmatrix} x(k) + \begin{bmatrix} 0.1749 & 0.1609 \\ -0.08306 & 2.153 \end{bmatrix} \sigma(u(k)) \tag{76}$$

$$y(k) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x(k) \tag{77}$$

where $x(k) = [x_1(k), x_2(k)]^T$ and $x_1(k)$ is the angular velocity and $x_2(k)$ is the armature current $u(k)$ is the input voltage.

For the discrete time SOF the following values of F and the gain matrices k_p, k_I and k_D are obtained by solving the LMI (73) with a Γ value of

$$\Gamma = \begin{bmatrix} 0.002 & 0 \\ 0 & 0.002 \end{bmatrix} \tag{78}$$

$$F = \begin{bmatrix} -3.1623 & 0 & -3.1623 & 0 & -3.1623 & 0 \\ 0 & -3.1623 & 0 & -3.1623 & 0 & -3.1623 \end{bmatrix} \times 10^8 \tag{79}$$

and the following PID controller gain matrices are:

$$\begin{aligned} k_p &= \begin{bmatrix} -3.1623 & 0 \\ 0 & -3.1623 \end{bmatrix} \times 10^8 \\ k_D &= \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \times 10^8 \\ k_I &= \begin{bmatrix} -50 & 0 \\ 0 & -50 \end{bmatrix} \times 10^8 \end{aligned} \tag{80}$$

The same PID controller gains are implemented for the system when there is no anti windup compensation. In the case of the PID anti windup controller by H_∞ synthesis the following matrix F is obtained by solving the LMI shown in (75)

$$F = \begin{bmatrix} -3.1604 & 0 & -3.1604 & 0 & -3.1604 & 0 \\ 0 & -3.1604 & 0 & -3.1604 & 0 & -3.1604 \end{bmatrix} \times 10^8 \tag{81}$$

$$\begin{aligned} k_p &= \begin{bmatrix} -3.1604 & 0 \\ 0 & -3.1604 \end{bmatrix} \times 10^8 \\ k_D &= \begin{bmatrix} -1.5802 & 0 \\ 0 & -1.5802 \end{bmatrix} \times 10^8 \\ k_I &= \begin{bmatrix} -1.5802 & 0 \\ 0 & -1.5802 \end{bmatrix} \times 10^8 \end{aligned} \tag{82}$$

With these results, a numerical simulation of the DC motor with anti windup and no anti windup compensation was done with the PID anti windup controller gain matrices achieving the following outcome.

In Fig. 23 the respective angular velocities when a anti windup and no anti windup controllers are implemented in the feedback loop of the DC motor, the system is stabilized at the nominal speed 1,750 RPM (183.26 rad/s) when a

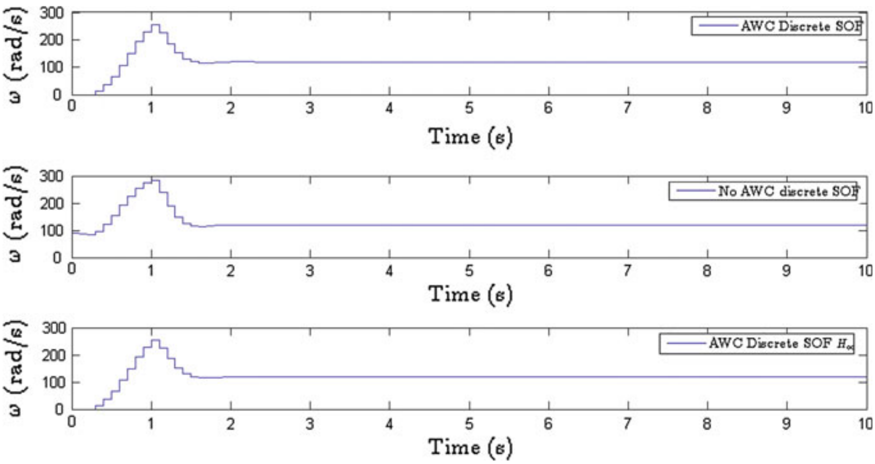


Fig. 23 Angular velocities of the DC motor

disturbance torque of 10 Nm is applied to the motor. It is verified that in the case when a SOF and H_∞ SOF the performance of the system is better than when no anti windup controller is implemented. The overshoot is smaller and a small settling time is obtained in the first mentioned cases, in comparison when only a PID controller, with no AW compensation, is implemented. This fact occurs due to the better controller and compensation characteristics when a SOF and H_∞ SOF PID anti windup compensators are implemented.

In Fig. 24 the input voltages of the DC motors in the three cases are depicted, where the voltages for the SOF and H_∞ SOF yields a more regular results than when no anti windup compensation is implemented. It can be noticed also that the input voltages are greater in the first cases than when no AWC is implemented, but

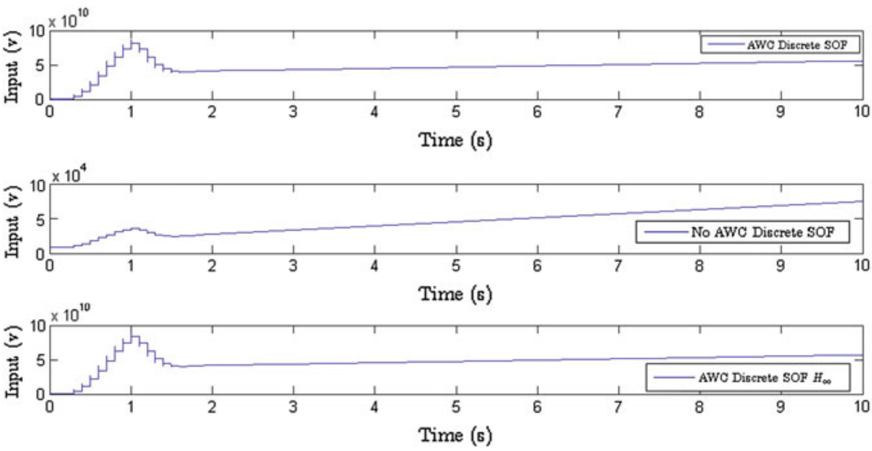


Fig. 24 Input voltage of the DC motor

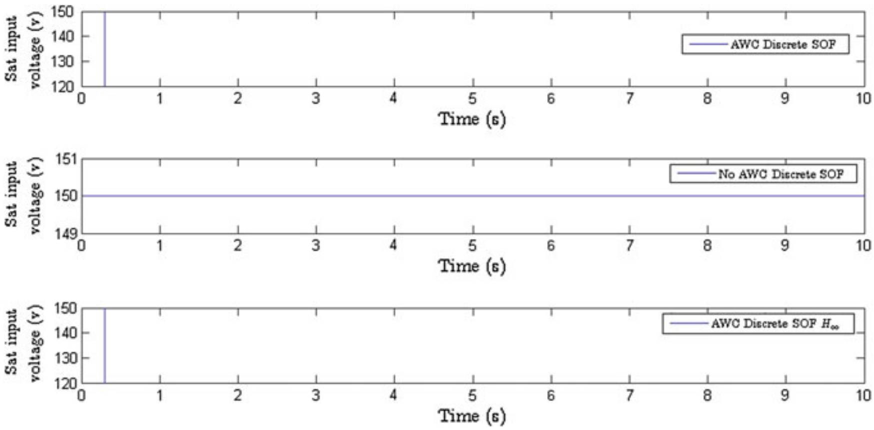


Fig. 25 DC motor saturated input voltages (v)

these values reach a steady state value in comparison with the increasing values when no AWC is implemented.

In Fig. 25 the saturated input voltages are obtained where as explained before, these values affects the system performance when the actuator, or in this case the DC motor input, is saturated due to the physical limits and properties of this model.

In Fig. 26 the armature currents of the DC motor are shown for the three cases in which the expected values are reached when a disturbance input torque is applied to the model. In Fig. 27 the mechanical torque of the model is depicted in the three cases, so as it is noticed the final value of 10 Nm, which is the value of the disturbance torque applied at 0 s is reached in the three cases but with a higher undershoot when there is no anti windup compensation.

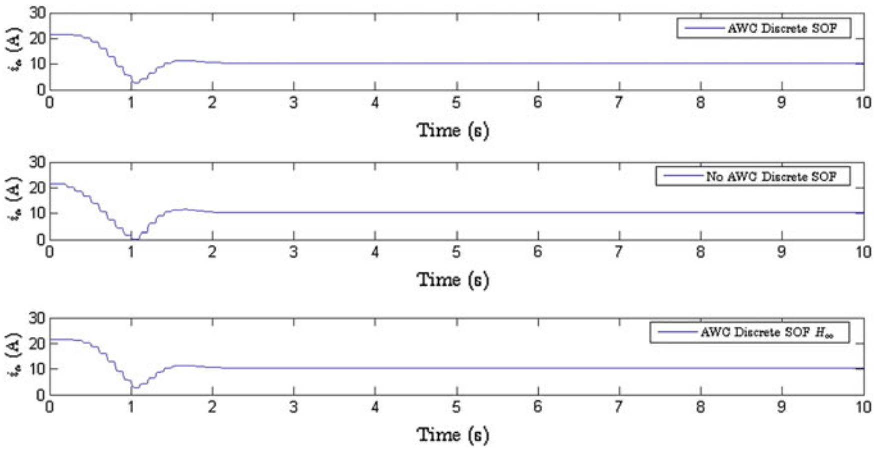


Fig. 26 DC motor armature current i_a

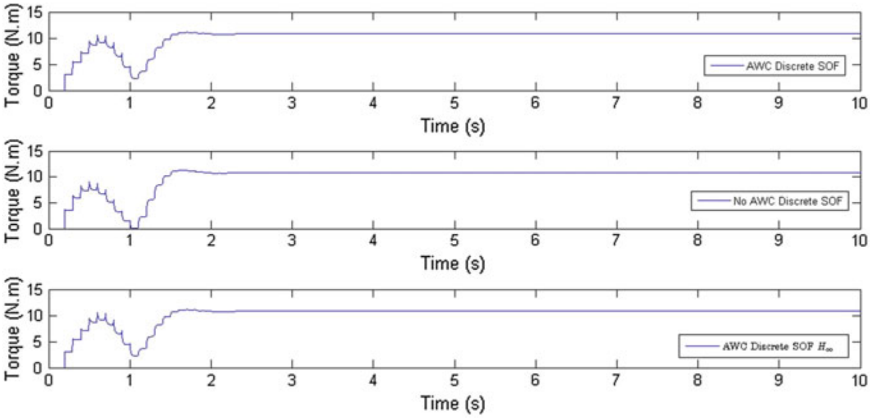


Fig. 27 DC motor torque

With the results obtained in this section, it is proved that a PID anti windup controller by SOF and H_∞ SOF can be derived for the control of MIMO systems when the inputs are saturated or constrained. In the following section a discussion of all the proposed anti windup controllers derived in this chapter are shown in order to analyze the advantages and disadvantages of these control techniques.

7 Discussion

In this chapter four anti windup controllers for SISO and MIMO continuous and discrete time systems are shown. For the first two cases, the anti windup controllers derived in the respective sections, it is shown that when the plant system is represented by a first order plus time delay model, the PID anti windup internal model controller techniques can be achieved meeting the internal stability and robustness requirements. A back calculation filter was implemented in order to suppress the integrator action when the input of the system is saturated, this compensation strategy reduces the increasing of the integrator output when the system saturates while ensuring the system stability.

It is proved that the advantages of the anti windup SISO continuous time systems is that they reduce the unwanted effects produced by windup, obtaining better results such as small overshoot and small settling time. It is proved that when a PID controller is implemented with no anti windup back calculation the performance of the system deteriorates due to the windup phenomena.

In the case of the discrete time SISO systems, a similar approach such as the SISO continuous time system where a back calculation filter is implemented to improve and avoid the deterioration of the system performance. The advantages of this control strategies is that the overshoot, settling time and other characteristics of the system performances are improved due to the filter included in the additional loop reduces the integral action when the input of the system saturates. Similar to the continuous time counterpart, an internal model controller (IMC) is implemented with an additional loop which includes a filter as a compensator while the internal stability and robustness of the system are ensured to guarantee an appropriate system performance.

In the case of the design of an anti windup controller for continuous and discrete time MIMO systems, an anti windup compensation approach is implemented where the saturated and non saturated signals are added by a feedback loop to the PID controller to eliminate the windup phenomena when the input of the system is saturated, as occurs in SISO systems. For continuous and discrete time systems, an anti windup PID controller by static output feedback was implemented, converting the PID controller into an static output feedback law and adding the difference of the saturated and non saturated input to compensate the windup effects. The advantages of the PID anti windup controller for continuous time systems, is that the system performance is not deteriorated by the influence of windup when the input is constrained or saturated, improving the controller action and preventing a high overshoot, settling time and other performance properties. In the case of the anti windup control of discrete time systems, the same advantages and properties are proved theoretically similar as the continuous time case; the system performance is improved by the addition of the difference of the saturated and no saturated signal that improves the system performance avoiding the deterioration of the system output by decreasing the integral action when the input of the system are constrained or saturated.

8 Conclusions

In this chapter some anti windup control strategies for SISO and MIMO system for discrete and continuous time models are shown. In the SISO cases an internal model anti windup PID controller is implemented by a back calculation algorithm in order to suppress the unwanted effects yielded by windup, when the system input is saturated and the integral action of the PID controller is increased. It was proved that the system performance is improved by the implementation of the back calculation loop that includes a compensator filter. The performance of the system with anti windup and no anti windup compensation was tested, and it was proved that in the first case the system performance is not deteriorated producing a smaller overshoot and settling time reducing the integral action in the discrete and continuous time SISO systems.

In the MIMO cases, a continuous and discrete time anti windup PID controllers are implemented in order to eliminates the performance deterioration caused by the integral action of the controller when the input of the system is saturated. It is proved that in the discrete and continuous time cases, the PID controllers can be achieved by a static output feedback control law (SOF) and by a H_∞ controller synthesis ensuring the system stability and robustness properties of the closed loop system. The PID controller gain matrices, in both cases, are found by solving the respective linear matrix inequalities LMI's in order to obtain the required gains to stabilize the system. In both cases it is proved that the system performance is not deteriorated by the windup phenomena when the input of the system is constrained or saturated, then in comparison when only a PID controller is implemented, the settling time and overshoot are smaller due to the anti windup characteristics of the PID controller.

Appendix 1

In this appendix the internal model PID controller, explained in Sect. 3 the gain and time constants are found with the following equations.

Define:

$$D(s) = ((\lambda s + 1)^r - P_{1A}(s))/s \tag{83}$$

and

$$K_p = p_{1m}(0) \tag{84}$$

Then the following gain and time constants are obtained using (15) with the following equations of the function $f(s)$ and its derivatives (Lee et al. 1998):

$$f(0) = \frac{1}{K_p D(0)} \tag{85}$$

where $D(0)$ is

$$D(0) = r\lambda - \dot{P}_{1A}(0) \tag{86}$$

the derivatives of $D(0)$, $\dot{D}(0)$ and $\ddot{D}(0)$ are shown in (Lee et al. 1998). Then the derivative of $f(0)$ is given by:

$$\begin{aligned} \dot{f}(0) = &\left(\frac{K(\alpha + \beta\Delta_\phi)a_1}{a_0 - \alpha - \beta\Delta_\phi} - \frac{K(\alpha + \beta\Delta_\phi)a_0 a_1}{(a_0 - \alpha - \beta\Delta_\phi)^2} - \frac{K(\alpha + \beta\Delta_\phi)a_0\tau}{a_0 - \alpha - \beta\Delta_\phi}\right)(r\lambda + \theta)^{-1}K_p^{-2} \\ &+ \frac{1/2r(r-1)\lambda^2 - 1/2\theta^2}{K_p(r\lambda + \theta)^2} \end{aligned} \tag{87}$$

$$\ddot{f}(0) = \dot{f}(0)\left(\left(\frac{\ddot{p}_{1m}(0)D(0) + 2\dot{p}_{1m}(0)\dot{D}(0) + K_p\ddot{D}(0)}{\dot{p}_{1m}(0)D(0) + K_p\dot{D}(0)}\right) + 2\dot{f}(0)/f(0)\right) \tag{88}$$

Appendix 2

In this appendix the internal model PID controller, explained in Sect. 4 the gain and time constant are found and shown in the following equations. Consider the following representation in Taylor series of the digital PID controller (31) based on the analog controller design shown in (Lee et al. 1998)

$$G_c(s) = \frac{f(z)}{z-1} = \frac{1}{z-1}\left(f(1) + f'(1)(z-1) + \frac{f''(1)}{2}(z-1)^2 + \cdots\right) \tag{89}$$

Due to $G_c(s) = \frac{f(z)}{z-1}$ the following equation can be considered:

$$D(z) = \frac{(z-\alpha) - P^*_{\gamma A}(1-\alpha)z}{z-1} \tag{90}$$

because of (30) can be represented by:

$$G_c(z) = \frac{(1-\alpha)zP^{*-1}_{\gamma M}}{(z-\alpha) - P^*_{\gamma A}(1-\alpha)z} \tag{91}$$

The design procedure of the discrete time SISO controller is similar to the continuous time SISO case, (Lee et al. 1998) where (90) can be represented by:

$$D(z) = \frac{N(z)}{z-1} \tag{92}$$

where

$$N(z) = (z-\alpha) - P^*_{\gamma A}(1-\alpha)z \tag{93}$$

Then by the Taylor series expansion of $D(z)$ the following equation is obtained:

$$D(z) = \frac{1}{z-1}\left(N(1) + N'(1)(z-1) + \frac{N''(1)}{2}(z-1)^2 + \frac{N'''(1)}{6}(z-1)^3 + \cdots\right) \tag{94}$$

Considering that $N(1) = 0$, (94) becomes in:

$$D(z) = N'(1) + \frac{N''(1)}{2}(z-1) + \frac{N'''(1)}{6}(z-1)^2 + \cdots \tag{95}$$

Expanding $D(z)$ in Taylor series expansion as an only term, the following result is obtained:

$$D(z) = D(1) + D'(1)(z-1) + \frac{D''(1)}{2}(z-1)^2 + \cdots \tag{96}$$

Then associating the similar terms of (95) and (96) the following values for $D(1)$ and its derivatives are obtained:

$$\begin{aligned} D(1) &= N'(1) \\ D'(1) &= N''(1)/2 \\ D''(1) &= N'''(1)/3 \end{aligned} \tag{97}$$

the values of $D(1)$ and its derivatives can be found by:

$$D(1) = 1 + (N-1)(1-\alpha) \tag{98}$$

$$D'(1) = (-N(N-1)(1-\alpha))/2 \tag{99}$$

$$D''(1) = ((N+1)N(N-1)(1-\alpha))/3 \tag{100}$$

Then the values for $f(1)$ and its derivatives are found by (Lee et al. 1998):

$$f(1) = \frac{1}{(p^*_{\gamma M}(1)/(1-\alpha))D(1)} \tag{101}$$

$$f'(1) = -\frac{(p'^*_{\gamma M}(1)/(1-\alpha))D(1) + (p^*_{\gamma M}(1)/(1-\alpha))D'(1)}{((p^*_{\gamma M}(1)/(1-\alpha))D(1))^2} \tag{102}$$

$$f''(1) = f'(1)\left(\frac{(p''^*_{\gamma M}(1)/(1-\alpha))D(1) + 2(p'^*_{\gamma M}(1)/(1-\alpha))D'(1) + (p^*_{\gamma M}(1)/(1-\alpha))D''(1)}{(p'^*_{\gamma M}(1)/(1-\alpha))D(1) + (p^*_{\gamma M}(1)/(1-\alpha))D'(1)}\right) + 2f'^2(1)/f(1) \tag{103}$$

With $f(1)$ and its respective derivatives, the parameters of the digital PID controllers can be found using (32).

References

Baheti, R. S. (1989). Simple anti-windup controllers. In *American Control Conference*, (pp. 1684–1686) June 21–23, 1989.

Bateman, A., & Zongli, L. (2002). An analysis and design method for discrete-time linear systems under nested saturation. *IEEE Transactions on Automatic Control, 47*(8), 1305–1310.

Bohn, C., & Atherton, D. P. (1995). An analysis package comparing PID anti-windup strategies. *Control Systems Magazine, IEEE, 15*(2), 34–40.

Cao, Y.-Y., Lin, Z., & Ward, D. (2002). An antiwindup approach to enlarging domain of attraction for linear systems subject to actuator saturation. *IEEE Transactions on Automatic Control, 47* (1), 140–145.

Chen, Y. S., Tsai, J. H., Shieh, L., & Moussighi, M. (2003). Digital redesign of anti-wind-up controller for cascaded analog system. *ISA Transactions, 42*(1), 73–88.

Cockbum, J. C., & Bailey, F. N. (1991). Loop gain-phase shaping design of SISO robust controllers having mixed uncertainty. In *American Control Conference*, (pp. 1981–1986).

Doyle III, F. J. (1999). An anti-windup input/output linearization scheme for SISO systems. *Journal of Process Control, 9*(3), 213–220.

Fujimori, A. (2004). Optimization of static output feedback using substitutive LMI formulation. *IEEE Transactions on Automatic Control, 49*(6), 995–999.

He, Y., & Wang, Q.-G. (2006). An improved ILMI method for static output feedback control with application to multivariable PID control. *IEEE Transactions on Automatic Control, 51*(10), 1678–1683.

Henrion, D., Tarbouriech, S., & Garcia, G. (1999). Output feedback robust stabilization of uncertain linear systems with saturating controls: An LMI approach. *IEEE Transactions on Automatic Control, 44*(11), 2230–2237.

Kwan Ho, L., Joon-Hwa, L., & Wook-Hyun, K. (2006). Sufficient LMI conditions for H infinity output feedback stabilization of linear discrete-time systems. *IEEE Transactions on Automatic Control, 51*(4), 675–680.

Lambeck, S., & Sawodny, O. (2004). Design of anti-windup-extensions for digital control loops. *Proceedings of the* 2004 *American Control Conference*, (pp. 5309–5314). 2004.

Lee, Y., Park, S., Lee, M., & Coleman, B. (1998). PID controller tuning for desired closed-loop responses for SI/SO systems. *AIChE Journal, 44*(1), 106–115.

Lim, J. S., & Lee, Y. I. (2008). Design of discrete-time multivariable PID controllers via LMI approach. *International Conference on Control, Automation and Systems (ICCAS 2008)*, (pp. 1867–1871).

Li-Sheng, H., Biao, H., & Yong-Yan, C. (2004). Robust digital model predictive control for linear uncertain systems with saturations. *IEEE Transactions on Automatic Control, 49*(5), 792–796.

Matsuda, Y., & Ohse, N. (2006). An approach to synthesis of low order dynamic anti-windup compensators for multivariable PID control systems with input saturation. *International Joint Conference SICE-ICASE* 2006, (pp. 988–993). doi:10.1109/SICE.2006.315736.

Mayer, S., Dehnert, R., & Tibken, B. (2013). Controller synthesis of multi dimensional, discrete LTI systems based on numerical solutions of linear matrix inequalities. American *Control Conference (ACC*) (pp. 2386–2391). June 17–19, 2013.

Morales, R. M., Heath, W. P., & Li, G. (2009). Robust anti-windup against LTI uncertainty using frequency dependent IQCs. ICCAS-SICE (pp. 3329–3334), August 18–21, 2009.

Morari, M., & Zafiriou, E. (1989). *Robust Process Control*. New Jersey: Prentice Hall.

Neto, A. T., & Kucera, V. (1991). Stabilization via static output feedback. In: *Proceedings of the* 30*th IEEE Conference on Decision and Control* 1991 (pp. 910–913). doi:10.1109/CDC.1991.261451.

Rehan, M., Khan, A. Q., Abid, M., & Iqbal, N. (2013). Anti-windup-based dynamic controller synthesis for nonlinear systems under input saturation. *Applied Mathematics and Computation, 220*(1), 382–393.

Saeki, M., & Wada, N. (1996). Design of anti-windup controller based on matrix inequalities. In: *Proceedings of the 35th IEEE Conference on Decision and Control* 1996 (pp. 261–262). doi:10.1109/CDC.1996.574310.

Shamsuzzoha, M., & Lee, M. (2007). IMC-PID controller design for improved disturbance rejection of time-delayed processes. *Industrial and Engineering Chemistry Research, 46*(7), 712–749.

Shuping, M., & Boukas, E. K. (2009). Stability and H infinity control for discrete-time singular systems subject to actuator saturation. In: *American Control Conference ACC* '09, (pp. 1244–1249). doi:10.1109/ACC.2009.5159906.

Taylor, J. H., & O'Donnell, J. R. (1990). Synthesis of nonlinear controllers with rate feedback via sinusoidal-input describing function methods. In: *American Control Conference* (pp. 2217–2222).

Tu, Y.-W., & Ho, M.-T. (2011). Synthesis of low-order anti-windup compensators for PID control. In: 2011 8th *Asian Control Conference* (*ASCC*), (pp. 1437–1442).

Wittenmark, B. (1989). Integrators, nonlinearities, and anti-reset windup for different control structures. In: *American Control Conference*, (pp. 1679–1683).

Wu, F., Lin, Z., & Zheng, Q. (2005). Output feedback stabilization of linear systems with actuator saturation. *Proceedings of the 2005 American Control Conference*, (pp. 3385–3390). doi:10.1109/ACC.2005.1470494.

Zhang, J., Wu, J., Zhang, Y., & Hopwood, F. (2011). Design and applications of an optimal anti-windup digital controller using scalar sign function approach. In *IEEE International Conference on Control Applications* (*CCA*) (pp. 94–101). doi:10.1109/CCA.2011.6044418.

Zongli, L., & Liang, L. (2006). Set invariance conditions for singular linear systems subject to actuator saturation. In *Chinese Control Conference* (*CCC*) (pp. 2070–2074). doi:10.1109/CHICC.2006.280919.